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An Algorithm of Filtering Noises in Multi-beam Data Based on Rolling Circle Transform

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***Abstract*—After having analyzed the disadvantage of existing automatic filtering algorithm, which means deleting the micro morphology or obstacle caused by devilishly pursuing the smoothing effect of artificial filtering method, aims at Ping as the processing cell of multi-beam bathymetric survey data, and by means of rolling circle transform's characteristic of identifying and analyzing the concave (convex) part quantitatively under specific precision, the paper details an algorithm for filtering noises in multi-beam data based on a rolling circle transform, and details the keystone and solution steps of the model. At last, in a VC++ environment, some experiments were done to validate the algorithm's validity. The experiments show that the algorithm can filter noises in multi-beam data; maintaining marine topography completeness and enhancing the efficiency in gross error detection.**

Keywords—rolling circle transform; multi-beam data; filtering; algorithm

I. INTRODUCTION

The use of multi-beam bathymetric system to obtain submarine terrain information is one of the important means of ocean surveying and marine research[1]. Compared with terrestrial measurements, there are many unstable factors in ocean surveying, such as instrument noise, sea condition factors or unreasonable setting of multi-beam parameters, which make the measurement data contain a small amount of abnormal data. In addition, there are smaller random noises in the data, influenced by environmental and other factors[2-5]. In order to improve the accuracy of submarine topography, multi-beam bathymetric data must be filtered to eliminate false signals, restore and retain real information. Traditional multi-beam bathymetric data filtering usually uses manual filtering method, but because the amount of multi-beam bathymetric data is very large, the task of data processing is more and more onerous, the traditional manual operation mode can not meet the requirements of massive multi-beam bathymetric data processing. In addition, manual filtering is a large number of similar "weeding" operation, with strong subjectivity, so that the treatment of the seabed terrain is very smooth, but this treatment method may lead to the emergence of benthic geomorphology or small obstacles are unreasonably deleted situation[5-6]. Therefore, it is necessary to study the fast and reliable data filtering method.

Scholars at home and abroad have made a lot of achievements in the detection of bathymetric anomaly data. According to the statistical characteristics of bathymetric data in neighborhood, a depth gate filtering algorithm based on threshold value is proposed in [5]. The algorithm has been improved by the Simrad Corp, but the threshold value of this kind of algorithm mostly depends on the experience of marine surveyors, and the manual intervention is larger[6]. According to the depth and plane position of the measured beam footprints, the multi-type surface function is used to fit the terrain, and a trend surface filtering algorithm for multi-beam bathymetric data is proposed in [7]. This kind of algorithm is relatively simple to operate, for the terrain fluctuation is relatively flat situation is more applicable, but if the terrain changes complex will lead to incomplete filtering[8]. In [9], on the basis of summing up the previous algorithms, a hybrid algorithm based on median filtering, local variance detection and wavelet analysis is proposed according to the distribution characteristics of bathymetric data in larger regions. It is used to locate the abnormal data, and filter the random noise out, and a good filtering effect is obtained, but for the phase of the algorithm medium and small ripple median filtering, the selection and setting of the off parameters is not discussed too much[10]. CUBE algorithm has a good detection effect of abnormal data, and has become an important module in the software CARIS HIPS. However, for some reasons, the research on CUBE algorithm in China mostly stays in theoretical level. In addition, due to the particularity of multi-beam measurement, in the absence of prior information, for the measurement data in the existence of various types of abnormal data, it is often difficult to distinguish whether it belongs to a false signal or from a certain class of special terrain or artificial elements (such as undersea geomorphology or derrick, etc.) echo signal, which results in the problem of ambiguity of abnormal signals in multi-beam bathymetric data processing[6-10]. On the one hand, the problem of ambiguity makes it difficult to form a clear evaluation criterion for the effect of multi-beam filtering algorithm, on the other hand, under the condition that there is no prior information, the existing automatic filtering algorithm has the tendency of excessive pursuit of manual filtering effect, which makes the underwater terrain of the filtered sea area often too smooth.

