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Decision-Making Based on Multi-Attribute Value Theory Under Preference Uncertainty

Vladislav Shakirov
Faculty of Power Engineering and Automation
Bratsk State University
Bratsk, Russia
mynovember@mail.ru

***Abstract*—This paper advances the multi-attribute value theory to take into account the uncertainty of the decision-maker's preferences with respect to the value of assessments and importance of attributes. It considers the basic steps of the original method of multi-attribute value theory against the modified one. The modified method can use the decision-maker's responses as fuzzy triangular numbers to construct single-attribute value functions and to find the scaling coefficients. The paper presents an approach to finding fuzzy scaling coefficients by comparing paired matches and generating a system of linear equations with fuzzy coefficients. It is proposed to solve the system of equations by representing it as a set of systems of interval equations derived by splitting fuzzy sets by α levels. Multi-attribute fuzzy assessment of alternatives is carried out by solving linear programming problems based on systems of equations pertaining to individual α levels. The paper provides an example of using the modified method for multi-attribute comparison of alternatives.**

Keywords—decision-making, value theory, fuzzy number, interval

I. Introduction

Multi-attribute utility theory and multi-attribute value theory (MAVT) have long been used to support decision-making on numerous systemic problems in power engineering, healthcare, transport security, urban infrastructure [1, 2, 3], emergency response [4], and environmental protection [5].

Researchers have modified MAVT in various ways to make decisions on the basis of incomplete or inaccurate data. The modifications were purposed to enable the decision-maker (DM) to use fuzzy data so as to represent their preferences. What necessitates this ability is the fact that making decisions on complex systemic problems is associated with high uncertainty in the input as well as in the possible future conditions, coupled with the need to account for the interests of various stakeholders. This makes it difficult for the DM to represent their preferences accurately [6, 7]. The classical tool (analyzing the sensitivity of alternative rankings to changes in the inputs or in the preferences) cannot cover all the possible changes.

One modification of this method enabled using fuzzy attribute weights and fuzzy values of attribute-based assessments within the FMAVT. That method employed ordinary single-attribute value functions [8, 9]. A further modification enabled using fuzzy single-attribute value functions as part of the FFMAVT [10, 11]. Using fuzzy single-attribute value functions complicates the problem of finding the scaling coefficients and multi-attribute values. This paper considers the steps of a modified method as well as approaches to implementing such steps.

II. Multi-Attribute Value Theory Method Steps

Consider the basic steps of the original MAVT [1, 2], see Fig. 1.

Step 1 is for the DM to state the principal objective and the composite objectives to be pursued when solving the problem, as well as the set of alternatives. Then generate the attributes to assess the extent of objective attainment when comparing the alternatives.

Step 2 is to collect data for attribute-based assessment of alternatives involving objective or subjective models.

Step 3 is to verify the conditions of MAVT applicability [1, 2]. Meeting the conditions of mutual independence of attributes by preference enables deriving the value function as an additive function [1]:

$$v(y) = v(y_1, y_2, ..., y_n) = \sum_{i=1}^{n} k_i v_i(y_i), \qquad (1)$$

where $v_i(y_i)$ is a single-attribute value function; y_i is the evaluation of an alternative by the i-th attribute; k_i is the scaling coefficient of the i-th attribute, $\Sigma k_i = 1$; n is the number of criteria.

Step 4 is to construct single-attribute value functions on several reference points, see Fig. 2 [1, 2]. The first step here is to normalize, i.e. to assign the value $v_i(y_i^1)$ =1 to the best assessment by the i-th attribute y_i^1, and to assign the value $v_i(y_i^0)$ =0 to the worst one y_i^0. Then involve another attribute j to find a point that has subjectively average value within y_i^0 to y_i^1. This assessment will have a value of 0.5. Similarly, find points of value 0.25, 0.75. These five points will usually suffice to construct a value function.

