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Test and Application of Persistence Point of Long Memory Time Series

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Abstract. This paper presents two new simple statistic respectively to detect the long memory time series of the changes from smooth to non-stationary and persistence in smooth change from non-stationary variation point, two statistics under the null hypothesis is given the limits of the distribution, and proves that under the alternative hypothesis testing method. The consistency of the data simulation results show that the proposed method not only can good control inspection level, and has high potential test. At last, by a group of us inflation data to illustrate the practicability of this method.

Key words: persistent variable point; a long memory; Hypothesis testing.

1. Introduction

Many hydrology, meteorology, economic and financial time series such as there is the phenomenon of long dependency, namely the current observation data with a very long period of time have correlation between observed data, while with the increase of time interval, the correlation will reduce gradually, but still cannot be ignored, so can't use some traditional time series model to model. In the modeling of time series data, should not only consider the data is stationary or non-stationary, sometimes also need to consider whether there is a persistent change points in the data. If a time series $X_t\ t=1,2...$ changes from a stationary sequence to a non-stationary sequence or from a non-stationary sequence to a stationary sequence at a certain moment, it is called a time series $X_t\ t=1,2...$ and a persistence point of change occurs at the time.

The research on persistent variation point can be traced back to the 1990s. De long and his colleagues found that the studied data changed from stationary sequence to unit root sequence after world war ii when they studied the actual export data of the United States and European Union countries. Hakkio and based on the analysis of the us fiscal deficit data, it found that the same problem, although the author points out the research the importance of persistent changing point, but as a result of stationary sequence and statistical properties of unit root sequence has a completely different, lead to the inspection of persistent changing point is relatively difficult. At present, the time sequence of persistent changing point for short memories ($I(0)-I(1)$ The persistence point of the structure), There have been a large number of studies in the literature. For example, Kim and Busetti proposed the ratio method to test the persistent variable point under the stationary (I (0)) null hypothesis, and Banerjee and Leyboune studied the test problem of persistent variable point under the non-stationary null hypothesis. These two kinds of inspection methods in a sequence is a stable and smooth sequences but there is no persistence variation point, gives the existence of persistent changing point error statistical conclusions, in order to overcome this problem, Leybourne proposed inspection square accumulation and methods of the persistent changing point. Although square accumulation and the method is proposed under the null hypothesis, the simulation results show that the method from the I (0) to change and from I (1) to (0) the test potential change persistent change point better.

These studies consider is persistent change point in the time series of short memories. On long memory time series of persistent changing point early research literature [8], Sibberttsen and Kruse adopts Leybourne square tired and methods of testing long memory time series of persistent changing point, found that there will be a serious level of distortion, therefore has carried on the correction to the critical value, when using fixed threshold, accumulated and square method can better control the test level not only, also have high potential test. Martins, Rodrjgues, Hassler and Meller studied the problem of testing the persistence variability of long memory time series and applied the proposed method to the analysis of some fiscal deficit data. Chen online long memory time series are studied, such as persistent changing point from smooth to non-stationary monitoring problem, put forward the monitoring the persistence of even change the ratio of the change point method, and put forward the critical value of the bootstrap method is used to approximate statistics, such as the problem of monitoring the ARCH model of the distribution of the changing point of change from short to long memory persistence variation point problem of inspection and online monitoring of research can be found in literature [14] and [15].

In this paper, at the same time in stationary and non-stationary long memory time series under the null hypothesis of persistent changing point inspection problems, is used to test long memory time series was developed based on the Shao mean change point inspection ratio statistics, puts forward two modified simple ratio statistics were used to test changes from smooth to non-stationary persistence variation point and from the non-stationary to the persistence of even change point. Compared with the method of literature in square accumulated and the proposed method does not need to estimate the long memory time series variance, due to the long memory time series variance in hours it is difficult to accurately estimate the sample size, so the proposed approach is more easy to practical application. This paper not only in the original assumptions on the limiting distribution of the two test statistics, proved the consistency of the testing method and through the numerical simulation shows the proposed method has good potential experience level and experience. Finally the application of this method to the United States from May 1952 to April 1977 inflation data showed the practicability of the method.

