

Dynamic Pricing of Obsolete Mobile Phones for Online Recycling Platforms based on Optimal Control Theory

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Abstract. Currently, the number of obsolete mobile phones are being generated in China. However, the obsolete mobile phones are not effectively recycled through online recycling platforms. According to consumers' recycling willingness, this paper shows how the dynamic optimal control model was established. On the basis of this model, a pricing strategy was proposed, which set an optimal recycling price. The result shows that (1) the information leakage is the most crucial to pricing strategy; (2) consumers' sensitivity to service level is the next. (3) environmental awareness will greatly promote consumers to recycle obsolete mobile phones voluntarily.

Keywords: Online recycling platform, Optimal control theory, Dynamic pricing, Recycling willingness.

1. Introduction

In recent years, a various of online recycling platforms are emerging [1], which provides a new channel to recycle obsolete mobile phones effectively. However, it fails to release the storage of obsolete mobile phones well [1]. Faced with potential recycling market, setting an appropriate recycling price is particularly important for the recycling platform to expand market share. Different from positive sales enterprises, consumers are the original point in recycling transactions. And their recycling willingness has a significant impact on the profit of the online recycling platform [2]. For example, if the online recycling platform provides a more convenient recycling services such as door-to-door collection, it will attract more consumers to recycle obsolete mobile phones. Thus, it will improve the revenue. However, this will increase the cost and reduce the revenue. Therefore, according to consumers' recycling willingness, this paper establishes the optimal control model to seek the optimal dynamic pricing.

2. Literature Review

In the following, we review the literatures related to our research from two aspects. Firstly, the optimal control theory can reflect the market demand variation in continuous process, and has been widely used in dynamic pricing research. The dynamic pricing control of process-product innovation is solved by this theory [3]. The recycling growth rate of online recycling platform will dynamically change [4]. Hence, the optimal control theory is suitable for our problem. Secondly, consumers' recycling demand is not only regulated by the recycling price, but influenced by time cost, privacy security and other factors. Tan et al., found that the recycling price was far lower than consumers' expectations, which is the primary reason affecting consumers' recycling willingness [5]. Zhou et al., pointed out that consumers' extreme sensitivity to information leakage leads to their fear of mobile phone recycling [6]. Wang et al., found that environmental education has a positive impact on consumers' intention. Therefore, this paper will integrate the influencing factors of consumers' recycling willingness into the optimal control model to obtain the optimal pricing strategies.

3. Model Establishment and Solution

3.1 Problem Description

We consider the profit maximization problem of a single online recycling platform in the planning horizon. The online recycling platform purchases the obsolete mobile phones from consumers at a certain recycling price and processes them to obtain a revenue. At the same time, the recycling platform considers the impact of consumers' recycling willingness on the growth rate of recycling quantity, and then dynamically adjusts the recycling price to improve the performance.

Detailed parameters and variables used in this paper are shown in table 1, in which we make a necessary explanation.

Table 1. Model parameters and variables

| Notations | Definition | Notations | Definition |
|---------------|--|-----------|---|
| T | Planning horizon | φ | The sensitivity to service* |
| $q(t)$ | The recycling quantity at time t | s | Service quality or level |
| $p(t)$ | The recycling price at time t | m | The sensitivity to information leakage* |
| R | The unit revenue (earning from processors) | d | The probability of information leakage |
| α | Unit cost of service | θ | The sensitivity to price |
| β | Unit cost associated with price change | $r(t)$ | The rate of price change at time t |
| ε | Natural growth rate* | | |

Introduction: Stems* denote the consumers' recycling willingness.

Several significant factors are introduced to our model, including price change, service level and information leakage. Some mathematical equations are given below under the reasonable assumption.