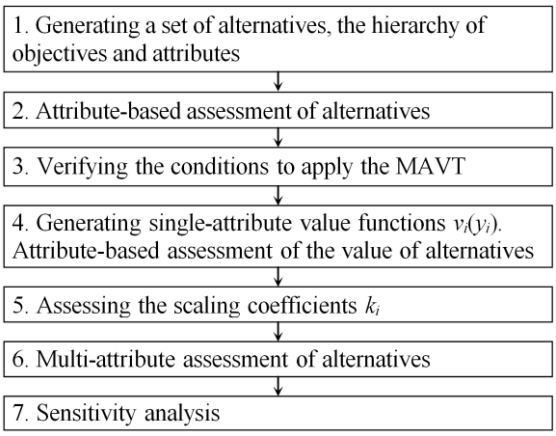

Fig. 1. Multi-attribute value theory method steps

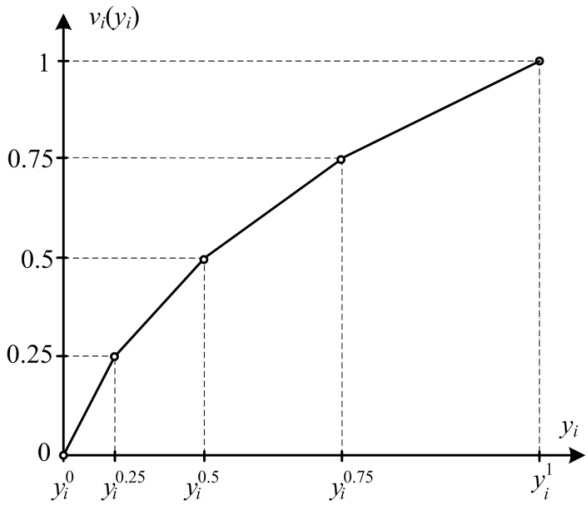

Fig. 2. Single-attribute value function

Note that when constructing a value function, finding the value-average point might be difficult for the DM [1, 2]. A person doing an analysis can make mistakes or answer inconsistently [12]. When constructing a function, a decision maker may have uncertainty regarding the value of criterion assessments due to several possible scenarios for the development of the problem being solved, the presence of several opinions in group decision making and other factors. To overcome the difficulties of assigning accurate estimates when constructing value functions, it is advisable to give the DM the opportunity to specify the estimates inaccurately, using intervals of possible deviations of estimates or fuzzy numbers [10, 11].

Step 5 in Fig. 1 shows two-step assessment of the scaling coefficients k_i. The first step is to order the attributes by significance. To that end, record the worst assessment by each attribute and ask the DM to order the assessment by priority of improvement. The second step is to generate a system of linear equations with scaling coefficients evaluated by comparing the synthetic alternatives of equal value.

To make the analysis simpler for the DM, generate equivalent alternatives that would differ in assessment by only two attributes: *i*, *j*. Assessment of the other criteria are fixed at the worst level, which makes it possible to exclude them from expression (1). For example, for the decision maker, two alternatives *A*, *B* are formed. Alternative *A* has the best assessment y_i^1 by the *i*-th criterion and the worst assessment y_j^0 by the *j*-th criterion. Alternative *B* has the worst assessment y_i^0 by the *i*-th criterion and the best assessment y_j^1 by the *j*-th criterion.

If the DM decides the alternative *A* is preferable, then the *i*-th attribute is more important. Then the DM must identify such alternative *C* which has the worst assessment y_j^0 in terms of the less important attribute and has an assessment y_i^c in terms of the more important attribute; this alternative must be equivalent to the alternative *B* that lost the preceding contest (Fig. 3). This response can be used to make an equation based on (1):

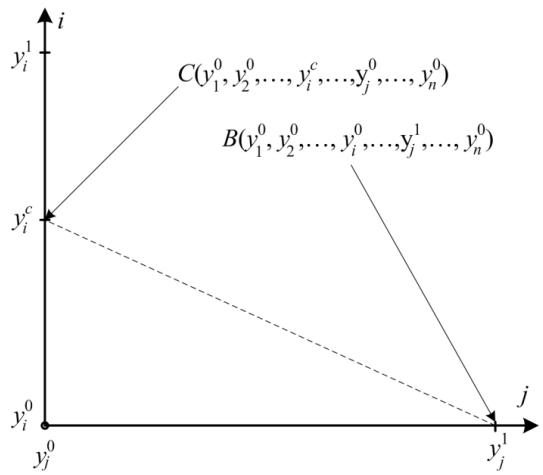

Fig. 3. Standard procedure to find equivalent alternatives to compute the scaling coefficients

$$k_i v_i(y_i^c) = k_j \,. \tag{2}$$

Given that the sum of scaling coefficients must be 1, it suffices to make *n*-1 such equations.

Given the preference uncertainty, it might be difficult for the DM to point out the equivalent alternatives. This is why it is advisable to let the DM make the assessment y_i^c in terms of a fuzzy number or an interval [10, 11].