In "Specification for Hydrographic Surveys", it is stipulated that, in principle, 3~5 echo signals must be recorded continuously in order to be able to identify a small target on the recording paper[6,14]. This provides a certain criterion for solving the two semantic problem of the above anomaly signal, that is, in addition to considering the difference between the abnormal signal and the statistical characteristics of the bathymetric data, the filter of the multi-beam bathymetric data needs to be further judged according to the number of continuous echo signals. The possibility that a constant signal belongs to a special terrain or artificial element. Buffer analysis is one of the basic spatial analysis functions in GIS, and it is an important method to measure the characteristics of spatial elements[11]. Rolling circle transform algorithm based on buffer analysis function is widely used in the analysis of concave and convex of planar elements and the extraction of minimum convex packets[12]. Based on the analysis of the essence of rolling circle transform algorithm, this paper quantitatively identifies and analyzes the concave and convex properties of the multi-beam bathymetric data with Ping as the processing unit by using rolling circle transform combined with the number of continuous echo signals, and takes into account the statistical characteristics of the bathymetric data in Ping, and establishes the criterion for determining the anomaly signal. On the basis of maintaining the terrain integrity of the seafloor, the purpose of fast filtering of multi-beam bathymetric data is realized.

II. Construction Principle and Geometric Characteristic Analysis of Rolling Circle Transform

A. Line Feature Buffer Boundary Transform

For line element T with a distribution axis of $\{Q_1....Q_{N+1}\}$, the mathematical definition of the left and right buffer boundary transform $K_L(r)$ and $K_R(r)$ is[12-13]:

$$\begin{cases} T\cdot K_L(r)=\{P_l \mid \{d_e(P_l,T_i) \mid \left|\overline{T_i}\times\overline{P_lQ_i}\right|\le 0\}=r\} \\ T\cdot K_R(r)=\{P_r \mid \{d_e(P_r,T_i) \mid \left|\overline{T_i}\times\overline{P_lQ_i}\right|\ge 0\}=r\} \end{cases} \quad (1)$$

where r represents a buffer distance; $T\cdot K_L(r)$ ($T\cdot K_R(r)$) represents the left (right) buffer boundaries of line feature T; P_l (P_r) represents any point on $T\cdot K_L(r)$ ($T\cdot K_R(r)$); T_i represents any segment on line feature T; d_e represents a two-dimensional Euclidean distance operation; × represents a vector product operation.

B. The Construction Principle of Rolling Circle Transform

Rolling circle transform refers to a geometric transformation that forms a trace line by scrolling on one side of the element along an infinitely smooth ring on a two-dimensional plane, and rolling circle transform has a left and right side[12]. In mathematical form, rolling circle transform is equivalent to the transform combination of the left (right) side buffer boundary of the line element[12-13]. The mathematical definition of rolling circle transform is[12]:

$$\begin{cases} T\cdot V_L(r)=T\cdot K_L(r)\cdot K_R(r) \\ T\cdot V_R(r)=T\cdot K_R(r)\cdot K_L(r) \end{cases} \quad (2)$$

where $V_L(r)$ ($V_R(r)$) represents left (right) side rolling circle transform. As shown in Figure 1, for a given line feature T, the curve $T\cdot V_L(r)$ obtained by the left rolling circle transform

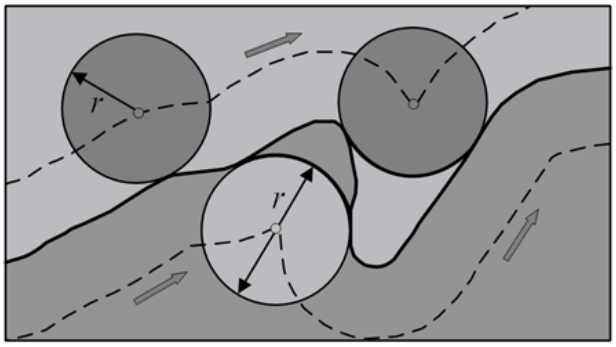


FIGURE I. DIAGRAM MAP OF ROLLING CIRCLE TRANSFORM

$V_L(r)$ has the characteristics of keeping the convex part constant, narrowing or filling the concave part; while the curve $T\cdot V_R(r)$ obtained by the right rolling circle transform $V_R(r)$ has the characteristics of keeping the recess constant, narrowing or flattened convex part.

