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Algorithm of Sequential Improving the Size Coefficient for Solving the Problem of Partitioning the Multiple Connected Orthogonal Polygon

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***Abstract*—The problem of geometrical partitioning the multiple connected orthogonal polygon is considered in the given paper. The problem refers to the NP-hard class of problems because it is necessary to fulfill exhaustive search for assured finding the optimal solution. It stipulates the interest for developing efficient heuristic methods for solving the above problem. Some multiply-connected orthogonal polygon is supposed to be parted into a set of rectangles avoiding their intersecting and their crossing the polygon borders.**

The goal function represents certain minimization of summarized length of boundary junctions in the process of partitioning. The mathematical model of the problem is offered. The algorithm based on improving the size coefficient, characterizing the degree of rectangle elongation, has been developed. The algorithm consists of two procedures applied sequentially: the first one is for generating the primary polygon partitioning, and the second is for primary partitioning transformation taking into account the adjacent elements and the compound adjacent elements. The computing experiment has been carried out and the results are shown.

Keywords*—problem of geometrical partitioning, multiple connected orthogonal polygon, size coefficient, primary partition, composite united, minimization of joints length partition.

I. Preface

The partition problems refer to the class of discrete optimization problems. Actually, there are many problems based on the similar models, for example, scheduling cutting-packing problems, nesting and many others. The solving of such problems is aimed at resource saving. The problems of orthogonal packing into a quadrant [7,8], on sheets [8,10] and into a semi-endless strip [9,14,15,17] are of special importance. No less important is determining of the lower and upper boundaries in cutting-packing problems [8]. Besides, there are classes of regular and non-regular packing problems [11]. All the above mentioned problems are considered for 2D and 3D space [12, 13, 16]. The researchers in this field have been developing new approaches to solving different specific problems that arise in practice.

The partitioning problems have been known and are being researched to present day [1, 2]. In these problems it is required to divide some object into parts to reach definite aim. The partitioning problems are considered to be problems of rational use of resources, therefore the solving of the above problems results in real economic effect for business. The efficient solution to such problems is urgent from both theoretical and practical points of view. The partitioning problem can be considered as a separate problem as well as a sub-problem of some technological process [3, 4].

Technological processes in different applied branches of industry are known to use the stage of cutting or the stage of element placement taking into account their geometrical properties [3]. The process is very important for resource saving but it is difficult for decision making. The classification of the basic models for geometrical placement problems is given in papers [2, 5].

The process of creating a card for geometrical partitioning the multiple connected orthogonal polygon into rectangular domains is considered in the given paper.

II. The Problem of Geometrical Partitioning the Multiple Connected Orthogonal Polygon

Let us consider in greater detail the problem of geometrical partitioning the multiple connected orthogonal polygon (MOP), where it is required to divide some domain into minimum number of elements (rectangles). At that, the total length of boundary junctions of the elements can be considered as an important criterion of partitioning. When solving such problems, the existence of the so-called prohibited zones in the area to be divided is of great importance because the search for the algorithm depends on it. The proposed mathematical model of the problem of geometrical partitioning MOP is a modification of the sub-problem to the complex problem of geometrical covering and orthogonal cutting [3, 6].

III. Mathematical model of the problem of partitioning the multiple connected orthogonal polygon into rectangular elements.

Given: a MOP P presented by $< W, H, B >$, where W is the width of the ambient rectangle, and H is its height, $B = \{B_v\}$, $v = \overline{1, m}$ denotes the set of prohibited rectangular zones, $B_v = \{\chi_v, \eta_v, \omega_v, \lambda_v\}$, where χ_v, η_v are the bottom left corner coordinates in the MOP P coordinate system, ω_v is the width and λ_v is the height of the prohibited zone.

Fig.1 shows the example of MOP P.

