The Game Analysis of Compensation for Traffic Accidents Based on Pedestrian Vehicle Collision

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Abstract. Taking the traffic accident compensation based on pedestrian vehicle collision as example, this paper carries out modeling description on final-offer arbitration model. And then the Nash equilibrium solution of the traffic accident compensation is analyzed in the final-offer arbitration model. Finally, The Analysis of Nash Equilibrium on Final-offer Arbitration Model is given under the Cauchy distribution and the Laplace distribution. The high-yielding is obtained by the parties to the dispute rationally.

1. Introduction

With the continuous development of the current economy, automobiles are becoming more and more popular, and the number of automobiles is also increasing. Road traffic accidents also occur frequently, among which vehicle-pedestrian collision accidents[1] are common and the consequences are very serious. In the compensation of car-person collision accident, when the higher price of the victim and the lower compensation of the perpetrator can not reach an agreement, reasonable compensation can be obtained by resorting to law, but in real life, the arbitration system is often used for traffic accident compensation. Traditional arbitration and final offer arbitration are two common ways of arbitration. Traditional arbitration [2] is a compromise solution formed by the arbitrators according to their own preferences after they ask for the price. Under this arbitration system, the participants of both parties can anticipate the arbitrators' preferences, thus the participants will motivate the more extreme price. Therefore, economists have proposed another arbitration system called final offer arbitration [3], which requires arbitrators to choose only one of the participants' bid as the final result, thus reducing the chilling effect of agreement arbitration. Many papers use the final offer arbitration mechanism model to analyze the problems of employee welfare, length of service buyout and power market [4-10] under the condition that the arbitrator's preference scheme obeys the probability distribution of normal distribution, so as to obtain Nash equilibrium solution. However, in practical problems, arbitrators' preference schemes not only obey normal distribution, but also other probability distributions such as Cauchy distribution and Laplace distribution. In this paper, the Nash equilibrium of the final bid arbitration mechanism model under Cauchy distribution and Laplacian distribution is discussed, taking the compensation for traffic accidents caused by car-pedestrian collision as an example.

It is necessary to have some knowledge of probability theory to discuss the final bid arbitration mechanism model.

2. Preparatory knowledge

Let $S=\{e\}$ be the sample space of random experiment E. $X=X(e)$ is a real-valued singlevalued function defined in sample space S. For any real number x, $X = X(e)$ is random variable when the set $\{e \mid X(e) \le x\}$ has definite probability. Let X be a random variable, $F(x) = P\{X \le x\}$ is a distribution function of random variables for any real number x.

Theorem 1 The distribution function $F(x)$ of a random variable X has the following properties:

(1) For any real number x ,we have $0 \le F(x) \le 1$,and $F(-\infty) = \lim_{x\to-\infty} F(x) = 0,\ F(+\infty) = \lim_{x\to+\infty} F(x) = 1$.

(2) $F(x)$ is a monotone undiminished function, that is, if $x_1 < x_2$,then $F(x_1) \le F(x_2)$.

(3) $F(x)$ is right continuous.

Definition 1 If the distribution function $F(x)$ of a random variable X can be expressed as an integral of a nonnegative integrable function $F(x) = \int_{-\infty}^{x} f(x)dx$, then X is called continuous random variable and $f(x)$ is the probability density function of random variable X.

Theorem 2 The probability density function $f(x)$ and distribution function $F(x)$ of continuous random variables have the following properties:

(1) The probability density function $f(x)$ satisfies $\int_{-\infty}^{+\infty} f(x)dx = 1$ on $(-\infty, +\infty)$.

(2) $F(x)$ is derivable at a continuous point of $f(x)$, and $F'(x) = f(x)$.

(3) For any real number $a, b(a < b)$,we have

$$P\{a < X < b\} = P\{a \le X < b\} = P\{a < X \le b\} = P\{a \le X \le b\} = F(b) - F(a) = \int_a^b f(x)dx.$$

Definition 2 Let $f(x)$ be a probability density of a continuous random variable X, $\int_{-\infty}^{+\infty} xf(x)dx$ is called the mathematical expectation of a random variable X when the integral $\int_{-\infty}^{+\infty} xf(x)dx$ absolutely converges. Denoted EX, that is, $EX = \int_{-\infty}^{+\infty} xf(x)dx$.

Definition 3 Let X be a continuous random variable, the integral $\int_{-\infty}^{+\infty} (x - EX)^2 f(x)dx$ is called the variance of a random variable X when the integral $\int_{-\infty}^{+\infty} xf(x)dx$ exists and absolutely converges. Denoted DX, that is, $DX = \int_{-\infty}^{+\infty} (x - EX)^2 f(x)dx$.

