Advances in Economics, Business and Management Research, volume 82
International Conference on Management, Education Technology and Economics (ICMETE 2019)

Stochastic Computing and Intelligent Optimization Analysis for a Generalized Information Delivery System

Xiao Xiaonan
Xiamen University Tan Kah Kee College
Zhangzhou, 363105

***Abstract*—In this paper, we use the optimal nonlinear random filtering method and intelligent optimization algorithm to study the optimal control problem of a kind of incomplete data and continuous nonstationary stochastic information delivery system. We obtain the two optimal control mathematical models in these two situations; illustrate how to establish the optimal encoding and decoding of the nonstationary stochastic process; and provide an effective and reliable approach for the optimal control of such a process.**

Keywords—Generalized information; Delivery system; Stochastic analysis; Intelligent optimization algorithm; Optimal control

I. INTRODUCTION

Information system search, analysis and application is a very complicated issue for information transfer optimization. So is the stochastic dynamic optimization simulation and control. They include a comprehensive analysis of many aspects of information filtering, information processing, and information statistical analysis [1-4]. A great number of facts indicate how filtering information and the result of information processing and analysis play a vital role in the improving of the signal transfer system and the efficiency of information decision [5-9]. In order to effectively solve the optimization modeling and control of random signal of a kind of multi-dimensional, general part in the measurable, stable process, this paper adopts random optimization analysis and stochastic recursive filtering method. It undertakes an in-depth study in the optimal filtering and estimation of random signal in a partially observational generalized process with a fractional rational spectral density [10-15]. Furthermore it leads to the optimal filter equation and optimal estimation equation of random signals in the generalized process. Hence it provides a reliable theoretical foundation and an efficient mathematic method for further research of the optimal control of such a process and for further improvement of the efficiency of the sort of information transmission system [16-20].

II. SYSTEM DESCRIPTION AND INFORMATION SEARCH

For a class of fractional rational spectral density and some observational generalized process stochastic signal transfer system $\eta(t), t=0,\pm 1,\pm 2,\cdots$, Its spectral representation formula is

$$\eta(t)=\int_{-\pi}^{\pi} e^{i\lambda t}\,\frac{P_{n-1}(e^{i\lambda})}{Q_n(e^{i\lambda})}\,\phi(\mathrm{d}\lambda) \tag{1}$$

In it, $\phi(\mathrm{d}\lambda)$ is the orthogonal random measure, and there is

$$E\phi(\mathrm{d}\lambda)=0, \qquad E|\phi(\mathrm{d}\lambda)|^2=\frac{\mathrm{d}\lambda}{2\pi},$$

$$P_{n-1}(Z)=\sum_{k=0}^{n-1} b_k Z^k, \quad Q_n(Z)=\sum_{k=0}^{n} a_k z_k, \quad a_n=1,\ a_k, b_k\in R^1$$

All of the roots of equation $Q_n(Z)=0$ are located in the unit.

From (1) we can infer that the process of $\eta(t)$ has a fractional rational spectral density

$$f_\eta(\lambda)=\left|\frac{P_{n-1}(e^{i\lambda})}{Q_n(e^{i\lambda})}\right|^2 \tag{2}$$

Now according to the constituting process of the measure $\phi(d\lambda)$

$$\varepsilon(t)=\int_{-\pi}^{\pi} e^{i\lambda(t-1)}\phi(d\lambda) \tag{3}$$

then $$E\varepsilon(t)=0, \qquad E|\varepsilon(t)|^2=\int_{-\pi}^{\pi}\frac{\mathrm{d}\lambda}{2\pi}=1$$

and $$E\varepsilon(t)\bar{\varepsilon}(s)=\int_{-\pi}^{\pi} e^{i\lambda(t-s)}\frac{1}{2\pi}\mathrm{d}\lambda=\delta(t,s) \tag{4}$$

Wherein $\delta(t,s)$ is a Kronecker symbol.

From (4), we can argue that a variable sequence $\varepsilon(t),\ t=0,\ \pm 1,\ \pm 2,\cdots,$ is a sequence of non-relational value.