Step 6 (Fig. 1) is to assess the alternatives by means of the generated multi-attribute value function (1).

Step 7 (Fig. 1) is to analyze the sensitivity of these multi-attribute assessments to changes in the inputs or in the DM's preferences [1, 2]. Analyze the alterations in the ranking of the alternatives resultant from varying the scaling coefficients, the assessment of alternatives, their values, or sundry parameters. The alternatives that stably rank high must be deemed most preferred.

III. MAVT Modification to Handle DM Preference Uncertainty

As noted above when describing MAVT steps 4 and 5 (Fig. 1), it is advisable to enable the DM to describe his preferences by fuzzy numbers. Consider the basic changes in the method that could give such an option.

Consider the proposed procedure of constructing a fuzzy single-attribute value function (FSAVF) using triangular fuzzy numbers. Further, the following denotation is used for attribute-based assessments *y* and the values of assessments *v*. The superscript *W* corresponds to the best assessment, while *B* corresponds to the worst one. The superscript *R* denotes the right boundary of value (the higher value). The superscript *L* denotes the left boundary of value (the lower value). The superscript *C* corresponds to the core of fuzzy assessment.

Building FSAVF is also carried out on five reference points. After normalization, find the mid-value point $y_i^{0.5C}$ as well as the possible boundary values $y_i^{0.5W}$, $y_i^{0.5B}$ within [y_i^0, y_i^1]. The DM believes that altering the attribute-based assessment result from y_i^0 to $y_i^{0.5C}$ is equivalent to alteration from $y_i^{0.5C}$ to y_i^1 while not excluding changing the equivalent-value point from $y_i^{0.5W}$ to $y_i^{0.5B}$. Similarly, find the mid-value point within [y_i^0, $y_i^{0.5C}$] to find, respectively, $y_i^{0.25W}$, $y_i^{0.25C}$, $y_i^{0.25B}$, as well as the mid-value point within [$y_i^{0.5C}$, y_i^1] to find, respectively, $y_i^{0.75W}$, $y_i^{0.75C}$, $y_i^{0.75B}$. Fig. 4 presents an example of constructing a fuzzy single-attribute value function.

The FSAVF gives attribute-based assessments a fuzzy value. To find the fuzzy value by these functions, there are procedures for finding the maps in case of fuzzy mapping of crisp and fuzzy attribute-based assessments [10].

The membership functions of the maps for the case of fuzzy mapping of a crisp or fuzzy assessment will insignificantly differ from the triangular functions [10], therefore the following expressions can be used. In case the attribute-based assessment is crisp y_i^k:

$$\tilde{v}_i(y_i^k) = f_\Delta(v_i, v_{i0}^L(y^k), v_{i1}^C(y^k), v_{i0}^R(y^k)). \quad (3)$$

In case the attribute-based assessment is a triangular fuzzy number $\tilde{y}_i = f_\Delta(y_i, y_i^W, y_i^C, y_i^B)$:

$$\tilde{v}_i(\tilde{y}_i) = f_\Delta(v_i, v_{i0}^L(y_i^W), v_{i1}^C(y_i^C), v_{i0}^R(y_i^B)). \quad (4)$$

A special case of the fuzzy value function is the interval value function. In contrast to the FSAVF, on the interval from [y_i^0, y_i^1] only the boundaries of the average value point [$y_i^{0.5W}$, $y_i^{0.5B}$] are defined. Estimates within the interval [$y_i^{0.5W}$, $y_i^{0.5B}$] have the same value for the decision maker, equal to 0.5, and the same degree of belonging μ. The boundaries of the other average values of the points [$y_i^{0.25W}$, $y_i^{0.25B}$] are defined on the interval [y_i^0, $0.5(y_i^{0.5W} + y_i^{0.5B})$], and [$y_i^{0.75W}$, $y_i^{0.75B}$] – on the interval [$0.5(y_i^{0.5W} + y_i^{0.5B})$, y_i^1] respectively.