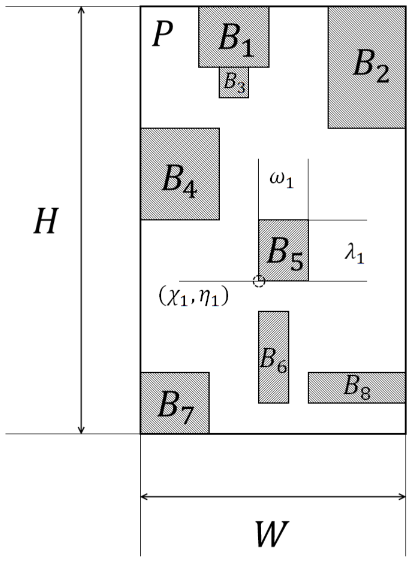

Fig. 1. The MOP P example with the mentioned mathematical notations

It is required to find a rectangular partitioning R, that is a set of rectangles $R = \{r_i\}$, $|R| = n$, $r_i = \{x_i, y_i, w_i, h_i\}$, where x_i, y_i are the bottom left corner coordinates in the MOP P coordinate system, w_i, h_i are the width and the height of the rectangle r_i correspondingly, so that :

$$\begin{cases} 0 \le x_i \le W \\ 0 \le x_i + w_i \le W \\ 0 \le y_i \le H \\ 0 \le y_i + h_i \le H \end{cases}, \ i = \overline{1, n} \tag{1}$$

$$\begin{gathered}(x_i + w_i \le x_j) \vee (y_i + h_i \le y_j) \vee \\ \vee (x_j + w_j \le x_i) \vee (y_j + h_j \le y_i), \\ i, j = \overline{1, n}, i \ne j\end{gathered} \tag{2}$$

$$\begin{gathered}(x_i + w_i \le \chi_v) \vee (\chi_v + \omega_v \le x_i) \vee \\ \vee (y_i + h_i \le \eta_v) \vee (\eta_v + \lambda_v \le y_i), \\ i = \overline{1, n}, v = \overline{1, m}\end{gathered} \tag{3}$$

$$\begin{gathered}\sum_{i=1}^{n} w_i \cdot h_i = W \cdot H - \sum_{v=1}^{m} \omega_v \cdot \lambda_v, \\ i = \overline{1, n}, v = \overline{1, m}\end{gathered} \tag{4}$$

The set of conditions (1) implies that the rectangle $r_i \in R$ must not overstep the bounds of the MOP P.

The set of conditions (2) denotes that the rectangle $r_i \in R$ must not intersect with any other rectangle $r_i \in R$.

The set of conditions (3) means that the rectangle $r_i \in R$ must not intersect with any prohibited zone $B_v \in B$.

The set of conditions (4) means that the rectangles from the set R must cover the whole inner area of the MOP excluding the prohibited zones. From this point on, the domain $W \cdot H - \sum_{v=1}^{m} \omega_v \cdot \lambda_v$ is called the area of the inner part of MOP or the useful area of MOP.

Fig. 2 shows the example of partition.

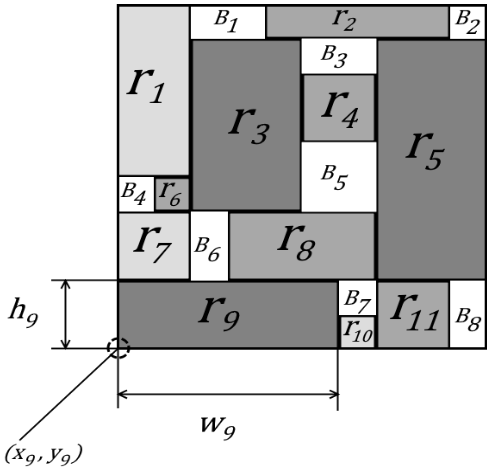

Fig. 2. The example of partitioning MOP with the mentioned mathematical notations

The objective function is aimed at minimizing the boundary junction length:

$$\sum_{i=1}^{n} w_i + h_i \to \min,$$

that is, the sum of the elements' half-perimeters in the resulting partition must tend to minimum.

