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The Analysis of Nash Equilibrium on second-hand housing transactions Based on the Final-offer Arbitration Model

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Abstract. Taking the second-hand housing transactions as example, this paper carries out modeling description on final-offer arbitration model. And then the Nash equilibrium solution of a bargaining problem between sellers and buyers in the final price of the houses for sale in the second-hand house market is analyzed in the final-offer arbitration model. Finally, The analysis of Nash Equilibrium on Final-offer Arbitration Model is given under the Weibull distribution, the Rayleigh distribution and Pareto distribution. The high-yielding is obtained by the parties to the dispute rationally.

1. Introduction

With the rapid development of urbanization in China, urban construction land is increasingly scarce, new commercial housing cycle is long and supply is insufficient, which makes more and more second-hand housing sales increase year by year. The price of second-hand housing has become the focus of attention of all parties. But the final transaction price is determined not only by the quality of the house for sale, but also by the bargaining between buyers and sellers [1] in the transaction process. In the transaction of second-hand housing, when the buyer and seller can not reach an agreement on the price, besides resorting to law, the arbitration system can play a very important role. One is traditional arbitration [2], which is a common way of arbitration, that is, after bidding by both parties, the arbitrator determines the final solution according to his own preferences. This kind of arbitration generally considers compromise to form a final result. Under the traditional arbitration system, participants can anticipate the arbitrator's preference, so that participants can motivate more extreme prices. Therefore, in this context, another system called final offer arbitration [3] has been proposed by economists, which requires arbitrators to choose only one of the participants' bid as the final result, thus reducing the chilling effect of agreement arbitration. Most of the articles[4-10] are based on the probability distribution of arbitrators' preference schemes obeying normal distribution. The final bid arbitration mechanism model is applied to analyze and study the problems of employee welfare, length of service buyout, power market and so on. However, in practical problems, arbitrators' preference schemes may also obey other probability distributions such as Weibull distribution, Rayleigh distribution and Pareto distribution. In the case of two risk-neutral participants, the Nash equilibrium of the final offer arbitration mechanism model under Weibull distribution, Rayleigh distribution and Pareto distribution is discussed, taking the bargaining problem between buyers and sellers of second-hand housing transactions as an example.

2. Nash equilibrium solution of the game under Multi-probability distribution

2.1 Nash equilibrium solution of final request arbitration model

Taking the bargaining between buyers and sellers in the second-hand housing transaction as an example, this paper discusses the arbitration mechanism model of final offer. The two parties

involved in bargaining in the transaction are buyers and sellers. The dispute is caused by the price of second-hand house. Then the game is carried out in two steps: first, the two sides negotiate the price in the transaction under the supervision of the real estate intermediary (arbitrator). That is, the buyer and the seller simultaneously issue their respective accepted real estate amounts, which X_1 and X_2 are expressed in terms of their respective amounts (if $X_1 \geq X_2$, there is no need for arbitration). Secondly, the arbitrator chooses one of the two as the final solution.

Assuming that the arbitrator himself has his own reasonable plan for the price of the house with ϕ expressing this ideal value and further assumes that after observing the bids X_1 and X_2 of both parties, the arbitrator simply chooses X that the closest bid. Let $X_1 < X_2$,if $X < \frac{X_1 + X_2}{2}$, then the arbitrator will choose X_1. If $X > \frac{X_1 + X_2}{2}$,then the arbitrator will choose X_1. If $X = \frac{X_1 + X_2}{2}$,then the arbitrator tossed a coin to decide. The arbitrator knows the ideal value ϕ, but neither of the participants knows it. The participants consider X as a random variable whose distribution function is $F(x) = P\{X \leq x\}$.

If the event A indicates that the amount of property given by the buyer is selected by the arbitrator, the event B indicates that the amount of property required by the seller is selected by the arbitrator. Then

$$\begin{aligned} P(A) &= P(X < \frac{X_1 + X_2}{2}) + \frac{1}{2} P(X = \frac{X_1 + X_2}{2}) \\ &= P(X \leq \frac{X_1 + X_2}{2}) - P(X = \frac{X_1 + X_2}{2}) + \frac{1}{2} P(X = \frac{X_1 + X_2}{2}) \\ &= P(X \leq \frac{X_1 + X_2}{2}) - \frac{1}{2} P(X = \frac{X_1 + X_2}{2}) \\ &= F(\frac{X_1 + X_2}{2}) - \frac{1}{2} F(\frac{X_1 + X_2}{2}) + \frac{1}{2} F(\frac{X_1 + X_2}{2} - 0) \\ &= \frac{1}{2}[F(\frac{X_1 + X_2}{2}) + F(\frac{X_1 + X_2}{2} - 0)] , \end{aligned} \tag{1}$$

$$P(B) = 1 - \frac{1}{2}[F(\frac{X_1 + X_2}{2}) + F(\frac{X_1 + X_2}{2} - 0)]. \tag{2}$$

And then the expected amount of compensation is

$$\begin{aligned} W(X_1, X_2) &= X_1 \cdot P(A) + X_2 \cdot P(B) \\ &= X_2 + \frac{1}{2}(X_1 - X_2)[F(\frac{X_1 + X_2}{2}) + F(\frac{X_1 + X_2}{2} - 0)]. \end{aligned} \tag{3}$$

If the bids (X_1^*, X_2^*) of both parties are Nash equilibrium of the game between the buyer and the seller,then X_1^* must satisfy: $\min_{X_1} W(X_1, X_2^*)$ and X_2^* must satisfy: $\max_{X_2} W(X_1^*, X_2)$.