The interval value of an alternative with a crisp assessment at interval mapping:

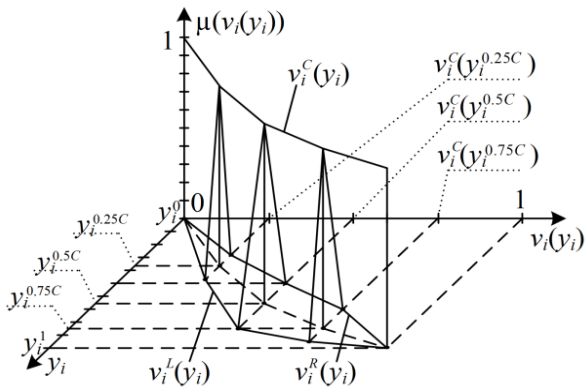

Fig. 4. FSAVF comprising triangular membership functions

$$\bar{v}_i(y_i^k) = [v_i^L(y_i^k), v_i^R(y_i^k)]. \quad (5)$$

The interval value of an alternative with an interval assessment at interval mapping:

$$\bar{v}_i(\bar{y}_i^k) = [v_i^L(y_i^W), v_i^R(y_i^B)]. \quad (6)$$

The DM can also use fuzzy assessments in his responses when finding the scaling coefficients for Step 5 (Fig. 1). Fig. 5 presents the procedure of finding an alternative *A* that is equivalent to the alternative *B*; the DM gives a fuzzy assessment by the *i*-th attribute.

Resultant is the equation:

$$\tilde{k}_j = \tilde{k}_i \tilde{v}_i(\tilde{y}_i^j), \quad (7)$$

where $\tilde{k}_i$, $\tilde{k}_j$ are fuzzy scaling coefficients; $\tilde{y}_i^j$ is the DM's assessment by the attribute *i* obtained by comparison against the attribute *j*; $\tilde{v}_i(\tilde{y}_i^j)$ is the map obtained by the fuzzy mapping of a fuzzy attribute-based assessment, as found per (4).

Equations like (7) for *n*-1 pairs of attributes can be used to find the values of fuzzy scaling coefficients given that their sum must equal 1.

To simplify the solution of the system, the most important attribute is assigned the index 1; this attribute is then used as reference, against which other attributes are compared. The result is a system of equations:

$$\begin{cases} \tilde{k}_2 = \tilde{k}_1 \tilde{v}_1(\tilde{y}_i^2), \\ \tilde{k}_3 = \tilde{k}_1 \tilde{v}_1(\tilde{y}_1^3), \\ \ldots, \\ \tilde{k}_n = \tilde{k}_1 \tilde{v}_1(\tilde{y}_1^n), \\ \sum_{i=1}^{n} \tilde{k}_i = 1. \end{cases} \quad (8)$$

The specific feature of solving equations with fuzzy numbers is that the distribution law generally does not hold; there are no reciprocal or opposite numbers [13]. The first approaches to solving fuzzy equations consisted in implementing additional fuzzy-number subtractions and divisions [14]. Analysis of the applicability of classical approaches based on the principle of extension [13] shows it is often impossible to find a solution [15]. Widespread are approaches based on representing a system of fuzzy linear equations as a set of systems of interval equations derived by splitting fuzzy sets by α levels [15, 16].

The α-level set of the fuzzy set A in X is a set consisting of the elements $x \in X$, whose degree of membership in the fuzzy set A is no less than α [17].

$$A_\alpha = \{x | x \in X, \mu_A(x) \geq \alpha\} \quad (9)$$

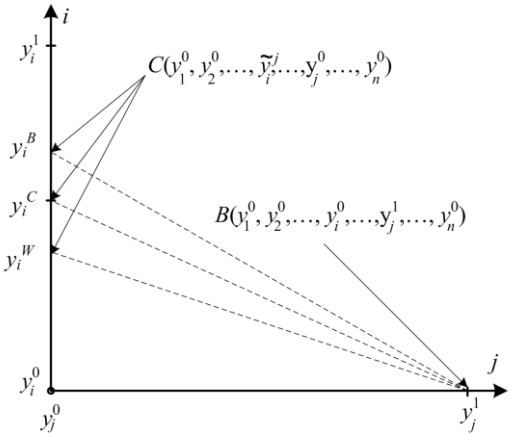

Fig. 5. Fuzzy matching of equivalent alternatives

This can produce a set of interval systems of linear algebraic equations (ISLAE):

$$\begin{cases} \bar{k}_{2,\alpha} = \bar{k}_{1,\alpha} \bar{v}_{1,\alpha}(\bar{y}_{1,\alpha}^2), \\ \bar{k}_{3,\alpha} = \bar{k}_{1,\alpha} \bar{v}_{1,\alpha}(\bar{y}_{1,\alpha}^3), \\ \ldots, \\ \bar{k}_{n,\alpha} = \bar{k}_{1,\alpha} \bar{v}_{1,\alpha}(\bar{y}_{1,\alpha}^n), \\ \sum_{i=1}^{n} \bar{k}_{i,\alpha} = 1. \end{cases}, \quad (10)$$

where $\alpha \in [0;1]$.