2. Model and Test Statistics

The following integral model is considered in this paper:

$$(1-L)^d X_t = \varepsilon_t \quad t = 1,2...$$

Where L is a postposition operator, ε_t is an independent identically distributed random variable with mean 0 and variance 0, X_t is related to the memory parameter d, $0 \le d < 3/2$ and it has the following form:

$$X_t = \sum_{j=0}^{\infty} w_j(d)\varepsilon_{t-j}$$

$w_j(d) = \dfrac{\Gamma(d+j)}{\Gamma(d)\Gamma(1+d)}$, if $0 \le d < 1/2$, X_t is a smooth long memory process, if $1/2 < d < 3/2$, X_t is a non-stationary long memory process. For the sake of simplicity, $X_t \sim I(d_1)$ $(0 \le d_1 < 1/2)$, $X_t \sim I(d_2)$ $(1/2 < d_2 < 3/2)$.

In this paper, two kinds of hypothesis testing problems are considered as follows: the first is the hypothesis testing from stationary to non-stationary change persistent point, that is, the original hypothesis is

$$H_0^{[1]}: X_t \sim I(d_1) \quad t = 1,2,\dots,n.$$

The alternative hypothesis is

$$H_1^{[1]}: X_t \sim \begin{cases} I(d_1), & t = 1,\dots,k^* \\ I(d_2), & t = k^*+1,\dots,n \end{cases}$$

The second is the hypothesis test of the persistence change point from non-stationary to stationary, that is, the original hypothesis is

$$H_0^{[2]}: X_t \sim I(d_2) \quad t = 1,2,\dots,n.$$

The alternative hypothesis is

$$H_1^{[2]}: X_t \sim \begin{cases} I(d_2), & t=1,\dots,k^* \\ I(d_1), & t=k^*+1,\dots,n \end{cases}$$

n Is Sample size, k^* is some unknown variable.

$$\overline{X}_{j,k} = (k-j+1)^{-1}\sum\nolimits_{t=j}^{k} X_t \ , S_t(j,k) = \sum\nolimits_{h=j}^{t}(X_h - \overline{X}_{j,k}) \quad t=j,\dots,k, 1\le k\le n$$

For the first hypothesis testing problem, the following statistics (1) are used:

$$F_n(k) = \frac{\sqrt{k}\left|\overline{X}_{1,k} - \overline{X}_{k+1,n}\right|}{k^{-1}\left\{\sum\nolimits_{t=1}^{k} S_t^2(1,k)\right\}^{1/2}} \quad 1\le k\le n \tag{1}$$

For the second hypothesis test problem, the following statistic (2) was used for the test

$$M_n(k) = \frac{\sqrt{n-k}\left|\overline{X}_{1,k} - \overline{X}_{k+1,n}\right|}{(n-k)^{-1}\left\{\sum\nolimits_{t=k+1}^{n} S_t^2(k+1,n)\right\}^{1/2}} \quad 1\le k\le n \tag{2}$$

Shao [16] makes use of the following statistics

$$G_n(k) = \frac{\sqrt{n}\left|\overline{X}_{1,k} - \overline{X}_{k+1,n}\right|}{n^{-1}\left\{\sum\nolimits_{t=1}^{k} S_t^2(1,k) + \sum\nolimits_{t=k+1}^{n} S_t^2(k+1,n)\right\}^{1/2}}, \ 1\le k\le n. \tag{3}$$

Theorem 2: when the alternative $H_1^{[1]}$ hypothesis is true, that is, when there is a change point from stationary to non-stationary in the sequence.

$$F_n(k) = O_P(n^{d_1-d_2})$$

Proof: it can be assumed that there is a change point k^* from stationary to non-stationary in the sequence.