3. Main results

3.1 Nash equilibrium solution of final request arbitration model

Taking the bargaining between buyers and sellers in the second-hand housing transaction as an example, this paper discusses the arbitration mechanism model of final offer. The two parties involved in bargaining in the transaction are buyers and sellers. The dispute is caused by the price of second-hand house. Then the game is carried out in two steps: first, the two sides negotiate the price in the transaction under the supervision of the real estate intermediary (arbitrator). That is, the buyer and the seller simultaneously issue their respective accepted real estate amounts, which ξ_1 and ξ_2 are expressed in terms of their respective amounts (if $\xi_1 \ge \xi_2$, there is no need for arbitration). Secondly, the arbitrator chooses one of the two as the final solution.

Assuming that the arbitrator himself has his own reasonable plan for the price of the house with ϕ expressing this ideal value and further assumes that after observing the bids ξ_1 and ξ_2 of both parties, the arbitrator simply chooses X that the closest bid. Let $\xi_1 < \xi_2$, if $\xi < \frac{\xi_1 + \xi_2}{2}$, then the arbitrator will choose ξ_1. If $\xi > \frac{\xi_1 + \xi_2}{2}$, then the arbitrator will choose ξ_1. If $\xi = \frac{\xi_1 + \xi_2}{2}$, then the arbitrator tossed a coin to decide. The arbitrator knows the ideal value ϕ, but neither of the participants knows it. The participants consider ξ as a random variable whose distribution function is $F(x) = P\{X \le x\}$.

If the event A indicates that the amount of property given by the buyer is selected by the arbitrator, the event B indicates that the amount of property required by the seller is selected by the arbitrator. Then $P(A) = P(\xi < \frac{\xi_1 + \xi_2}{2}) + \frac{1}{2} P(\xi = \frac{\xi_1 + \xi_2}{2})$

$$
\begin{aligned}
&= P(\xi \le \frac{\xi_1 + \xi_2}{2}) - P(\xi = \frac{\xi_1 + \xi_2}{2}) + \frac{1}{2} P(\xi = \frac{\xi_1 + \xi_2}{2}) \\
&= P(\xi \le \frac{\xi_1 + \xi_2}{2}) - \frac{1}{2} P(\xi = \frac{\xi_1 + \xi_2}{2}) \\
&= F(\frac{\xi_1 + \xi_2}{2}) - \frac{1}{2} F(\frac{\xi_1 + \xi_2}{2}) + \frac{1}{2} F(\frac{\xi_1 + \xi_2}{2} - 0) \\
&= \frac{1}{2} [F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] \ , \qquad (1)
\end{aligned}
$$

$$P(B) = 1 - \frac{1}{2} [F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] . \qquad (2)$$

And then the expected amount of compensation is

$$
\begin{aligned}
W(\xi_1, \xi_2) &= \xi_1 \cdot P(A) + \xi_2 \cdot P(B) \\
&= \xi_1 \cdot \frac{1}{2} [F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] + \xi_2 \cdot \{1 - \frac{1}{2} [F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)]\} \\
&= \xi_2 + \frac{1}{2} (\xi_1 - \xi_2) [F(\frac{\xi_1 + \xi_2}{2}) + F(\frac{\xi_1 + \xi_2}{2} - 0)] . \qquad (3)
\end{aligned}
$$

Assuming that the buyer's goal is to minimize the arbitration result of the amount of the property, the seller tries to maximize the amount of the property. If the bids (ξ_1^*, ξ_2^*) of both parties are Nash equilibrium of the game between the buyer and the seller,then X_1^* must satisfy: $\min_{X_1} W(\xi_1, \xi_2^*)$ and ξ_2^* must satisfy:. $\max_{X_2} W(\xi_1^*, \xi_2)$.

Let the arbitrator's preference scheme ξ be a continuous random variable and the corresponding probability density function be $f(x)$ .Then $P(A) = F(\frac{\xi_1 + \xi_2}{2})$, $P(B) = 1 - F(\frac{\xi_1 + \xi_2}{2})$,and then

$$W(\xi_1, \xi_2) = \xi_1 \cdot P(A) + \xi_2 \cdot P(B) = \xi_1 \cdot F(\frac{\xi_1 + \xi_2}{2}) + \xi_2 \cdot [1 - F(\frac{\xi_1 + \xi_2}{2})] . \qquad (4)$$