It is worth considering that the objective function aimed at minimizing the summarized boundary junction length generally is not equivalent to the function of minimizing the number of rectangles in partitioning. But having modified the objective function, it is possible to gain the minimum boundary junction length for minimum number of the elements composing the partition.

IV. Algorithm for sequential improving the size coefficient (SISC)

For solving the problem of MOP geometrical partition we offer a two-step algorithm based on evaluation of the size coefficient for rectangular partition. The SISC algorithm creates a limited sorting out of the MOP partition.

First step implies the search for the initial partition of the MOP. To describe the procedure of initial partition it is necessary to define some additional mathematical notations.

We scrutinize some rectangle $F = \{x_f, y_f, w_f, h_f\}$. It is chosen to limit the searching area. We will describe its usage below.

Besides, we introduce a set of rectangles $E=\{e_k\}, e_k=\{x_k^e, y_k^e, w_k^e, h_k^e\}$, where x_k^e, y_k^e are coordinates of the rectangle's left bottom corner e_k in the MOP P coordinate system, w_k^e, h_k^e are the width and length of e_k rectangle correspondingly. It incorporates all elements of the set of prohibited zones B.

The chart in Fig. 3 shows the procedure of the initial partition.

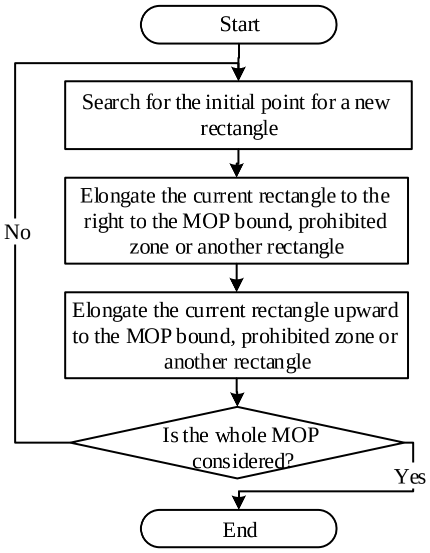

Fig. 3. The procedure of the initial partition

Step-by-step description of the initial MOP partition

1. $F = \{0, 0, W, 0\}$.
2. Find a rectangle r_i:
   2.1. C – is such a set of points that if $y_k^e = y_f + h_f$, then $x_k^e + w_k^e \in C; x_f \in C$.
   2.2. $x_i = x_z^e, y_i = y_f + h_f$;
   2.3. if $\exists e_k : \begin{cases} x_i < x_k^e < x_f + w_f \\ y_k^e = y_f + h_f \end{cases}$, then $w_i = x_k^e - x_i$, otherwise $w_i = W - x_i$.
   2.4. if $\exists e_k : \begin{cases} x_i < x_k^e < x_i + w_i \\ y_k^e > y_i \end{cases}$, then $h_i = y_k^e - y_i$, otherwise $h_i = H - y_i$.
   2.5. Add the found rectangle r_i to the set E.
   2.6. Delete the coordinate x_z^e from the set C.
   2.7. If $C = Ø$, then return to step 2.1.
   2.8. Otherwise
   2.8.1. If $\exists e_k : y_k^e + h_k^e > y_f$, choose a new rectangle F' so that: $\begin{cases} F' \in E, \\ y_{f'} + h_{f'} > y_f, \\ y_{f'} + h_{f'} \to \min \end{cases}$ and believe $F = F'$.
   2.8.1.1. Come back to step 2.1.
   2.8.2. Otherwise the end.

The second step of the SISC algorithm is a procedure of the initial partition reorganization taking into account the size coefficient. The size coefficient makes it possible to evaluate the ratio of the sides of a partition element.