C. Analysis of Geometric Characteristics of Rolling Circle Transform

In rolling circle transform, the size of the threshold (buffer distance r) determines the degree of its influence on the spatial geometry of the line elements. For the two-dimensional plane to take on the first line of elements, the degree of filling (flattened) of its concave (convex) part has the following function relationship with the buffer distance[13]:

$$r-r^{'}=r-\sqrt{r^2-l^2}\le\Phi \quad (3)$$

where $r^{'}$ represents the distance from the center of the rolling circle center and the center of the concave (convex) part after completely filling (flattened); l represents half the width of a concave (convex) part; Φ ($\Phi\ge 0$) represents the filling degree of the concave (convex) part. The smaller the Φ, the higher the filling (flattened) of the concave (convex) part, and, conversely, the lower the filling (flattened) degree of the concave (convex) part.

As shown in Figure 2, P is a concave feature point, P_l is the left rolling circle center, P_{lr} , P_{rl} respectively for the concave through the left (right) side of rolling circle transform $V_L(r)$ ($V_R(r)$) corresponding to the feature point. The size of $P_{lr}P_{rl}$ can be understood as a quantitative description of the degree of fluctuation at the concave feature point P . P_lP_{lr} intersects with line segment AB at $P_{lr}^{'}$. Φ , the size of $P_{lr}P_{lr}^{'}$, reflects the filling degree of the concave, that is, the quantitative description accuracy of the fluctuation degree of the concave feature point P .

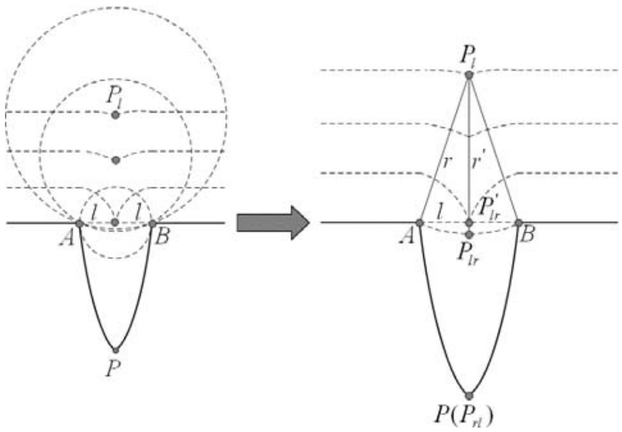


FIGURE II. GEOMETRY CHARACTERISTIC OF ROLLING CIRCLE TRANSFORM

III. MULTI-BEAM BATHYMETRIC DATA FILTERING ALGORITHM BASED ON ROLLING CIRCLE TRANSFORM

A. Data Structure Organization of Ping

The multi-beam sounding system uses two sets of transducer arrays, which transmit and receive directivity orthogonal, to obtain a series of narrow beams perpendicular to the heading distribution, which are represented by Ping. In the bathymetric central coordinate system, the mathematics of Ping is defined as[6-7]:

$$T = \{Q_i \mid (x_i, y_i, z_i), i = 1, 2, \ldots, N\} \tag{4}$$

where Q_i represents any beam bathymetric point in Ping; x_i, y_i and z_i represents the three-dimensional component of the beam bathymetric point Q_i in the central coordinate system of the bathymetric; N represents the number of beam bathymetric point Q_i in Ping. Under the condition that the acoustic bending is ignored, x_i, y_i and z_i can be solved by (5):

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} d_i \sin\theta_i \\ 0 \\ d_i \cos\theta_i \end{bmatrix} \tag{5}$$

where d_i ($d_i = Ct_i/2$) represents half of the product of echo time t_i and system sound speed C; θ_i represents the incident angle of the i th beam, that is:

$$\theta_i = \theta_{bw}(i - M_i/2) \tag{6}$$

where θ_{bw} represents a beam angle; $M_i/2$ represents the central beam number of a stripe.

As shown in Figure 3, in the bathymetric central coordinate system, the three-dimensional components x_i, y_i, z_i of the beam bathymetric point Q_i are only related to the x-axis and y-axis ($y_i \equiv 0$), and there is a functional relationship between the x_i and z_i components, such as $z_i = f(x_i)$ ($f(x_i) = x_i/\tan\theta_i$). Therefore, the processing unit Ping of the multi-beam bathymetric data conforms to the mathematical definition of the offline elements of the two-dimensional condition.