First of all to prove $k\le k^*$, if $k\le k^*$, $X_{1,k} \sim I(d_1)$ $X_{k+1,k^*} \sim I(d_1)$ $X_{k^*+1,n} \sim I(d_2)$

$$F_n(k^*) = \frac{k^{1/2}\left|\frac{1}{k}\sum\nolimits_{t=1}^{k} X_t - \frac{1}{n-k}\sum\nolimits_{t=k+1}^{k^*} X_t - \frac{1}{n-k}\sum\nolimits_{t=k^*+1}^{n} X_t\right|}{k^{-1}\left\{\sum\nolimits_{t=1}^{k}[\sum\nolimits_{h=1}^{t}(X_h - \frac{t}{k}\sum\nolimits_{t=1}^{k} X_t)]^2\right\}^{1/2}}$$

$$n^{1+d_2}\frac{\lfloor nr\rfloor^{1/2}}{n}\left|\frac{n}{\lfloor nr\rfloor}\sum\nolimits_{t=1}^{\lfloor nr\rfloor} X_t - \frac{n}{n-\lfloor nr\rfloor}\sum\nolimits_{t=\lfloor nr\rfloor+1}^{\lfloor nr''\rfloor} X_t - \frac{1}{1-\lfloor nr\rfloor}\sum\nolimits_{t=\lfloor nr''\rfloor+1}^{n} X_t\right|$$

$$\Rightarrow r^{1/2}\left|B_{d_1}(r)/r - \{B_{d_1}(r''-r) - B_{d_2}(1-r'')\}/(1-r)\right|,$$

$$\frac{n}{\lfloor nr\rfloor}\left\{n^{-2-2d}\sum\nolimits_{t=1}^{\lfloor nr\rfloor}[\sum\nolimits_{h=1}^{\lfloor nr'\rfloor}(X_h - \frac{\lfloor nr'\rfloor}{\lfloor nr\rfloor}\sum\nolimits_{t=1}^{\lfloor nr\rfloor} X_t)]^2\right\} \Rightarrow r^{-1}\int_0^r w_d(r';0;r)^2 dr',$$

$$(n^{d_2-d_1})F_n(k) \to_p \infty, F_n(k) = O_P(n^{d_1-d_2}).$$

$$k^* \le k \ \ X_{1,k} \sim I(d_1), X_{k^*+1,k} \sim I(d_2), X_{k+1,n} \sim I(d_2),$$

$$F_n(k^*) = \frac{k^{1/2}\left|\frac{1}{k}\sum\nolimits_{t=1}^{k^*} X_t - \frac{1}{n-k}\sum\nolimits_{t=k^*+1}^{k} X_t - \frac{1}{n-k}\sum\nolimits_{t=k+1}^{n} X_t\right|}{k^{-1}\left\{\sum\nolimits_{t=1}^{k^*}[\sum\nolimits_{h=1}^{t}(X_h - \frac{1}{k}\sum\nolimits_{t=1}^{k^*} X_t)]^2 + \sum\nolimits_{t=k^*+1}^{k}[\sum\nolimits_{h=1}^{t}(X_h - \frac{1}{k-k^*}\sum\nolimits_{t=k^*+1}^{k} X_t)]^2\right\}^{1/2}}$$

Similar to the above proof, in conclusion

$$(n^{d_2-d_1})F_n(k) \to_p \infty \qquad n \to \infty \quad F_n(k) = O_P(n^{d_1-d_2})$$

Theorem 3: when the null hypothesis H_0[2] When, namely $X_t \sim I(d_2)$, $1/2 \le d_2 < 3/2$, when $n \to \infty$

$$Q_n(\tau_1,\tau_2) \to_D \sup_{r\in[\tau_1,\tau_2]} \frac{(1-r)^{3/2}\left|B_d(r)/r - \{B_d(1)-B_d(r)\}/(1-r)\right|}{\left\{\int_r^1 W_d(r';r,1)^2 dr'\right\}^{1/2}},$$

Among them $w_d(r';r,1) = B_d(r') - B_d(r) - \{B_d(1)-B_d(r)\}(r'-r)/(1-r)$, $r \in (r_1, r_2)$, $0 \le r_1 < r_2 \le 1$.

The proof of this theorem is similar to the proof of theorem 1, which is omitted here.