Let $q(t)$ be the recycling quantity of online recycling platform at time t and $\dot{q}(t) = dq(t)/dt$ denotes the rate of change of recycling quantity i.e. recycling growth rate. This growth rate depends on natural growth rate, service level, information leakage and price change. As in the studies of Jiang et al., [8], We assume that the recycling growth rate is a linear function of these factors and can be expressed as $\dot{q}(t) = \varepsilon + \varphi s - md + \theta r(t)$. Obviously, the recycling quantity increases with increase in recycling price and platform's service level. This phenomenon is captured by φs and $\theta r(t)$, respectively. As we all know, the probability of information leakage has a negative impact on recycling growth rate. It is captured by $-md$. Moreover, there is also a natural growth rate in the recycling quantity due to various of factors such as platform image, environmental awareness. This growth can be brought into model through ε .

Let $p(t)$ be the recycling price paying for consumers at time t , so $r(t) = \dot{p}(t)$ denotes the rate of price change at time t . The decision-makers of online recycling platform need to decide the $p(t)$ in each planning horizon. Revenues of platform comes from transactions, in which recycling platform buy obsolete mobile phones from consumers and then process it to get a revenue R . Below are some costs related to our dynamic pricing model.

I. Total cost of the recycling platform is $p(t)q(t)$ at time t . II. Total service cost is $\alpha s q(t)$ at time t . III. Cost of the recycling price change, which is a positive value regardless of the rise or fall of the recycling price. Referring to the research by Muller et al. [9], the cost is considered to being set as a quadratic function of the price change rate, i.e. $\beta r(t)^2$. Hence, the profit of online recycling platform at time t can be written as $Profit(t) = (R - p(t))q(t) - \alpha s q(t) - \beta r(t)^2$.

3.2 Model Establishment

The online recycling platform can optimize his profit by making the decision on variables $r(t)$ over time. The optimal control formulation can be written as

$$\max \Pi = \max_{r(t)} \int_0^T (R - p(t)) q(t) - \alpha s q(t) - \beta r(t)^2 dt \quad (1)$$

$$s. t. \begin{cases} \dot{q}(t) = \varepsilon + \varphi s - md + \theta r(t), q(0) = q_0 \\ \dot{p}(t) = r(t), p(0) = p_0 \\ q(t) \geq 0, \forall t \end{cases} \quad (2)$$

$$(3)$$

$$(4)$$

Where q_0 and p_0 are the initial values of the recycling quantity and the recycling price at the beginning of the planning horizon, respectively. Here, $p(t)$ and $q(t)$ are the state variables, $r(t)$ are the control variable. Hereafter, Equations (2) and (3) are referred to the state equations.

3.3 Solution Methodology

In this subsection, we present an optimal solution to the model. A Hamiltonian function is utilized to gain a necessary condition for optimal control model:

$$H = (R - p(t))q(t) - \alpha s q(t) - \beta r(t)^2 + \lambda_q(t)(\varepsilon + \varphi s - \psi d + \theta r(t)) + \lambda_p(t)r(t) \quad (5)$$

where $\lambda_q(t)$ and $\lambda_p(t)$ are the adjoint variables corresponding to the state variable $q(t)$ and $p(t)$ at time t respectively. These adjoint variables can be interpreted as the shadow price related to a unit change in state variables at time t . For example, $\lambda_p(t)$ is the marginal value of a change in $p(t)$ at time t , So is the $\lambda_q(t)$.

According to maximum principle, the necessary conditions for optimal solution can be obtained

$$\frac{\partial H}{\partial r(t)} = -2\beta r(t) + \theta \lambda_q(t) + \lambda_p(t) = 0 \quad (6)$$

And the adjoint formulation with the terminal conditions is:

$$\dot{\lambda}_q(t) = -\frac{\partial H}{\partial q(t)} = -(R - p(t)), \lambda_q(T) = 0 \quad (7)$$

$$\dot{\lambda}_p(t) = -\frac{\partial H}{\partial p(t)} = -(-q(t)), \lambda_p(T) = 0 \quad (8)$$

$$\text{From the equation (6), We can derive } r^*(t) = \frac{\theta}{2\beta} \lambda_q + \frac{1}{2\beta} \lambda_p \quad (9)$$