The initial partition obtained as the result, most probably will not satisfy the objective function, therefore it is required to introduce some changes, reorganization, transformations into the MOP partition chart.

We introduce several mathematical notations.

The set $A_i = \{a_l^i\}$ of adjacent rectangular elements for the rectangle r_i is a set of such rectangles $\in R$, that one of the following conditions is fulfilled:

$$(x_i \geq x_l^i) \wedge (x_i + w_i \leq x_l^i + w_l^i) \wedge \wedge (y_l^i = y_i + h_i) \vee (y_i = y_l^i + h_l^i) \quad (5)$$

$$(y_i \geq y_l^i) \wedge (y_i + h_i \leq y_l^i + h_l^i) \wedge \wedge (x_l^i = x_i + w_i) \vee (x_i = x_l^i + w_l^i) \quad (6)$$

$$i = \overline{1, n}, \quad 1 \leq l \leq 4$$

The above conditions allow us to find rectangles which are candidates to be joined to the rectangle r_i. Condition (5) takes account of the situation when some candidate a_l^i is located above the rectangle r_i or under it; condition (6) does the same when a_l^i is on the left or right of the rectangle r_i.

We know that $A_i \subset R$, so when any element $a_l^i \in A_i$ changes, the corresponding element $r_j \in R, i, j = \overline{1,n}, i \neq j$ changes too. We also say that the rectangle a_l^i is adjacent to the rectangle r_i.

Then we introduce the size coefficient to define the ratio of the sides of partition elements $\gamma(r_i)$:

$$\gamma(r_i) = \begin{cases} \frac{w_i}{h_i}, & when\ w_i \leq h_i \\ \frac{h_i}{w_i}, & when\ w_i > h_i \end{cases}, i = \overline{1, n}$$

The size coefficient value is in the interval from 0 to 1.

Then we introduce another value ψ: it is defined by an expert, default value $\psi = 2$. This value is bound to the ratio of the sides and it allows us to neglect certain elements.

We introduce an operation of joining rectangular elements r_i and the rectangle a_l^i, $\theta(r_i, a_l^i)$:

- If a_l^i is located above r_i. that is $y_l^i = y_i + h_i$, then $h_i = h_i + h_l^i$ or, if a_l^i is under r_i, that is $y_l^i + h_l^i = y_i$, then $h_i = h_i + h_l^i$, $y_i = y_l^i$. Besides, if the corresponding left or right sides of rectangles r_i and a_l^i are on the same line, then the following is fulfilled:
$$\begin{cases} a_l^i \notin A_i, \mathit{when}\, x_l^i = x_i \wedge w_l^i = w_i \\ w_l^i = w_l^i - w_i, \mathit{when}\ x_l^i + w_l^i = x_i + w_l^i \wedge w_l^i > w_i \\ x_l^i = x_l^i + w_i, w_l^i = w_l^i - w_i, \mathit{when}\ x_l^i = x_i \wedge w_l^i > w_i \end{cases}$$
Otherwise a_l^i is divided into two parts.
- If a_l^i is located to the right of r_i, that is $x_l^i = x_i + w_i$, then $w_i = w_i + w_l^i$ or, if a_l^i is to the left of r_i, that is $x_l^i + w_l^i = x_i$, then $w_i = w_i + w_l^i$, $x_i = x_l^i$. Besides, if the corresponding top or bottom sides of the rectangles r, and a_l^i are on the same line then the following operations are fulfilled:
$$\begin{cases} a_l^i \notin A_i, \mathit{if}\ \ y_l^i = y_i \wedge h_l^i = h_i \\ h_l^i = h_l^i - h_i, \mathit{if}\ \ y_l^i + h_l^i = y_i + h_l^i \wedge h_l^i > h_i \\ y_l^i = y_l^i + h_i, h_l^i = h_l^i - h_i, \mathit{if}\ \ y_l^i = y_i \wedge h_l^i > h_i \end{cases}$$
Otherwise a_l^i is divided into two parts.
- $i = \overline{1, n}, 0 \le l \le 4$