Theorem 4: when alternative hypothesis H_1[2], That is, the sequence changes from non-stationary to stationary ($I(d_2)$ to $I(d_1)$), available

$$M_n(k) = O_P(n^{d_2-d_1})$$

When a change occurs in a long memory sequence k^* Makes the sequence change fromnon-stationary to stationary. $k \le k^*$, $X_{1,k} \sim I(d_2)$,

$X_{k+1,k^*} \sim I(d_2)$, $X_{k^*+1,n} \sim I(d_1)$ if $k^* \le k$, $X_{1,k} \sim I(d_2)$, $X_{k*+1,k} \sim I(d_1)$, $X_{k*+1,k} \sim I(d_1)$, $X_{k+1,n} \sim I(d_1)$.

The proof process similar to theorem 2 can be obtained $M_n(k) = O_P(n^{d_2-d_1})$.

3. Numerical Simulation

In this section, the finite sample properties of the two revised test statistics are analyzed through numerical simulation, focusing on the analysis of the influence of factors such as parameters and the position of change points on the test results, all of which are realized by R language. Since the critical value of the proposed statistics is unknown, the critical value needs to be simulated first $(\tau_1,\tau_2) = (0.15, 0.85)$ ((τ_1,τ_2) Choice by Andrews in a convention) was formed after the first use, take a sample size of n = 5000, loop 10000 times, calculate the quantile statistics experience as a critical value, the part of long memory parameters are listed in table 1 d corresponding to 90% and 95% of the critical value. The actual problem long memory parameter d is usually unknown, but d consistent estimates can be used in place of, in this paper, with the expansion of Whittle method to estimate the d.

For research experience level, sample size n = 100 and 500 respectively, the long memory parameter respectively 0.0, 0.1, 0.25, 0.4, 0.6, 0.75, 0.9, 1.0, after a 5000 - cycle experience levels shown in table 2. Can be seen from table 2, under all kinds of long memory parameter selection, experience level are very close to the test level, and with the increase of sample size, experience level distortion rate decreases.

Next, we analyze the test potentials of two test statistics under the alternative hypothesis, and consider three different locations of change points, $\tau = 0.3, 0.5, 0.7$. Because of the sample sizewhen n=100. For the most part, the empirical potential is too low

Table 1. Critical value of correction test:

d	90%	95%	d	90%	95%
0.00	19.24	23.18	0.60	38.43	48.19
0.10	20.85	25.23	0.75	47.53	61.14
0.25	24.10	28.81	0.90	58.41	74.83
0.40	28.19	36.25	1.00	68.71	97.75

Table 2. Experience level of revised test (%)

d	$n=100$		$n=500$	
	10%	5%	10%	5%
0.00	8.4	4.1	10.3	5.2
0.10	8.7	4.0	8.9	4.2
0.25	8.9	4.2	9.7	5.6
0.40	11.7	3.6	10.5	4.4
0.60	9.4	3.8	10.1	5.6
0.75	9.5	5.5	10.9	4.8
0.90	11.8	6.4	11.1	6.1
1.00	12.9	5.7	10.8	4.8

When n = 500. In addition, in order to illustrate the improvement effect of the correction method, the before correction test statistics (3) have done the simulation test potential. Table 3 is by the test statistics (3) to simulate the experience, we can find that, on the persistence of inspection from smooth to non-stationary change change point, the more variable point location, the higher the test potential, and in the inspection from the durability of the non-stationary to smooth change point, before changing point position to the test potential is higher. This is mainly because when data from smooth to non-stationary variation in the durability of the change point, the more variable point location, non-stationary parts when calculating the statistics of the denominator of relatively smaller, which will not make the denominator increases rapidly, so that the entire value of the statistic is relatively large. This also happens to verify the test in this paper from smooth to non-stationary changing variable point correction statistics to delete the second half of the sample accumulating the rationality of the peace party. Otherwise exist in the data from the durability of the non-stationary to smooth change point, modified statistic deleted first half sample cumulative party so are the cause of peace. Although this uncorrected statistic is still able to test the persistent variable point, it can be seen that the test potential is generally low, that is, the test efficiency is low, so it is not a better test method.