And along with the process $\eta(t)$ for the spectrum of expression (1), we can also define a new set of procedures $\eta_1(t),\ \eta_2(t),\cdots,\ \eta_n(t)$ with the following formula:

$$\eta_j(t)=\int_{-\pi}^{\pi} e^{i\lambda t} W_j(e^{i\lambda})\phi(\mathrm{d}\lambda), \qquad j=1,2,\cdots,n \tag{5}$$

Wherein the frequency characteristics $W_j(Z),\ j=1,2,\cdots,n,$ is

$$W_j(Z)=Z^{-(n-j)}W_n(Z)+\sum_{k=j}^{n-1}\beta_k Z^{-(k-j+1)}, \qquad j=1,2,\cdots,n-1 \tag{6}$$

$$W_n(Z)=-Z^{-1}\sum_{k=0}^{n-1}\alpha_k W_{k+1}(Z)+Z^{-1}\beta_n \tag{7}$$

and $\beta_1=b_{n-1},\ \beta_j=b_{n-j}-\sum_{i=1}^{j-1}\beta_i\alpha_{n-j+i}, \quad j=2,\cdots,n$ (8)

From (6), (7) we can obtain respectively

$$W_j(Z)=Z^{-1}[W_{j+1}(Z)+\beta_j] \tag{9}$$

$$W_n(Z)=Z^{-1}[-\sum_{k=0}^{n-1}\alpha_k W_{k+!}(Z)+\beta_n] \tag{10}$$

It is not difficult to deduce

$$W_n(Z)=Z^{-1}\left\{\sum_{k=0}^{n-1}\alpha_k[Z^{-(n-K-1)}W_n(z)+\sum_{j=k+1}^{n-1}\beta_j Z^{-(j-k)}]+\beta_n\right\} \tag{11}$$

so $$W_n(Z)=\frac{P_{n-1}^{(n)}(Z)}{Q_n(Z)} \tag{12}$$

In it $P_{n-1}^{(n)}(z)$ is the polynomial formula whose order is no more than n-1

From (9) to (12) we can also extrapolate

$$W_j(Z)=\frac{P_{n-1}^{(j)}(Z)}{Q_n(Z)} \tag{13}$$

In it, the order of the polynomial is no more than n-1, and according to (8), we have

$$P_{n-1}^{(1)}(z)\equiv P_{n-1}(Z) \tag{14}$$

Therefore, $\eta_1(t)=\eta(t)$

III. Optimal Recursive algorithms and analysis

Suppose the news which requires transition is what Gaussian random variable "θ" of $M_\theta=m, D\theta=r>0,$ holds, meanwhile parameter "m" and "r" are known no matter what they stand for at the beginning end or acceptance point.

The signals received at the outlet of the "transmission systems" $\xi=\xi(t), 0\le t\le T$

and suppose they satisfy the following stochastic equation:

$$d\xi_t=A(t,\theta,\xi)dt+d\omega_t, \xi_0=0 \tag{15}$$

where $\omega=(\omega_t), 0\le t\le T$, is Wiener process, which is not decided by θ, non-advanced functional $A=(A(t,\theta,\xi)), 0\le t\le T$. We give the code and suppose equation (15) has the only strong solution of being

$$p\{\int_0^T A^2(s,\theta,\xi)d\xi<\infty\}=1$$

we put the limited condition

$$\frac{1}{t}\int_0^t MA^2(s,\theta,\xi)ds\le p \tag{16}$$

to functional $A=(A(t,\theta,\xi)), 0\le t\le T$ (where p is a given constant).

Then at every moment t, according to acceptance signal $\xi_0^t=\{\xi_s, s\le t\},$ it can form "outlet news" $\tilde{\theta}_t(\xi)$. We make elective decode's non-advanced functional $\hat{\theta}=(\hat{\theta}_t(\xi)), 0\le t\le T$. Let the optimum form recovery news θ in certain sense.

While regenerating error

$$\Delta(t)=\inf\ M[\theta-\hat{\theta}_t(\xi)]^2, 0\le t\le T$$

where inf was elected based on all permissible code $A=(A(s,\theta,\xi)),s\ge 0$ and decode $\hat{\theta}(\xi)$.

When we give the code, owing to

$$M[\theta-\hat{\theta}_t(\xi)^2]^2 \ge M[\theta-m_t(\xi)]^2$$

where $m_t = M(\theta \mid \mathrm{F}_t^{\xi})$, we have $\Delta(t)=\inf_A \backslash M[\theta-m_t]^2$ and the optimum decode (for signal ξ_0^t) is a posteriori mean value $m_t = M(\theta \mid \mathrm{F}_t^{\xi})$.