To compute and describe the set of the ISLAE solutions directly is usually time-consuming, often impossible [18]. The set of solutions can be of a complex structure [18]. The practical implication here is that one must inevitably describe the set of solutions rather approximately, i.e. replace the sets of solutions with simpler sets (approximations of a lower descriptive complexity) [18]. In the modern interval analysis, the two main assessment methods are inner and outer estimation [18].

For a multi-attribute assessment of alternatives in the context of the DM's preference uncertainty, it is an outer estimation of the set of the ISLAE solutions that is of interest. There have been devised numerous methods named after Gauss, Gauss-Seidel, Krawczyk, Hansen-Bliek-Rohn, as well as a formal approach [18-20]. An outer estimation of the set of the ISLAE solutions helps specify the intervals, within which the scaling coefficients might be adjusted given the DM preference uncertainty; it further enables the researcher to derive fuzzy scaling coefficients for the multi-attribute value functions. Notably, an outer ISLAE estimation is easiest to find when it is of a special form [18]. The form of the system of equations (10) also enables an outer estimation of the set of the ISLAE solutions without use of special methods. One approach is covered in [11]. However, one can apply a different approach by using the system (10) as a basis for formulating the linear programming constraints.

Thus, to find the minimum and maximum multi-attribute value of an alternative given the preference uncertainty, one can generate two linear optimization problems:

$$v_\alpha^L(y_\alpha) = \sum_{i=1}^{n} v_{i,\alpha}^L(y_{i,\alpha}^W) k_{i,\alpha} \to min, \quad (11)$$

$$v_\alpha^R(y_\alpha) = \sum_{i=1}^{n} v_{i,\alpha}^R(y_{i,\alpha}^B) k_{i,\alpha} \to max, \quad (12)$$

with constraints:

$$\begin{cases} v_{2,\alpha}^L(y_{2,\alpha}^W) k_{1,\alpha} - k_{2,\alpha} \leq 0, \\ v_{2,\alpha}^R(y_{2,\alpha}^B) k_{1,\alpha} - k_{2,\alpha} \geq 0, \\ v_{3,\alpha}^L(y_{3,\alpha}^W) k_{1,\alpha} - k_{3,\alpha} \leq 0, \\ v_{3,\alpha}^R(y_{3,\alpha}^B) k_{1,\alpha} - k_{3,\alpha} \geq 0, \\ \ldots \\ v_{n,\alpha}^L(y_{n,\alpha}^W) k_{1,\alpha} - k_{n,\alpha} \leq 0, \\ v_{n,\alpha}^R(y_{n,\alpha}^B) k_{1,\alpha} - k_{n,\alpha} \geq 0, \\ \sum_{i=1}^{n} k_{i,\alpha} = 1. \end{cases}. \quad (13)$$

These problems are solvable by any known linear programming method, e.g. by the simplex method. As a result of solving two optimization problems, one can obtain interval multi-attribute assessment of the value of each alternative:

$$\bar{v}_\alpha(\bar{y}_\alpha) = [v_\alpha^L(y_\alpha^W), v_\alpha^R(y_\alpha^B)]. \quad (14)$$

Step 6 (Fig. 1) combines the interval assessments obtained at α levels to generate multi-attribute fuzzy assessments of alternatives $\tilde{v}(\tilde{y})$.

In case of using fuzzy single-attribute value functions and fuzzy scaling coefficients, analyzing the sensitivity to changes in the DM's preferences (Step 7, Fig. 1) is not necessary. The fuzzy multi-attribute assessments of alternatives obtained by means of (14) fully reflect the possible changes in the ranking of alternatives.

IV. Application example of the modified MAVT method

Consider a hypothetical example to separate the individual steps of a multi-attribute assessment of alternatives in the context of the DM preference uncertainty.

Let the DM be tasked with a three-attribute assessment of a set of alternatives. Table 1 presents the assessment data. As a result of verifying the MAVT application conditions, the value function has been found to be additive.