The assembly $\gamma(\theta(r_i, a_l^i))$ denotes the ratio of the sides $\gamma(r_i)$, where r_i is the result of fulfilling $\theta(r_i, a_l^i)$. We denote the summarized half-perimeter of partitioning MOP $\sum_{i=1}^{n}(w_i + h_i)$ with the help of Π, and the summarized half-perimeter of partitioning after fulfilling $\theta(r_i, a_l^i)$ with the help of $\Pi(\theta(r_i, a_l^i))$.

The set $Q_i = \{q_g^i\}$ of compound adjacent rectangles for the rectangle r_i is a set of such rectangles $\in R$, that if joining procedure θ is applied to it, we get a rectangle $a_l'^i$ satisfying one of the conditions (5), (6), $i = \overline{1, n}, 0 \le l \le 4$, $1 \le g \le n$, $i \ne g$. The process of finding the set Q_i and posterior construction of the rectangle $a_l'^i$ we name as follows – *search for compound adjacent rectangles for the rectangle r_i.*

It is worth noting that the partition which goes after fulfillment of the current step of the algorithm is denoted as R'.

The chart in Fig. 4 shows the procedure of the initial partition transformation.

V. FORMAL PRESENTATION OF THE ALGORITHM

1. For $\forall\ r_i \in R$ find the set A_i;
2. If $\gamma(r_i) < \psi$ and $\exists\ A_i$: $A_i \ne \varnothing$ and $\exists a_l^i \in A_i$ are such as $\Pi(\theta(r_i, a_l^i)) < \Pi$ and $\Pi(\theta(r_i, a_l^i)) < \Pi(\theta(r_i, a_{l'}^i))$, $1 \le l \le 4$, $1 \le l' \le 4$, $l \ne l'$, then fulfil $\theta(r_i, a_l^i)$. If $R \ne R'$, then go back to point 1.
3. For $\forall r_i \in R$ find the set of compound adjacent rectangles Q_i. If $R \ne R'$, then go back to point 1, otherwise the end.

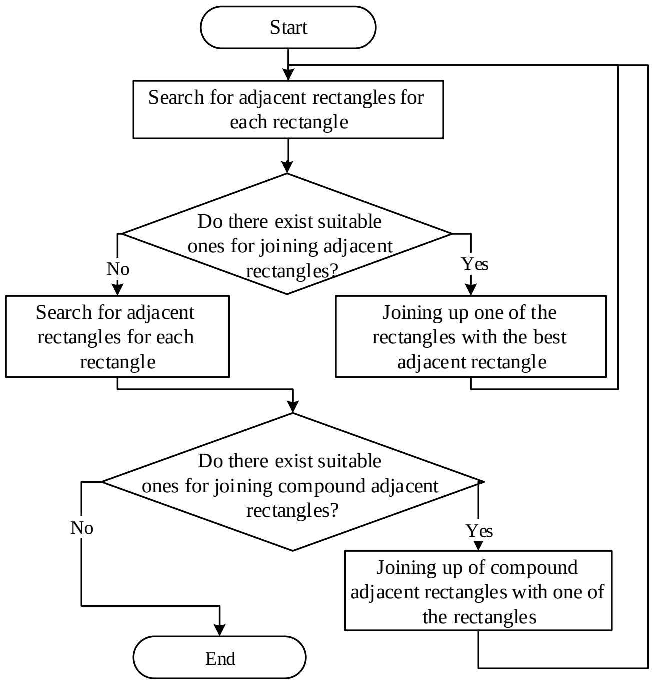

Fig. 4. The procedure of the initial partition transformation

Hence, the SISC algorithm is based on improving the size coefficient and sequential examining of vertexes of the initial MOP in one of the possible directions: horizontal or vertical. Thus, the offered solution to the problem of partitioning MOP consists of the following procedures:

1. Find the initial partition;
2. Transform the initial partition on the basis of the size coefficient:
   - taking into account the adjacent elements;
   - taking into account the compound adjacent elements.