In order to seek for the optimum code, the optimum and the minimal regenerating error $\Delta(t)$ of the news caused by transition within the time t, first we study subclass of linear's permissible code function $A(t,\theta,\xi)$ which depends on θ:

$$A(t,\theta,\xi)=A_0(t,\xi)+A_1(t,\xi)\theta \tag{17}$$

where $A_0=A_0(t,\xi),A_1=A_1(t,\xi),0\le t\le T$ are the non-advanced functional.

It can be

$$\Delta^*(t)=\inf \underset{(A_0,A_1)}{M}[\theta-m_t]^2 \tag{18}$$

also we can find variable $\Delta^*(t)$ and the optimum code function (A_0^*,A_1^*) corresponding to inf in(18).

Suppose a certain code (A_0,A_1) has been elected, $\xi=(\xi_t),0\le t\le T,$ and satisfies the following process of equation:

$$d\xi_t=[A_0(t,\xi)+A_1(t,\xi)\theta]dt+dw_t,\xi_0=0. \tag{19}$$

Consequently $m_t = M(\theta \mid \mathrm{F}_t^{\xi})$ and $r_t = M[(\theta-mt)^2 \mid \mathrm{F}_t^{\xi}]$ satisfy the equation

$$dm_t=r_tA_1(t,\xi)[d\xi_t-(A_0(t,\xi)+A_1(t,\xi)m_t)dt] \tag{20}$$

$$r=-r_t^2A_1^2(t,\xi) \tag{21}$$

meanwhile they fit the condition $m_0=m,r_0=r.$ While the solution of equation (21) is

$$r_t=\frac{r}{1+r\int_0^t A_1^2(s,\xi)ds}$$

also, we have $p(\inf_{0\le s\le T} r_t>0)=1.$ So according to (21)

$$\frac{\dot{r}_t}{r_t}=-r_tA_1^2(t,\xi),$$

therefore

$$\text{In}\, r_t-\text{In}\, r=-\int_0^t r_sA_1^2(s,\xi)ds$$

namely $$r_t=r\exp\{-\int_0^t r_sA_1^2(s,\xi)ds\}. \tag{22}$$

Because of

$$M[A_0(t,\xi)+A_1(t,\xi)\theta]^2$$
$$=M\{[A_0(t,\xi)+m_tA_1(t,\xi)]+[\theta-m_t]A_1(t,\xi)\}^2=M\{A_0(t,\xi)+A_1(t,\xi)m_t\}^2+Mr_tA_1^2(t,\xi) \tag{23}$$

According to the condition (16)

$$\int_0^t Mr_sA_1^2(s,\xi)ds\le P_t \tag{24}$$

According to HeHceH inequality $(Me^{-\eta}\ge e^{-M_m}),$ (22) and (24)

$$Mr_t\ge re^{-pt},0\le t\le T \tag{25}$$

so, to the given code (A_1,A_1)

$$M[\theta-m_t]^2=Mr_t\ge re^{-pt} \tag{26}$$

So (see (18))

$$\Delta^*(t)\ge re^{-pt} \tag{27}$$

To the optimum code $(A_0^*,A_1^*),$ inequality within (24) and (25) should become equality. If

$$A_1^*(t)=\sqrt{\frac{P}{r}}e^{pt/2} \tag{28}$$

and this transition can take place, because the corresponding r_t^* (see (21)) will be equal to re^{-pt} exactly.

Based on (23) and equality

$$\int_0^t Mr_s^*(A_1^*(s))^2ds=\int_0^t r_s^*(A_1^*(s))^2ds=pt$$

we can obtain

$$A_0^*(t,\xi^*+A_1^*(t)m_t^*(\xi^*)=0 \tag{29}$$

where according to (20), the optimum decode m_t^* is decided by equation

$$dm_t^* = \sqrt{pr}e^{-pt/2}d\xi_t^*, m_0^* = m \quad (30)$$

while transited signal $\xi^* = (\xi_t^*), 0 \le t \le T$ (see (19)), satisfies the equation

$$d\xi_t^* = \sqrt{\frac{p}{r}}e^{pt/2}(\theta - m_t^*)dt + dw_t$$

$$\xi_0^* = 0 \quad (31)$$

It's clear according to (30) that the optimum decode can also be written as

$$m_t^* = m + \sqrt{pr}\int_0^T e^{-pt/2}d\xi_s^*$$

$$= m + \sqrt{pr}[e^{-pt/2}\xi_t^* + \frac{p}{2}\int_0^t e^{-pt/2}\xi_s^* ds] \quad (32)$$

The equation (31) shows what the optimum operation of code depends on is not the news θ itself, but the error " $\theta - m_t^*$ "between θ and its optimum m_t^* times $\sqrt{\frac{p}{r_t}}$,which causes all time to transit.