TABLE I. Data on the assessments of alternatives by criteria

Attribute	Assessed worst	Assessed best
*K*1	50	150
*K*2	100	400
*K*3	150	900

Following Fig. 1, Step 4 is to construct single-attribute value functions. Consider the procedure of constructing a fuzzy single-attribute value function for the first attribute.

Construct the FSAVF upon five reference points. The worst assessment by the first attribute y_1^0 corresponds to the value $v_1(y_1^0) = 0$, while the best assessment y_1^1 corresponds to the value $v_1(y_1^1) = 1$. Then find the mid-value point $y_1^{0.5C}$ as well as the possible boundary values $y_1^{0.5W}$, $y_1^{0.5B}$. Ask the DM the question: " Suppose there is an alternative with an assessment according to the first criterion of 50. Which change of assessment would you value more: from 50 to 100 or from 100 to 150?" Suppose the DM prefers the former. Repeat the question until two equivalent changes have been found. For instance, let the DM equally value changes in assessment from 50 to 80 and from 80 to 150. Then ask the DM to propose expanding the boundaries of the assessment $y_1^{0.5C} = 80$ to obtain the assessments $y_1^{0.5W}$, $y_1^{0.5B}$. These assessments reflect the DM's uncertainty when it comes to expressing the accurate assessment $y_1^{0.5C} = 80$. E.g. $y_1^{0.5W} = 74$; $y_1^{0.5B} = 86$. Then ask the DM to set a point of equal value within [50, 80] to find $y_1^{0.25W}, y_1^{0.25C}, y_1^{0.25B}$, as well as within [80, 150] to find $y_1^{0.75W}, y_1^{0.75C}, y_1^{0.75B}$. After a few procedures of finding such points of equal value, the number of questions could be reduced, i.e. the DM could be offered to make fuzzy assessments without extra questions. Let the questioning produce the following assessments: $y_1^{0.25W} = 55$; $y_1^{0.25C} = 60$; $y_1^{0.25B} = 65$; $y_1^{0.75W} = 105$; $y_1^{0.75C} = 110$; $y_1^{0.75W} = 115$. Fig. 6 presents a FSAVF for the attribute K1. Similarly construct FSAVF for *K*2 and *K*3.

Step 5 in Fig. 1 is to find the scaling coefficients. Suppose that as a result of the survey it was established that the *K*1 criterion is more important than the *K*2 criterion, the *K*2 criterion is more important than the *K*3 criterion. Thus, *K*1 is the reference attribute for the pairwise comparison of alternatives. The DM is asked to compare alternatives in synthetic pairs. As a result of the survey, equivalent alternatives were established, see Fig. 7, 8.

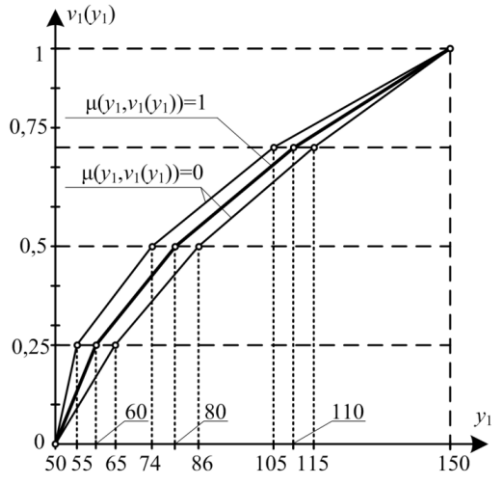

Fig. 6. Fuzzy single-attribute value function of the *K*1 criterion

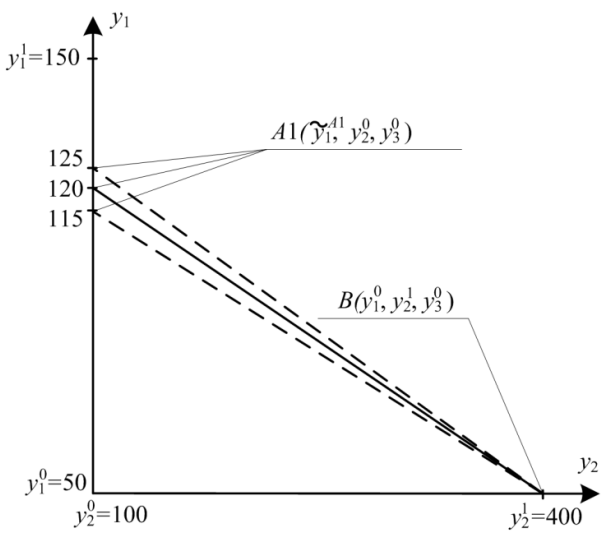

Fig. 7. The procedure for searching for equivalent alternatives differing in their assessments according to the criteria of *K*1 and *K*2