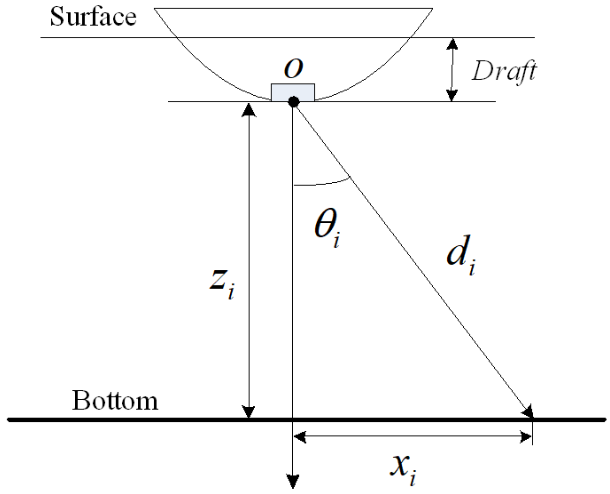


FIGURE III. BATHYMETRIC CENTER COORDINATE SYSTEM

B. Determination of Relevant Parameters in Rolling Circle Transform

The geometrical characteristics of rolling circle transform determine that it can quantitatively identify and analyze the concave (convex) part of a certain scale of the line element (determined by parameter l) under the given parameter Φ, that is, to meet certain precision conditions. In practical application, the key to select the value of parameter l is to determine how large the minimum range of an undersea concave (convex) part is, that is, the number of continuous echo signals in multi-beam measurement. In "Specification for Hydrographic Surveys", it is stipulated that, in principle, 3~5 echo signals must be recorded continuously in order to be able to identify a small target on the recording paper[6,14]. That is:

$$l = m\bar{\delta}/2 \tag{7}$$

where m ($3 \le m \le 5$) represents the number of echo signals recorded continuously; $\bar{\delta}$ represents the average width of a beam footprint. Combining the beam footprint calculation model in [15], the mathematical definition of $\bar{\delta}$ is:

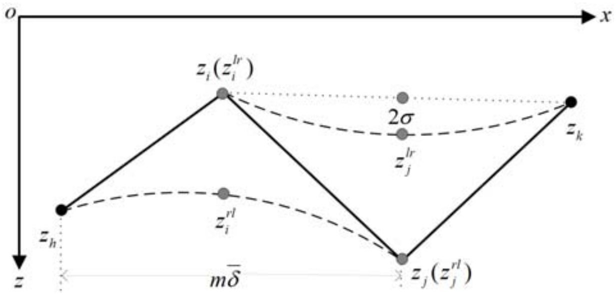
$$\bar{\delta} = \left(\sum_{i=1}^{N} d_i \theta_{bw} / \cos\theta_i\right) \Big/ N \tag{8}$$

For another important parameter Φ in rolling circle transform, the size determines the accuracy of rolling circle transform for the description of Ping inner concave (convex) part. In order to ensure that rolling circle transform has a good filtering effect, Φ can be determined according to the limit error (confidence 95%) of the soundings measurement in "Specification for Hydrographic Surveys" [15]. That is:

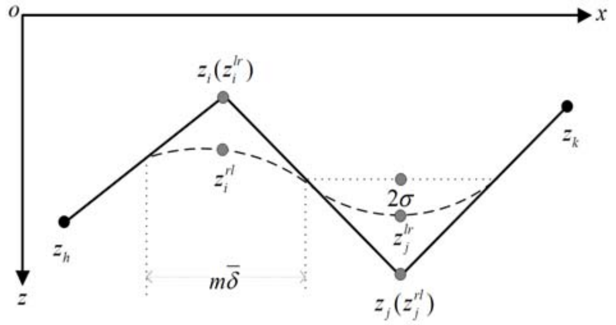
$$\Phi = 2\sigma \tag{9}$$

where σ represents the middle error of soundings measurement. Substitute (7) and (9) into (3) respectively, the threshold (buffer distance r) is further limited to $r \geq \sigma + \frac{(m\overline{\delta})^2}{16\sigma}$ .As shown in Figure 4, z_h , z_i , z_j , z_k ($h \leq i \leq j \leq k$) respectively represent the soundings corresponding to the beam bathymetric points Q_h , Q_i , Q_j and Q_k in Ping; Q_i and Q_j are adjacent convex and concave parts in Ping; z_i^{lr} (z_i^{rl}) and z_j^{lr} (z_j^{rl}) respectively represent the soundings of the characteristic points corresponding to the feature points Q_i and Q_j through the left (right) side rolling circle transform $V_L(r)$ ($V_R(r)$); $\left|z_i^{lr} - z_i^{rl}\right|$ and $\left|z_j^{lr} - z_j^{rl}\right|$ respectively represent the fluctuation degree of the undersea topography at the characteristic points Q_i and Q_j .