Now we check the two-order sufficient conditions for optimal control problem

$$\frac{\partial^2 H}{\partial r(t)^2} = -2\beta \leq 0 \quad (10)$$

By the equation (10), we know the solution for $r^*(t)$ provides us a maximum. Here, we directly present the optimal recycling price and optimal recycling quantity:

$$p^*(t) = \frac{((p_0 + \alpha s - R)\theta + q_0) \cosh\left(\sqrt{\frac{\theta}{\beta}}(t - T)\right)}{2\theta \cosh\left(\sqrt{\frac{\theta}{\beta}}T\right)} + \frac{(\varepsilon + \varphi s - md) \sinh\left(\sqrt{\frac{\theta}{\beta}}t\right)}{2\theta \sqrt{\frac{\theta}{\beta}} \cosh\left(\sqrt{\frac{\theta}{\beta}}T\right)} - \frac{(\varepsilon + \varphi s - md)t + q_0}{2\theta} + \frac{p_0 + R - \alpha s}{2} \quad (11)$$

$$q^*(t) = (\varepsilon + \varphi s - md)t + \theta(p^*(t) - p_0) + q_0 \quad (12)$$

Where $\sinh x = \frac{e^x - e^{-x}}{2}$, $\cosh x = \frac{e^x + e^{-x}}{2}$. Both are hyperbolic functions. The optimal price expressions derived in Equation (11) is the dynamic optimal price for decision-maker.

4. Sensitivity Analysis

In this section, sensitivity analysis for all parameters related to consumers' willingness is carried out to get some important pricing strategies based on the optimal price and the optimal recycling quantity trajectory $p^*(t)$ and $q^*(t)$ through numerical simulation. The basic parameters' values are set as follows: $T = 7$, $p_0 = 500$, $p_0 = 10000$, $R = 1200$, $\alpha = 120$, $\beta = 50$, $\varepsilon = 30$, $\varphi = 100$, $\theta = 35$, $s = 0.5$, $m = 1000$, $d = 0.05$. These parameters values and initial conditions are reasonable normalized values.

4.1 Impact of the Natural Growth Rate

The impact of the natural growth rate ε on the optimal recycling price and the optimal recycling quantity is discussed in figure 1, respectively. Only parameters ε is set different values ($\varepsilon = 1, 30, 100$).

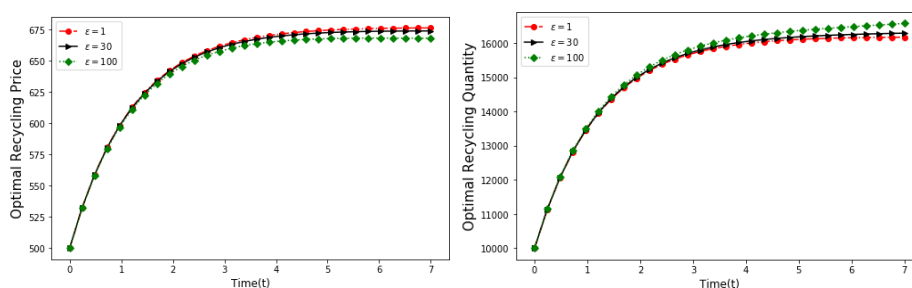


Figure 1. Impact of parameter ε on optimal recycling price and quantity

Figure 1 demonstrates that the optimal recycling price and recycling quantity gradually increase over time, and eventually tend to be a stable state. Compared figure 1-left with 1-right, an interesting phenomenon is that with the increase of the natural growth rate, there is a negative correlation between recycling price and recycling quantity. Our explanation is that consumers are more willing to recycle obsolete mobile phones due to the diffusion effect of online recycling mode and the enhancement of consumers' awareness of environmental protection. Therefore, the recycling platform can improve the operating performance by reducing the recycling price.

4.2 Impact of Consumers' Sensitivity to Service

By changing parameter values φ ($\varphi = 50, 100, 150$), we explore the impact of the consumers' sensitivity to service on the optimal recycling price and the optimal recycling quantity in figure 2.