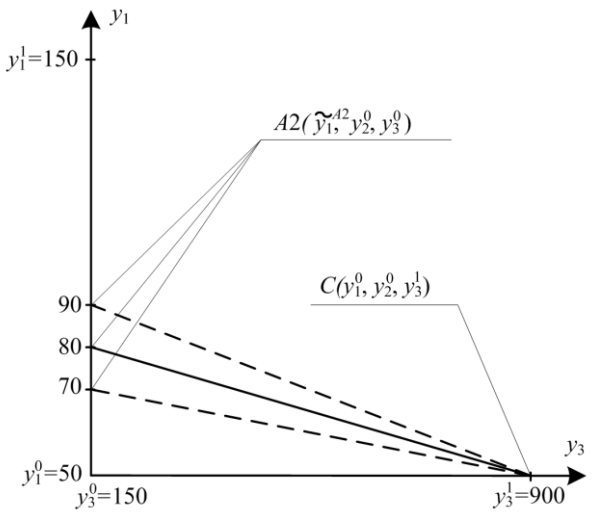

Fig. 8. The procedure for searching for equivalent alternatives differing in their assessments according to the criteria of *K*1 and *K*3

These responses can be used to make a system of equations based on (5):

$$\begin{cases} \tilde{k}_2 = \tilde{k}_1 \tilde{v}_1(\tilde{y}_i^{A1}), \\ \tilde{k}_3 = \tilde{k}_1 \tilde{v}_1(\tilde{y}_1^{A2}), . \\ \tilde{k}_1 + \tilde{k}_2 + \tilde{k}_3 = 1 \end{cases} \quad (15)$$

Write the resultant system of fuzzy linear equations as a set of systems of interval equations derived by splitting fuzzy sets by α levels:

$$\begin{cases} \bar{k}_{2,\alpha} = \bar{k}_{1,\alpha} \bar{v}_{1,\alpha}(\bar{y}_{1,\alpha}^{A1}), \\ \bar{k}_{3,\alpha} = \bar{k}_{1,\alpha} \bar{v}_{1,\alpha}(\bar{y}_{1,\alpha}^{A2}), , \\ \bar{k}_{1,\alpha} + \bar{k}_{2,\alpha} + \bar{k}_{3,\alpha} = 1 \end{cases} \quad (16)$$

where $\alpha \in [0;1]$.

Further, as an example, we will compare the two alternatives *D*, *E* with the assessments according to the criteria presented in Table 2.

We will carry out the analysis for the level α = 0. Find the interval value of the interval assessments $\bar{y}_{1,0}^{A1}$, $\bar{y}_{1,0}^{A2}$ in the system of interval equations (16) (Fig. 9).

We represent the system (16) as:

$$\begin{cases} \bar{k}_{2,0} = \bar{k}_{1,0}[0.75, 0.86], \\ \bar{k}_{3,0} = \bar{k}_{1,0}[0.31, 0.63], \\ \bar{k}_{1,0} + \bar{k}_{2,0} + \bar{k}_{3,0} = 1 \end{cases} \quad (17)$$

Define the maps with the interval mapping of crisp criterial assessments of alternatives *D*, *E* using expression (5) (table 3).

Generate two linear optimization problems based on (11), (12), (13).

TABLE II. DATA ON THE ASSASSMENTS OF ALTERNATIVES *D*, *E*

Alternative	Assessments by criteria		
	K1	***K2***	***K3***
D	75	250	400
E	60	350	500