To evaluate the algorithm we need to carry out a computing experiment that can help to analyze the initial partition procedure efficiency. It can also help to analyze the transformation procedure taking into account adjacent elements and compound adjacent elements as well. The software has been developed for this purpose. It is written in a high-level programming language C# in Microsoft Visual Studio 2010 development framework.

VI. COMPUTING EXPERIMENT

The computing experiment was carried out on different initial data sets, including those generated at random and those given by an expert. The described below classification of the initial data was introduced. The following notions were used in the process: 1-connected orthogonal polygon; multiple connected orthogonal polygon; prohibited zone quota. The prohibited zone quota is the ratio of their summarized area to the covering rectangle square of 1-connected orthogonal polygon or multiply connected orthogonal polygon, OOP or MOP correspondingly.

The following classes of input data have been singled out as a result in Table 1.

The computing experiment has been carried out on the PC with Intel® Core™ i5-4670K@3,40 GHz, RAM memory space 16 GB and OS Microsoft Windows 7 x64.

TABLE I. INPUT DATA CLASSES

Type of problem	**Type of polygon**	**Prohibited zone ratio**			**Number of examples**
		From	***To***	***Comment***	
1	1-connected orthogonal polygon	10%	20%	Small quota	50
2		21%	40%	Medium quota	50
3		41%	60%	Large quota	50
4		61%	80%	Very large quota	50
5	Multiply connected polygon	10%	20%	Small quota	50
6		21%	40%	Medium quota	50
7		41%	60%	Large quota	50
8		61%	80%	Very large quota	50

The given below tables show the results of the solving of mentioned examples with interim steps. Here are the names of all steps:

- *PP* (primary partition);
- *CU* (composite united);
- *MJLP* (minimization of joints length partition).

To evaluate the efficiency we use two notions:

- the number of elements to be partitioned;
- the summarized boundary junction length.

These notions are calculated as follows: a certain standard is created for the current output data. It is some partition that designedly does not satisfy the criterion of the boundary junction length minimization. For *PP*, *CU*, *MJLP* the notion of partition element number is equal to the ratio of the corresponding partition element number to the standard partition elements number. And the summarized boundary junction length equals the ratio of the corresponding summarized boundary junction length to the standard summarized boundary junction length. The more efficient is partitioning the less the notions become.

Experiment 1. In the given group of problems, 1-connected orthogonal polygons compose the area to be partitioned. Table 2 shows the information about the ratio of the number of elements to be partitioned.

TABLE II. INFORMATION ABOUT RATIO OF NUMBER OF ELEMENTS TO BE PARTITIONED FOR 1-CONNECTED ORTHOGONAL POLYGON

Partition step	**Number of elements to be partitioned**			
	Prohibited zone ratio			
	10%—20%	***21%—40%***	***41%—60%***	***61%—80%***
PP	0,2333804	0,279637	0,3108739	0,372728
CU	0,2233179	0,2705797	0,2946002	0,367728
MJLP	0,2284033	0,2755797	0,3014647	0,370728

Based on the above data (Table 2) we come to the conclusion that the *CU* step shows the best value of index but it gets worse when the prohibited zone ratio grows.

Table 3 shows the information about the summarized boundary junction length.

Based on the above data (Table 3) we come to the conclusion that the *MJLP* step shows the best value of index but it gets worse when the prohibited zone ratio grows.

TABLE III. INFORMATION ABOUT THE SUMMARIZED BOUNDARY JUNCTION LENGTH FOR 1-CONNECTED ORTHOGONAL POLYGON

Partition step	**Summarized boundary junction length**			
	Prohibited zone ratio			
	10%—20%	***21%—40%***	***41%—60%***	***61%—80%***
PP	0,5343825	0,5568235	0,6073746	0,6346959
CU	0,4315812	0,4582662	0,5288109	0,5708628
MJLP	0,4233203	0,4532662	0,5239638	0,5688628

Experiment 2. In the given group of problems multiple connected polygons are used as the area to be partitioned.