IV. Application

Assuming θ_t and $s_t, t = 0, \pm1, \pm2, \cdots$,is an unrelated generalized stable series,and $E\theta_t = ES_t = 0$ and the spectral density is

$$f_\theta(\lambda) = \left|1/e^{i\lambda} + c_1\right|^2, \quad f_\zeta(\lambda) = 1/\left|e^{i\lambda} + c_2\right|^2$$

Wherein $|c_i| < 1, \quad i = 1,2$.

If we consider θ_t as a random "valuable signal" , ζ as a "disturbance" and assuming the observation process as

$$\xi_t = \theta_t + \zeta_t \quad (33)$$

Then through the formula (1), we can find the unrelated sequences $\varepsilon_1(t) and\ \varepsilon_2(t)$, $t = 0, \pm1, \pm2, \cdots$ associated with $E\varepsilon_i(t) = 0,\ E\varepsilon_j(t)\ \varepsilon_i(s) = \delta(t,s), i = 1,2,$ Thereby,

$$\theta_{t+1} = C_1\theta_t + \varepsilon_1(t+1), \zeta_{t+1} = C_2\xi_t + \varepsilon_2(t+1) \quad (34)$$

Through(33)and(34)we can get

$$\xi_{t+1} = \theta_{t+1} + \zeta_{t+1} = (c_1 - c_2)\theta_t + c_2\xi_t + \varepsilon_1(t+1) + \varepsilon_2(t+1)$$

So the "unobservable" process θ_t and "observable" process ξ_t have satisfied equations

$$\theta_{t+1} = C_1\theta_t + \varepsilon_1(t+1)$$

$$\xi_{t+1} = (c_1 - c_2)\theta_t + c_2\xi_t + \varepsilon_1(t+1) + \varepsilon_2(t+1) \quad (35)$$

The best linear estimate m_t, $t = 0, 1, 2, \cdots$ of θ_t and the filtering MSE $r_t = E(\theta_t - m_t)^2$ satisfy the recurrence equation

$$m_{t+1} = c_1 m_t + \frac{1 + c_1(c_1 - c_2)r_t}{2 + (c_1 - c_2)^2 r_t}[\xi_{t+1} - (c_1 - c_2)m_t - c_2\xi_t] \quad (36)$$

$$r_{t+1} = c_1^2 r_t + 1 - \frac{[1 + c_1(c_1 - c_2)r_t]^2}{2 + (c_1 - c_2)^2 r_t} \quad (37)$$

Since the process $(\theta_t, \xi_t),\ t = 0, \pm1, \pm2, \cdots$ is a generalized stable process and there is $E\theta_t = E\xi_t = 0$ and covariance $d_{11} = E\theta_t^2, d_{12} = E\theta_t\xi_t, d_{22} = E\xi_t^2$, and they all satisfy the equations:

$$d_{11} = c_1^2 d_{11} + 1,$$

$$d_{12} = c_1(c_1 - c_2)d_{11} + c_1c_2d_{12} + 1,$$

$$d_{22} = (c_1 - c_2)^2 d_{11} + c_2^2 d_{22} + 2c_2(c_1 - c_2)d_{12} + 2$$

we can conclude that

$$d_{11} = \frac{1}{1 - c_1^2}, \quad d_{12} = \frac{1}{1 - c_1^2}, \quad d_{22} = \frac{2 - c_1^2 - c_2^2}{(1 - c_1^2)(1 - c_2^2)}$$

we can infer the initial conditions

$$m_0=\frac{d_{12}}{d_{22}}\xi_0=\frac{1-c_2^2}{2-c_1^2-c_2^2}\xi_0,$$

$$r_0=d_{11}-\frac{d_{12}^2}{d_{22}}=\frac{1}{1-c_1^2}-\frac{1-c_2^2}{(1-c_1^2)(2-c_1^2-c_2^2)}=\frac{1}{1-c_1^2-c_2^2}$$

So "the useful signal" θ_t can be determined by the equations (36) and(37)through the best linear estimation m_t and MSE r_t in the sense of mean square of $\xi_0,\cdots,\xi_t$. And this equation is based on the initial conditions

$$m_0=\frac{1-c_2^2}{2-c_1^2-c_2^2}\xi_0 \quad ,r_0=\frac{1}{2-c_1^2-c_2^2}$$

to solve the problem.