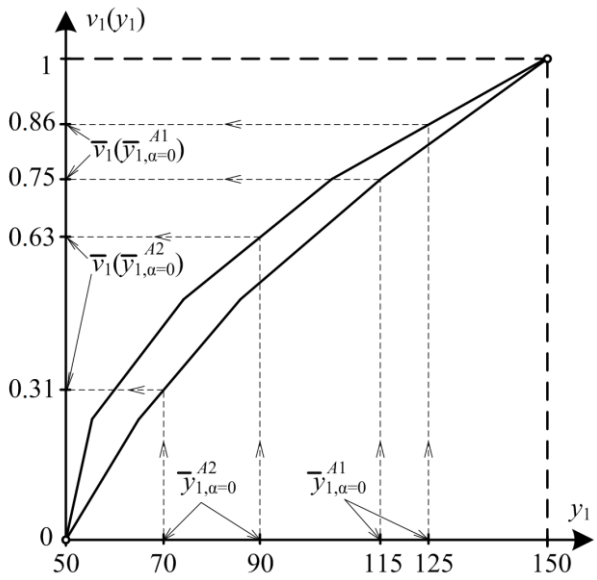

Fig. 9. Finding the interval value of the interval assessments of alternatives

TABLE III. VALUES OF ALTERNATIVES *D*, *E* BY CRITERIA

Alternative	Value of alternatives by criteria		
	K1	***K2***	***K3***
D	[0.367, 0.505]	[0.435, 0.558]	[0.223, 0.296]
E	[0.169, 0.316]	[0.788, 0.860]	[0.325, 0.443]

$$v_{D,0}^L(y_D) = \sum_{i=1}^{3} v_{i,0}^L(y_{Di}) k_{i,0} \to min, \quad (18)$$

$$v_{D,0}^R(y_D) = \sum_{i=1}^{3} v_{i,0}^R(y_{Di}) k_{i,0} \to max, \quad (19)$$

with constraints:

$$\begin{cases} 0.75k_{1,0} - k_{2,0} \le 0 \\ 0.86k_{1,0} - k_{2,0} \ge 0, \\ 0.31k_{1,0} - k_{3,0} \le 0 \\ 0.63k_{1,0} - k_{3,0} \ge 0, \\ k_{1,0} + k_{2,0} + k_{3,0} = 1. \end{cases} \quad (20)$$

As a result of solving the problems (18), (19), obtain multi-attribute interval assessments:

$$\bar{v}_{D,0}(y_D) = [0.350,\ 0.496].$$

Similar linear programming problems can be formulated to determine the multi-criteria interval value of the alternative *E*. As a result, can be obtained:

$$\bar{v}_{E,0}(y_E) = [0.405,\ 0.550].$$

Similarly, interval multi-criteria values corresponding to other α-levels can be obtained. As a result, a fuzzy multicriteria values of alternatives can be obtained under conditions of uncertainty of DM's preferences (Fig. 10).

To select the most preferred alternatives, standard approaches to comparing fuzzy numbers [21, 22] can then be applied.

The estimates obtained as a result of applying the modified MAVT method reflect possible deviations of the value of alternatives in accordance with the uncertainly expressed DM's preferences.

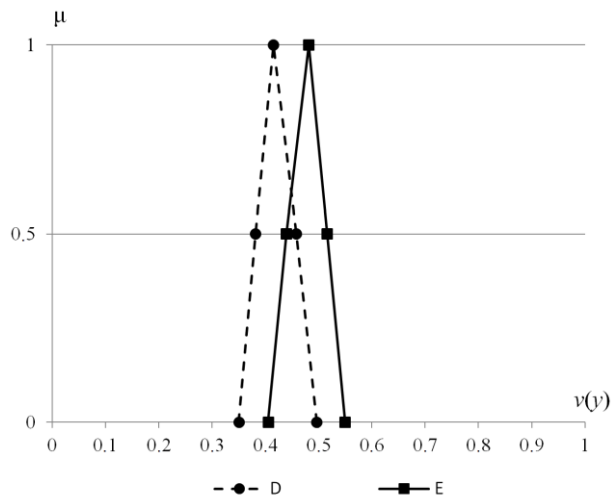

Fig. 10. Fuzzy multi-criteria values of alternatives *D*, *E*

V. Conclusion

The paper discusses an advancement of multi-attribute value theory for the case of uncertainty in the DM's preferences. It proposes procedures for constructing fuzzy singe-attribute value functions, finding the fuzzy value of alternatives, and obtaining fuzzy multi-attribute assessments of alternatives. The modified MAVT can consider the uncertainty in the DM's preferences, which is important when it comes to making decisions on complex problems with high degree of uncertainty in the future conditions, whereby different opinions on the importance of decision-making factors might be involved. The MAVT becomes even more complicated when the DM's responses are fuzzy. This is why it is advisable to apply the modified method to problems where poor decision-making might have severe long-term adverse effects.

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