Table 4 shows the information about the number of elements to be partitioned.

TABLE IV. INFORMATION ABOUT THE NUMBER OF ELEMENTS TO BE PARTITIONED FOR MOP

Partition step	**Ratio of number of elements to be partitioned**			
	Prohibited zone ratio			
	10%—20%	***21%—40%***	***41%—60%***	***61%—80%***
PP	0,2172549	0,2805929	0,2859708	0,3344213
CU	0,2037513	0,2689948	0,275375	0,3314213
MJLP	0,2122434	0,2760033	0,2820821	0,3335178

Based on the above data (Table 4) we come to the conclusion that the *CURP* step shows the best value of index but it gets worse when the prohibited zone ratio grows.

Table 5 shows the information about the ratio of summarized boundary junction length.

Based on the above data (Table5) we come to the conclusion that the *MJLP* step shows the best value of index but it gets worse when the prohibited zone ratio grows.

TABLE V. INFORMATION ABOUT THE INDEX OF SUMMARIZED BOUNDARY JUNCTION LENGTH

Partition step	**Index of summarized boundary junction length**			
	Prohibited zone ratio			
	10%—20%	***21%—40%***	***41%—60%***	***61%—80%***
PP	0,5249747	0,5756768	0,5952789	0,65327
CU	0,4496428	0,5102312	0,5460388	0,6221639
MJLP	0,4322081	0,4987315	0,5355071	0,6165466

On average, one set of data has been calculated for no longer than one minute. It is worth noting that the growth of problem dimension brings about the growth of working time.

VII. Summary and Conclusion

With regard to the computing experiment results we have drawn several conclusions about the efficiency of the offered way to solve the problem of geometrical partitioning MOP.

Firstly, when the quota of prohibited zones grows, the indexes of the number of elements to be partitioned and those of the summarized boundary junction length grow too. At the same time, for the steps *PP*, *CU* and *MJLP* the gap between their coefficients shrinks as well. It is explained by the fact that the more is the prohibited zone ratio the less freely the partitioning can go on.

Secondly, minimization of the summarized boundary junction length is not equal to minimization of the partition element number. Thus, trying to minimize the summarized boundary junction length we have to sacrifice the partition element number. Though certain compromise can be achieved, when necessary, and the experiment results show it as well.

On the whole, taking into account the results of the experiment we can recommend the *MJLP* modification to minimize the summarized boundary junction length. The procedure of *CU* can be the best choice when some compromise is needed between the summarized boundary junction length and the partition element number.

Besides, the described above algorithm can be modified. In particular, MOP can undergo some orthogonal rotation (90°, 180°, 270°) before the algorithm for the primary partitioning is applied. Another variant of modification is as follows: introduce such changes into the algorithm that the rectangle elongates to the right and upward simultaneously, that is, diagonally. Hence, the partitioning may consist of four-squares only. But they can outnumber the rectangles for the same area to be partitioned.

In summary the computing experiment proves that the algorithm SISC is quite advantageous to be used. And it is worth noting, there is much prospect in modification of the algorithm to gain an improved plan of MOP partition with regard to real usefulness of one of the efficiency index.

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References

[1] J. O'Rourke, G. Tewari "The structure of optimal partitions of orthogonal polygons into fat rectangles," in Computational Geometry: Theory and Applications, vol. 28, 2004, pp. 49–71.

[2] A. S. Filippova, J. I. Valiakhmetova "Optimal use of resources: cutting-packing problems." in: Petrosyan, L. A., Romanovsky, J. V., Wing-kay Yeung, D. Advances in economics and optimization: collected scientific studies dedicated to the memory of L. V. Kantorovich. Nova Science Publishers, Inc. NY, United States of America, 2014, pp. 35-48.