The buffer distance, on the premise of $r \geq \sigma + \frac{(m\overline{\delta})^2}{16\sigma}$ ($l = m\overline{\delta}/2$ and $\Phi=2\sigma$), for convex and concave feature points Q_i and Q_j , if the concave (convex) part is less than or equal to $m\overline{\delta}$ (as shown in Figure 4(a), $x_j - x_h \leq m\overline{\delta}$ and $x_k - x_i \leq m\overline{\delta}$), then the concave (convex) part is completely flattened (filled); otherwise (as shown in Figure 4(b), $x_j - x_h > m\overline{\delta}$ and $x_k - x_i > m\overline{\delta}$), the concave (convex) part is partially flattened (filled), and the concave (convex) part is flattened (filled) in the range of $m\overline{\delta}$. The above analysis shows that rolling circle transform has the characteristic of quantitatively identifying the small target of the seafloor when the number of the given continuous echo signal is m , and the recognition accuracy is less than 2σ , the limit error of the soundings measurement. This kind of characteristic of rolling circle transform provides a certain solution to solve the problem of ambiguity of abnormal signals in multi-beam bathymetric data processing, that is, in addition to considering the difference between the abnormal signal and the statistical characteristics of bathymetric data, the filtering of multi-beam sounder It is also necessary to further judge the possibility that the abnormal signal belongs to special terrain or artificial elements according to m , the number of continuous echo signals.



(a) The concave (convex) part is less than or equal to $m\overline{\delta}$.



(b) The concave (convex) part is larger than $m\overline{\delta}$.

FIGURE IV. CHARACTERISTIC OF IDENTIFYING MICRO MORPHOLOGY BASED ON ROLLING CIRCLE TRANSFORM

C. Establishment of Data Filtering Criterion in Rolling Circle Transform

According to the analysis in section *B* of part II, the size of $P_{lr}P_{rl}$ reflects the fluctuation degree of the concave (convex) feature point P relative to the bathymetric data in Ping, given the premise of l and Φ . Make $Error(i)$ represent the signal state value of beam bathymetric point Q_i (0 represents the abnormal signal, 1 represents the normal signal), and under the assumption that the variation of seafloor topography is smooth, continuous and gentle, the following multi-beam bathymetric data filtering criterion is established in this paper:

$$\begin{cases} \left|z_i^{lr} - z_i^{rl}\right| > k\sigma^{'}; then, Error(i) = 0 \\ \left|z_i^{lr} - z_i^{rl}\right| \leq k\sigma^{'}; then, Error(i) = 1 \end{cases} \tag{10}$$

where z_i^{lr} (z_i^{rl}) represents the corresponding soundings of the beam bathymetric point Q_i after the left (right) rolling circle transform; $\left|z_i^{lr} - z_i^{rl}\right|$ indicates the fluctuation degree of the undersea terrain at the beam bathymetric point Q_i ; k represents a gross difference decision factor; σ' indicates that the middle error of the fluctuation degree of the bathymetric data in the Ping test can be obtained by (11) solution, that is:

$$\sigma^{'} = \sqrt{\left(\sum_{i=1}^{N} (z_i^{lr} - z_i^{rl})^2\right) \Big/ N} \tag{11}$$

Figure 5 shows the schematic diagram of a multi-beam bathymetric data filter based on rolling circle transform. In the two-dimensional spatial right-angled coordinate system composed of $o-xz$, for the single Ping bathymetric data with a central axis of $\{Q_i \mid (x_i, z_i), i = 1, 2, \ldots, N\}$, the topographic fluctuation degree of any beam bathymetric point Q_i can be determined by the difference absolute value $\left|z_i^{lr} - z_i^{rl}\right|$, after the left (right) side rolling circle transform $V_L(r)$ ($V_R(r)$) by the buffer distance r. According to the law of “small probability event”, it can be considered as a small probability event in the statistical sense when the fluctuation degree of submarine topography at the beam bathymetric point Q_i is greater than 2 or 3 times of σ' [3,8]. So the crude difference determines the factor k =2 or 3.