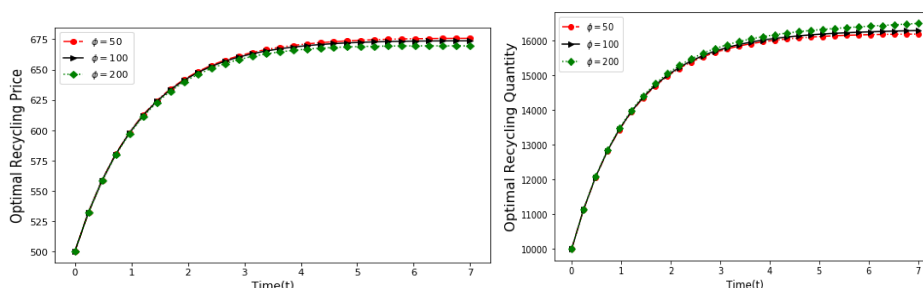


Figure 2. Impact of parameter φ on optimal recycling price and quantity

It can be seen from figure 2 that the optimal recycling price decreases with the consumers' sensitivity to service increases; On the contrary, the optimal recycling quantity increases with the consumers' sensitivity to service increase. One strange result is that there is a negative correlation between recycling price and recycling quantity. One possible explanation is that when consumers are more sensitive to the service, the recycling platform improves service level (resulting in the increase of service cost), and the recycling platform maximizes the platform's profit by lowering the price. Therefore, when consumers are highly sensitive to service level, lower recycling price strategy should be adopted to improve the performance of online recycling platform, and vice versa. So, consumers are paying more and more attention to service convenience and service quality.

4.3 Impact of Consumers' Sensitivity to Information Leakage

The analysis of consumers' sensitivity to information leakage is shown in figure 3. Parameters is set different values ($m = 100, 1000, 10000$), while others remain the basic.

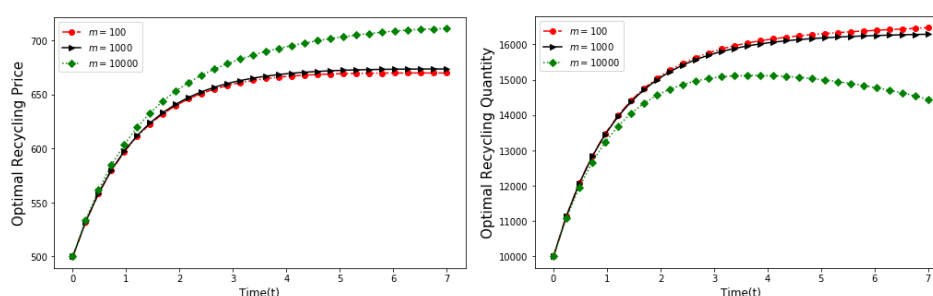


Figure 3. Impact of parameter m on optimal recycling price and quantity

From figure 3, we can see that the optimal recycling price decreases with the increase of consumers' sensitivity to information leakage. With the increase of the consumers' sensitivity to information leakage (the value of m increases from 100 to 1000), raising the recycling price is an effective measure to improve recycling quantity. However, when the consumers' sensitivity to information leakage is too high ($m = 10000$), high recycling price strategy fails to inhibit the decline of recycling quantity. Currently, consumers are extremely sensitive to the information leakage stored in obsolete mobile phones. If the privacy security cannot be guaranteed, consumers would rather not recycle their obsolete mobile phones for recycling value than leak their private data. In the big data era, the online recycling platform should develop more reliable information security technology to protect the security of privacy. It is better to win the trust of consumers, create a good platform image and improve the recycling performance. The government should issue the standards of information safety and supervise the recycling industry.

5. Conclusion

To fill this gap between the dynamic pricing strategy and consumers' recycling willingness, a dynamic pricing model based on the optimal control theory is developed to maximize the profits of online recycling platform. The results of this research are as follows: Of all the consumers' recycling willingness (i.e. consumers' sensitivity to price, service, information leakage.), the information leakage is the most crucial consumers' willingness, while service level is the next. The above two factor have a reverse correlation with optimal recycling price. Consequently, ensuring information security and moderately improving service level are of significance for improving performance in the long term. Some laws and regulations or related technical norms on the elimination of private information should to be issued to supervise the recycling industry. Environmental awareness will greatly promote consumers to recycle obsolete mobile phones voluntarily. Environmental education and advertising play a significant role in improving performance of the online recycling platform.

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