The simulation results of the corrected test statistic's empirical potential are shown in table 4. Compared to the results of table 3, the potential here has improved significantly, and in many cases to test potential increased by 30%, and when the inspection from non-stationary to smooth change effect is more apparent, this shows that the proposed correction method is very effective. By analyzing the experience the potential relation with variable point location is found when testing the persistence of changes from smooth to non-stationary variation point, change point location in the middle when testing the highest potential, but at 0.3 and 0.7 have similar test potential, the most variable point test statistics and literature concluded. When testing the persistence of a smooth change from non-stationary variation point, although the test potential and variable has a similar relationship between point location, but less regularity. To compare the change of the parameters, found that when the parameters of the greater the difference of d1 and d2 (that is, the greater the jump), experience the potential is higher, the testing method and the literature of various kinds of the changing point conclusion is consistent. Finally, it should be noted that when the long memory sequence changes from stationary to non-stationary, the test potential is higher than that when the sequence changes from non-stationary to stationary, which indicates that it is relatively difficult to test the persistence point of a type of variable. However, in general, the empirical potential results of the revised test statistics proposed in this paper are satisfactory.

4. Case Analysis

This section illustrates the validity of the test method with a set of actual data from the United States from May 1952 to 1977.

The 300 inflation figures for April 2016 (see chart 1). As you can see in figure 1, the figures appear in both the front and the back of the vertical bar.

Table 3. Statistics $G_n(k)$ Experience potential (%)

d	τ 0.3	τ 0.5	τ 0.7	d	τ 0.3	τ 0.5	τ 0.7
0.00→0.60	38.7	51.2	69.9	1.00→0.00	33.9	24.7	21.1
0.00→0.75	41.4	62.4	71.6	1.00→0.10	32.9	22.5	18.0
0.00→0.90	49.7	64.9	78.2	1.00→0.25	30.6	18.6	15.1
0.00→1.00	51.5	70.6	80.0	1.00→0.40	29.3	17.1	14.3
0.10→0.60	31.3	49.5	61.3	0.90→0.00	31.4	22.3	20.5
0.10→0.75	33.5	56.6	66.7	0.90→0.10	29.8	22.0	18.5
0.10→0.90	38.0	59.8	75.6	0.90→0.25	28.7	21.1	19.6
0.10→1.00	42.5	61.8	77.7	0.90→0.40	26.4	20.2	18.7
0.25→0.60	23.7	42.1	48.0	0.75→0.00	40.0	28.2	22.3
0.25→0.75	28.8	46.9	58.5	0.75→0.10	39.6	25.7	20.1
0.25→0.90	32.7	48.6	61.8	0.75→0.25	33.4	21.1	18.7
0.25→1.00	35.8	55.9	68.5	0.75→0.40	28.7	20.6	16.5
0.40→0.60	17.0	24.8	25.6	0.60→0.00	41.4	27.8	21.9
0.40→0.75	20.8	32.1	41.0	0.60→0.10	37.8	25.1	18.6
0.40→0.90	23.8	39.1	53.7	0.60→0.25	26.1	20.9	15.4
0.40→1.00	25.5	46.6	60.2	0.60→0.40	22.7	18.1	14.6

Table 4. Empirical potential of modified test (%)

d	τ 0.3	τ 0.5	τ 0.7	d	τ 0.3	τ 0.5	τ 0.7
0.00→0.60	84.3	89.6	87.5	1.00→0.00	82.5	83.4	82.7
0.00→0.75	89.8	92.7	91.3	1.00→0.10	74.8	78.8	77.5
0.00→0.90	95.9	98.5	95.7	1.00→0.25	60.2	67.3	67.7
0.00→1.00	96.8	99.3	96.2	1.00→0.40	38.8	48.5	54.8
0.10→0.60	77.4	82.7	81.8	0.90→0.00	80.9	82.7	79.8
0.10→0.75	87.1	88.3	87.7	0.90→0.10	73.3	74.4	73.3
0.10→0.90	93.3	96.7	92.6	0.90→0.25	53.6	59.5	60.9
0.10→1.00	95.4	99.1	94.5	0.90→0.40	30.3	41.6	44.2
0.25→0.60	66.0	71.5	70.5	0.75→0.00	71.1	73.5	67.7
0.25→0.75	76.3	80.7	77.8	0.75→0.10	62.3	63.9	58.7
0.25→0.90	89.2	91.3	87.6	0.75→0.25	41.8	46.7	43.5
0.25→1.00	92.4	93.3	91.8	0.75→0.40	29.7	38.9	28.4
0.40→0.60	42.4	43.7	38.6	0.60→0.00	72.4	69.6	58.7
0.40→0.75	59.3	62.6	56.2	0.60→0.10	57.9	62.1	53.5
0.40→0.90	76.7	77.2	72.7	0.60→0.25	39.4	41.5	37.2
0.40→1.00	84.5	86.9	81.2	0.60→0.40	23.5	27.0	22.2