[3] J. I. Valiahmetova, E. I. Hasanova, A. S. Filippova "Some approaches to solve a complex problem of Geometrical covering and orthogonal cutting" in Vestnik UGATU (scientific journal of Ufa State Aviation Technical University), vol. 17, no. 6 (59), 2013, pp. 88-91.

[4] K. S. Kulga, P. V. Menshikov "Optimization of a geometrical covering of the multicoherent orthogonal ground with boundary obstacles taking into account design-technology restrictions" (in Russian) in Vestnik RGTRU (Vestnik of Ryazan State Radio Engineering University), no. 4 (vol. 50), part 2, 2014, pp. 75-82.

[5] E. A. Mukhacheva et al. "Methods of local search in tasks of orthogonal cutting and packaging: review and prospects of development" (in Russian) in New Technologies. Information Technologies, no. 5, annex, Moscow, 2004, pp. 2-17.

[6] S. Muthukrishnan, V. Poosala, T. Suel "On rectangular partitionings in two dimensions: Algorithms, complexity and applications." in Database Theory – ICDT 99, vol. 1540 of Lecture Notes in Computer Science, Springer Berlin Heidelberg, 2002, pp. 236–256.

[7] L. Wei, W. Zhu, A. Lim, Q. Liu, X. Chen "An adaptive selection approach for the 2D rectangle packing area minimization problem." in Omega, 80, 2018, pp. 22-30.

[8] Y. Zhang, F.Y.L. Chin, H.-F. Ting, X. Han, C.K. Poon, Y.H. Tsin, D. Ye "Online algorithms for 1-space bounded 2-dimensional bin packing and square packing." in Theoretical Computer Science, 554, 2014, pp. 135–149.

[9] S. Imahori, M. Yagiura "The best-fit heuristic for the rectangular strip packing problem: An efficient implementation and the worst-case approximation ratio." in Computers & Operations Research, 37, 2010, pp. 325-333.

[10] E. Silva, J. F. Oliveira, G. Wäscher "The pallet loading problem: a review of solution methods and computational experiments." in International Transactions in Operational Research, 23, 2016, pp. 147–172.

[11] A. K. Sato, M.S.G. Tsuzuki, T.C. Martins, A.M. Gomes "Multiresolution based overlap minimization algorithm for irregular packing problems" in IFAC-PapersOnLine, 48-3, 2015, pp. 384–489.

[12] C. Paquay, M. Schyns, S. Limbourg "A mixed integer programming formulation for the three-dimensional bin packing problem deriving from an air cargo application" in International Transactions in Operational Research, 23, 2016, pp. 187–213.

[13] I. Araya, K.Guerrero, E. Nuñez "VCS: A new heuristic function for selecting boxes in the single container loading problem" in Computers and Operations Research, 82, 2017, pp. 27–35.

[14] J. Kallrath, S. Rebennack, J. Kallrath, R. Kusche "Solving real-world cutting stock-problems in the paper industry: Mathematical approaches, experience and challenges" in European Journal of Operational Research, 238, 2014, pp. 374–389.

[15] A. Bouaine, M. Lebbar, M.A. Ha "Minimization of the wood wastes for an industry of furnishing: a two dimensional cutting stock problem" in Management and Production Engineering Review, vol. 9, N. 2, 2018, pp. 42-51.

[16] T.A.M. Toffolo, E. Esprit, T. Wauters, G.V. Berghe "A two-dimensional heuristic decomposition approach to a three-dimensional multiple container loading problem" in European Journal of Operational Research, 257, 2017, pp. 526–538.

[17] M. Delormea, M. Iorib, S. Martello "Logic based Benders' decomposition for orthogonal stock cutting problems" in Computers & Operations Research, 78, 2017, pp. 290–298.