The first-order condition of the above optimization problem must be satisfied by the price combination of the two parties (ξ_1^*, ξ_2^*) for the amount of real estate. That is

$$
\begin{cases}
\dfrac{\partial W(\xi_1, \xi_2^*)}{\partial \xi_1} = F(\dfrac{\xi_1^* + \xi_2^*}{2}) - \dfrac{1}{2} (\xi_1^* - \xi_2^*) f(\dfrac{\xi_1^* + \xi_2^*}{2}) \\
\dfrac{\partial W(\xi_1^*, \xi_2)}{\partial \xi_2} = 1 - F(\dfrac{\xi_1^* + \xi_2^*}{2}) - \dfrac{1}{2} (\xi_2^* - \xi_1^*) f(\dfrac{\xi_1^* + \xi_2^*}{2})
\end{cases},
$$

and then $F(\frac{\xi_1^*+\xi_2^*}{2})=\frac{1}{2}$, $\xi_2^*-\xi_1^*=\frac{1}{f(\frac{\xi_1^*+\xi_2^*}{2})}$. (5)

3.2 Arbitrator's preferences subject to Normal distribution

Let the arbitrator's preference scheme obey the Normal distribution of expectation μ and variance σ^2. That is, $\xi \sim N(\mu,\sigma^2)$.Its probability density is $f(x)=\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$. Since the normal distribution is symmetrical on both sides of its expected value, the median is equal to its expected value μ. According to (5), we have $\frac{\xi_1^*+\xi_2^*}{2}=\mu$, $\xi_2^*-\xi_1^*=\frac{1}{f(\mu)}=\sqrt{2\pi}\sigma$.And then the Nash equilibrium bid of the game under Normal distribution is $\xi_1^*=\mu-\sqrt{\frac{\pi}{2}}\sigma$, $\xi_2^*=\mu+\sqrt{\frac{\pi}{2}}\sigma$.

The equilibrium bid of both parties is centrally symmetrical with the expectation μ of arbitrator's preference scheme, and the difference of bid increases with the uncertainty σ^2 of arbitrator's preference scheme.

3.3 Arbitrator's preferences subject to Cauchy distribution

Let the arbitrator's preference scheme obey the Cauchy distribution with parameters of θ and γ. That is, $\xi \sim C(\theta,\gamma)$.Its probability density is $f(x)=\frac{1}{\pi\gamma[1+(\frac{x-\theta}{\gamma})^2]}=\frac{1}{\pi}[\frac{\gamma}{(x-\theta)^2+\gamma^2}]$, the distribution function is $F(x)=\frac{1}{\pi}\arctan\frac{x-\theta}{\gamma}+\frac{1}{2}$. According to (5), we have $\frac{\xi_1^*+\xi_2^*}{2}=\theta$, $\xi_2^*-\xi_1^*=\pi\gamma$.And then the Nash equilibrium bid of the game under Weibull distribution is $\xi_1^*=\theta-\frac{\pi\gamma}{2}$, $\xi_2^*=\theta+\frac{\pi\gamma}{2}$.

The equilibrium asking price of both parties is centrally symmetric θ, and the difference of the asking price increases with the increase of uncertainty γ of the arbitrator's preference scheme.

3.4 Arbitrator's preferences subject to Laplace distribution

Let the arbitrator's preference scheme obey the Laplace distribution with parameters of μ and λ. That is, $\xi \sim L(\mu,\lambda)$. Its probability density is $f(x)=\frac{1}{2\lambda}e^{-\frac{|x-\mu|}{\lambda}}$, $-\infty<x<+\infty$, the distribution function is $F(x)=\frac{1}{2}[1+\mathrm{sgn}(x-\mu)(1-e^{-\frac{|x-\mu|}{\lambda}})$. According to (5),we have $\frac{\xi_1^*+\xi_2^*}{2}=\mu$, $\xi_2^*-\xi_1^*=\frac{1}{f(\frac{\xi_1^*+\xi_2^*}{2})}=2\lambda$.And then the Nash equilibrium bid of the game under Weibull distribution is $\xi_1^*=\mu-\lambda$, $\xi_2^*=\mu+\lambda$.

The equilibrium bid of both parties is centrally symmetric with the expectation value μ of arbitrator's preference scheme, and the difference of bid increases with the increase of uncertainty λ of arbitrator's preference scheme.

4. Conclusions

Taking the compensation for traffic accidents caused by human-vehicle collision as an example, the final bid arbitration model is established. Under the conditions of Cauchy distribution and Laplace distribution, Nash equilibrium analysis of game is discussed. It can be seen that the final bid arbitration mechanism model breaks the deadlock between the victim and the perpetrator, prompts both sides of the game to ask more seriously, the victim's higher bid or the perpetrator's lower compensation, once the arbitrator chooses, it will get higher profits, but the possibility of this bid being chosen will be greatly reduced. The final offer arbitration gives participants greater uncertainty through the arbitrator's preference scheme, which embodies the principle of coexistence of high risk and high profit. Therefore, both sides of the game will strive to reach consensus, thus contributing to the Nash equilibrium.

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