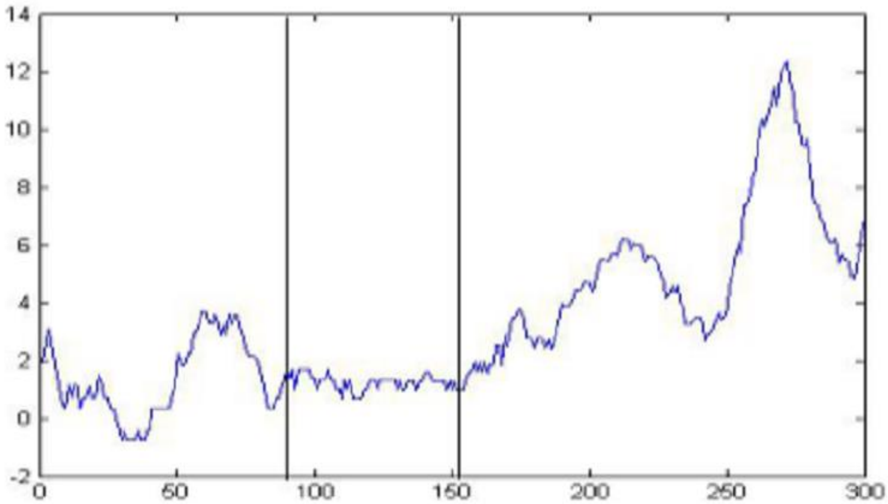

Fig. 1 Numerical simulation and actual data

Severe fluctuation, and the middle is relatively stable. Based on the short memory persistence time sequence variation point inspection methods to analyze the ratio of this set of data, found that exist in this group of data from the I (1) to (I) (0) and I (0) to (I) (1) two kinds of persistent changing point, changing point location respectively in July 1959 and May 1965. We first use of the proposed test statistics (2) before the test

Persistent change point in the 152 data, find the value of the test statistics in 78 samples (July 1958) maximum 214.31, using the method to estimate the 152 data of long memory parameter value is 1.029, the corresponding 95% critical value of 99.412, as a result of the statistic value is greater than the corresponding critical value, existing in the 152 data from the non-stationary to the persistence of even change point. Then we delete before 78 data, with statistics (1) check for the rest of the persistence of 222 data points, find the value of the test statistics in 156 samples (April 1965) reach the maximum value of 272.177, estimates from the 222 data have long memory parameter value is 1.201, 95% of its corresponding critical value of 127.22, as a result of the statistic value is greater than the corresponding critical value, existing in 222 after data from stable to non-stationary changing persistent changing point. The long memory parameter values of the first 78, the middle 78 and the last 144 data are estimated to be 1.103,0.467 and 1.37 respectively, which are consistent with our test results. The estimated results of the location of variation points obtained in this paper are somewhat different from those obtained by Chen, mainly because Chen treats this group of data as a short memory time series.

5. Summary

This article respectively in stationary and non-stationary long memory time series under the null hypothesis research persistence variation point inspection problem, put forward the two do not need to estimate the long memory time series variance and simple to calculate the new statistics, launched two statistic under the null hypothesis of limiting distribution, under the alternative hypothesis proved the consistency of statistic, and through the numerical simulation and actual data analysis shows the effectiveness and practicability of the method. As a result of long memory time series variance in sample size is small it is difficult to accurately estimate, so the proposed approach is more easy to practical application. Due to the method in detecting changes from smooth to non-stationary is superior to the effect of the persistent change point detection from non-stationary to the persistence of even change point of the effect, how to improve after a persistent change point detection effect is worth further research. In this paper, while considering the persistence single change point detection problem, easily by using the method of binary to persistent long memory time series variable point detection problem.

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