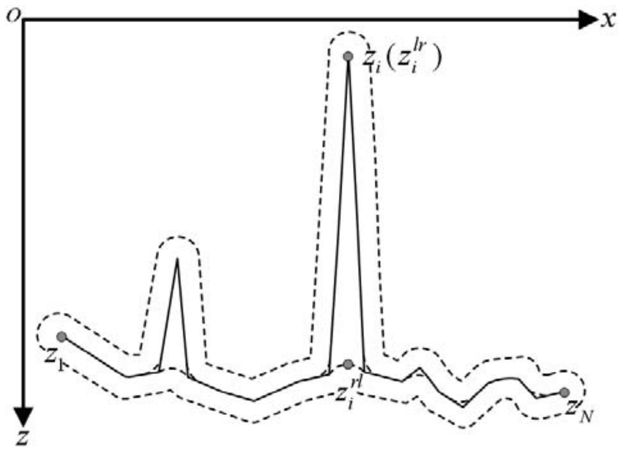
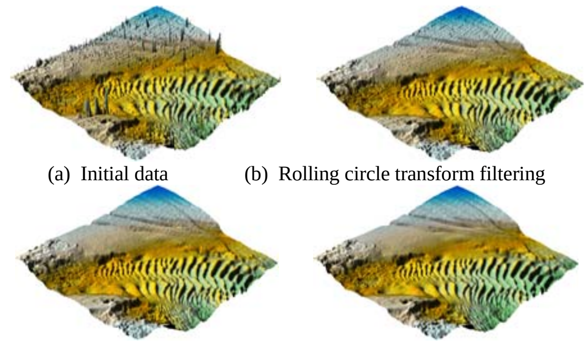


FIGURE V. PRINCIPLE OF FILTERING NOISES IN MULTI-BEAM DATA BASED ON ROLLING CIRCLE TRANSFORM

IV. EXPERIMENTAL RESULTS AND ANALYSIS

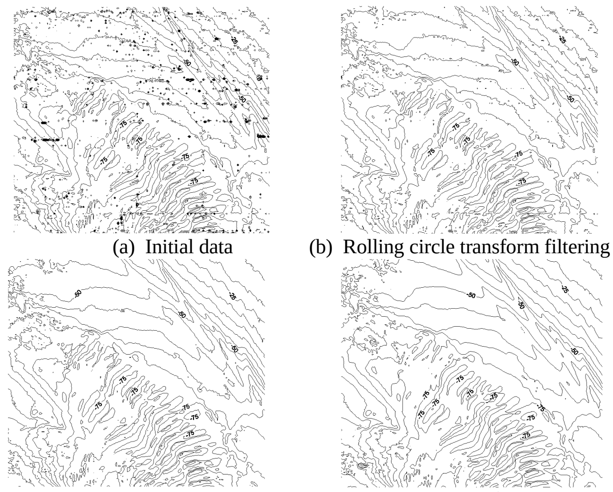
With the help of the construction principle of the midline element buffer in [11], this paper realizes the multi-beam bathymetric data filtering algorithm based on rolling circle transform by VC++ programming, and uses the software Surfer8.0 to visualize and analyze the generated experimental results. The experimental environment is Celeron(R) processor with a primary frequency of 2.53GHz, and a memory of 512M. The data used in the experiment are multi-beam bathymetric data of a sea area in the East China Sea, which contains 725,840 discrete water depth points, the limit error is 1m, the number m of continuous recorded echo signal and the difference determination factor k are set to 3 and 2 respectively, and the algorithm takes 2578s. In order to facilitate experimental comparison and analysis, using the weighted average method in Surfer8.0 (taking the distance inverse square right as the weight function), the original multi-beam bathymetric data is interpolated by the algorithm, CUBE algorithm (algorithm time 72s) and the multi-beam bathymetric data after manual filtering, with the formation of a 229×200 grid DDM. Figure 6 and Figure 7 show the comparison of the surface and isobath (equal depth of 5m) of the seafloor terrain based on the above different filtering methods respectively.