Let the arbitrator's preference scheme X be a continuous random variable and the corresponding probability density function be $f(x)$ .Then $P(A) = F(\frac{X_1 + X_2}{2})$, $P(B) = 1 - F(\frac{X_1 + X_2}{2})$,and then

$$W(X_1, X_2) = X_1 \cdot P(A) + X_2 \cdot P(B) = X_1 \cdot F(\frac{X_1 + X_2}{2}) + X_2 \cdot [1 - F(\frac{X_1 + X_2}{2})]. \tag{4}$$

The first-order condition of the above optimization problem must be satisfied by the price combination of the two parties (X_1^*, X_2^*) for the amount of real estate. That is

$$\begin{cases} \dfrac{\partial W(X_1, X_2^*)}{\partial X_1} = F(\dfrac{X_1^* + X_2^*}{2}) - \dfrac{1}{2}(X_1^* - X_2^*) f(\dfrac{X_1^* + X_2^*}{2}) \\ \dfrac{\partial W(X_1^*, X_2)}{\partial X_2} = 1 - F(\dfrac{X_1^* + X_2^*}{2}) - \dfrac{1}{2}(X_2^* - X_1^*) f(\dfrac{X_1^* + X_2^*}{2}) \end{cases},$$

and then $F(\dfrac{X_1^* + X_2^*}{2}) = \dfrac{1}{2}$, $X_2^* - X_1^* = \dfrac{1}{f(\dfrac{X_1^* + X_2^*}{2})}$. (5)

2.2 Arbitrator's preferences subject to Weibull distribution

Let the arbitrator's preference scheme obey the Weibull distribution with parameters of λ and k. Its probability density is $f(x) = \begin{cases} \dfrac{k}{\lambda}(\dfrac{x}{\lambda})^{k-1} e^{-(\frac{x}{\lambda})^k}, x \geq 0 \\ 0, x < 0 \end{cases}$,the distribution function is $F(x) = \begin{cases} 1 - e^{-(\frac{x}{\lambda})^k}, x \geq 0 \\ 0, x < 0 \end{cases}$. According to (5),we have $\dfrac{X_1^* + X_2^*}{2} = \lambda \sqrt[k]{\ln 2}$, $X_2^* - X_1^* = \dfrac{2\lambda}{k}(\ln 2)^{-\frac{k-1}{k}}$.And then the Nash equilibrium bid of the game under Weibull distribution is $X_1^* = \lambda \sqrt[k]{\ln 2}[1 - \dfrac{1}{k}(\ln 2)^{1-k}]$, $X_2^* = \lambda \sqrt[k]{\ln 2}[1 + \dfrac{1}{k}(\ln 2)^{1-k}]$.

2.3 Arbitrator's preference subjects to Rayleigh distribution

Let arbitrator's preference scheme obey Rayleigh distribution with parameter σ, and its probability density is $f(x) = \dfrac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, x > 0$, the distribution function is $F(x) = \begin{cases} 1 - e^{-\frac{x^2}{2\sigma^2}}, x > 0 \\ 0, x \leq 0 \end{cases}$. According to (5), we have $\dfrac{X_1^* + X_2^*}{2} = \sigma\sqrt{2\ln 2}$, $X_2^* - X_1^* = \sqrt{\dfrac{2}{\ln 2}}\sigma$. And then the Nash equilibrium bid of the game under Rayleigh distribution is $X_1^* = \sigma\sqrt{2\ln 2}(1 - \dfrac{1}{2\ln 2})$, $X_2^* = \sigma\sqrt{2\ln 2}(1 + \dfrac{1}{2\ln 2})$.

2.4 Arbitrator's Preference Scheme Subjects to Pareto Distribution

Let the arbitrator's preference scheme obey Pareto distribution with parameter k, and its probability density is $f(x) = \begin{cases} \dfrac{kC^k}{x^{k+1}}, x \geq C \\ 0, x < C \end{cases}$, the distribution function is $F(x) = 1 - (\dfrac{C}{x})^k$. According to (5), we have $\dfrac{X_1^* + X_2^*}{2} = C\sqrt[k]{2}$, $X_2^* - X_1^* = \dfrac{C2^{1+\frac{1}{k}}}{k}$. And then the Nash equilibrium bid of the game under Pareto Distribution is $X_1^* = C\sqrt[k]{2}(1 - \dfrac{1}{k})$, $X_2^* = C\sqrt[k]{2}(1 + \dfrac{1}{k})$.

3. Conclusions

Taking the bargaining problem between buyers and sellers in second-hand housing transactions as an example, the final offer arbitration model is established. Under the three probability distributions of

Weibull distribution, Rayleigh distribution and Pareto distribution, Nash equilibrium solution of the game is obtained. It can be seen that the final offer arbitration mechanism model breaks the deadlock of the competition between buyers and sellers, and promotes both sides of the game to make more serious bids. The seller's higher price or the buyer's lower price will get higher profits once the arbitrator chooses it, but the possibility of this bid being selected will be greatly reduced. The final offer arbitration gives participants greater uncertainty through the arbitrator's preference scheme, which embodies the principle of coexistence of high risk and high profit. Therefore, both sides of the game will strive to reach consensus, thus contributing to the Nash equilibrium.

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