REFERENCES

[1] Phng H Y, Ruan L Z, Xiang J L. A note on boundary layer of a nonlinear evolution system with damping and difuusions. [J].J Math Anal Appl, 2015, 426:1099-1129

[2] Ruan L Z, Zhu C J. Boundary layer for nonlinear evolution equations with damping and diffusion[J]. Discrete Contin Dyn Syst Ser A, 2012 , 32:331-352

[3] Xue L. Well-posedness and zero micro-rotation viscosity limit of the 2D micropolar fluid equations[J]. Math Meth Appl Sci, 2011, 34:1760-1777

[4] Isaacs I M. Finite group theory[M]. Providence, Rhode Island: American Mathematical Society, 2011:1-354

[5] Mo jiaqi. Singularly perturbed asymptotic solutions for higher order semilinear elliptic equations with two parameters[J]. Chinese Annals of Mathematics, 2010, 33A(1): 331-336

[6] Yang Y, Yang Q. Singular value of nonnegative rectangular tensor[J]. Front Math China, 2011, 6(2):363-378

[7] Qiu Jinming, Zhang Li. F-interfere law genercetion and its feature recognition [J]. Journal of Systems Engineering and Electronics, 2009, 20 (4):777-783.

[8] Ren Y, Lu S P, Xia N M. Remarks on the existence and uniqueness of the solutions to stochastic funtional differential equations with infinite delay [J]. Comput Appl Math, 2008, 220: 364-372.

[9] Shi Kaiquan, Yao Bingxue. Funtion S-rough sets and las identification [J]. Scince in China Series F: Information Sciences, 2008, 51 (5): 499-510.

[10] Yuan C G, William G. Approximate solutions of stochastic differential delay equations with Markovian switching [J]. Comput Appl Math, 2006, 194:207-226.

[11] Frauenfelder P, Schwab C, Todor R A. Finite elements for elliptic problems with stochastic coeffieients [J]. Computer Methods in Applied Mechnics and Engineering, 2005, 194 (2/5): 205-228.

[12] Wang Q H. Statistical Estimation in Partial Linear Models with Covariate Data Missing at Random[J]. Annals of the institute of Statistical Mathematics, 2009, 61:47-84.

[13] Zhang T, Ge S S, Hang C C. Stable Adaptive Control for a Class of Nonlinear Systems Using a Modified Lyapunov Function [A]. In the proceedings of the 14th IFAC World Congress. Beijing: 1999: 373-378.

[14] Yu L, Chen G D, Chu J. Optimal Guaranteed Cost Control of Linear Uncertain Systems: LMI approach[J]. IFAC, 1999, G-2e-21-1:541-546

[15] Anderson T W. An Introduction to Multivariate Statistical Analysis [M]. New York: John Wiley & Sons, 1990, 232-296

[16] Yan H,Wei Q L.Determining compromise weights for group decision making[J].Journal of Operational Research Society, 2002(53):680—687.

[17] Xiao Xiaonan. Absolute Continuity and Equivalence of Measure to Stochastic Diffusion Process in a kind of Measurable Space [J]. Journal of Shanxi Normal University (Natural Science Edition), 1995, 23 (4): 21-24

[18] Xiao Xiaonan. Characteristics Score and Statistics Analysis of Functional Structure of Generalized Nonstationary Random Process [J]. Journal of Xiamen University (Natural Science), 2006, 45(5).

[19] Xiao Xiaonan. The Minimum Estimation of Corresponding Risk of Statistical Model [J]. Journal of Xiamen University (Natural Science), 2008, 47(5).

[20] Xiao Xiaonan. Optimum Operation and Optimum Analysis to Premit Coding Function of a Kind of Signal Transitive Stochastic System [J]. Journal of Xiamen University (Natural Science), 2009, 48(2).