(a) Initial data (b) Rolling circle transform filtering

(c) CUBE algorithm (d) Manual filtering

FIGURE VI. MARINE TOPOGRAPHY SURFACE GENERATED BY DIFFERENT FILTERING NOISES ALGORITHM



(a) Initial data (b) Rolling circle transform filtering

(c) CUBE algorithm (d) Manual filtering

FIGURE VII. BATHYMETRIC CONTOUR GENERATED BY DIFFERENT FILTERING NOISES ALGORITHM

As can be seen from Figure 6 and Figure 7, the multi-beam bathymetric data filtering algorithm, CUBE algorithm and manual filtering method based on rolling circle transform can effectively eliminate the rough difference in the original multi-beam bathymetric data, which shows that the algorithm has good roughness discrimination and correction ability for multi-beam bathymetric data. Compared with the manual filtering method in days, its efficiency has been greatly improved. In order to further verify the filtering effect of the proposed algorithm on multi-beam bathymetric data, the minimum, maximum, and root mean square deviation of sounding in initial multi-beam bathymetric data after the proposed algorithm, CUBE algorithm and manual filtering are analyzed statistically. The results are shown in Table 1.

The experimental results show that:

- Due to the influence of rough difference data, the water depth value range and soundings mean square root difference of the original multi-beam bathymetric data are larger.
- In this paper, the depth and minimum values of the multi-beam bathymetric data after the algorithm, the CUBE algorithm and the manual filter are small, which shows

that all three filter out the large roughness in the original multi-beam bathymetric data.

- The soundings mean square root difference of the CUBE algorithm and the multi-beam bathymetric data after manual filtering is smaller than the corresponding soundings mean square root difference obtained by this algorithm, and the main reason for the above difference is that the soundings fluctuation of the multi-beam bathymetric data filtered by this algorithm is smaller than that of the undersea terrain.

Because the CUBE algorithm selects the soundings under the set grid, the soundings and its correlation error are estimated at the intersection of the grid, and finally the submarine trend surface is fitted, which makes the undersea terrain smoother after treatment. In addition, manual filtering has more smooth surface topography because of its more prior information and greater subjectivity.

It should be noted that the analysis of the experimental results in Table 1 is only statistically from the point of view of the bathymetric value of the multi-beam bathymetric data, which reflects the degree of seafloor topography after treatment by three types of methods. It is still a problem worth discussing whether the smoothing effect is an advantage or a shortcoming to maintain the accuracy of submarine terrain expression after CUBE algorithm and manual filtering of multi-beam bathymetric data are smoother. If the topography of the seafloor changes more complex or there are obstacles (such as reefs, shipwrecks, etc.), such a smoothing effect may cause obstacles to be unreasonably removed. In this paper, the algorithm takes full account of the rolling circle change based on the design. In exchange for the identification characteristics of the seafloor, the benthic geomorphology of a given range is preserved under certain precision conditions. In the statistical results, although the algorithm changes less than the CUBE algorithm and the manual filtering method in the degree of seafloor topography fluctuation, it shows that the algorithm is beneficial to the retention of seafloor geomorphology, and solves the two semantic problem of abnormal signal in multi-beam bathymetric data processing.

TABLE I. EXPERIMENT ANALYSIS

Sounding statistics(m)	*Initial data*	*Rolling circle transform filtering*	*CUBE algorithm*	*Manual filtering*
Minimum	-103.329	-99.437	-99.071	-98.774
Maximum	-1.553	-13.204	-13.263	-12.836
Root mean square deviation	18.542	16.732	16.174	15.979

V. Conclusion

Based on the principle of rolling circle transform and the analysis of the geometric characteristics of rolling circle transform, a multi-beam bathymetric data filtering algorithm based on rolling circle transform is proposed for multi-beam bathymetric data with Ping as processing unit, and the influence of buffer distance on the filtering effect of rolling circle transform is analyzed theoretically. Experimental results show that the algorithm can filter out the rough difference data in the multi-beam bathymetric data on the basis of maintaining the terrain integrity of the seafloor, and the efficiency is greatly improved compared with the manual filtering method. However, it should be pointed out that in the process of calculating the algorithm, the number of echo signals and crude difference determination factors involved in the continuous record are used in "Specification for Hydrographic Surveys" to give the empirical parameters, theoretical and universal need to be strengthened, it is necessary to further study how to according to the actual multi-beam bathymetric data in each Adaptive estimation of the soundings distribution of Ping. In addition, considering the large scale (or even massive) of multi-beam bathymetric data, it is necessary to study the optimization of the algorithm.

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