Performance Enhancement of IDMA System by Power and LDPC Code Optimization

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Abstract—**A novel way is presented to improve the performance of system in multiple access channel (MAC) by means of power allocation and low-density parity-check codes (LDPC) design. Power profiles are obtained by data optimization method based on the criteria of maximization of mutual information. Gaussian approximation (GA) method is used as a tool to approximate message the probability density function (PDF) and message updating formulas are derived. Optimized power profile and degree profiles are obtained for 2-user scenario. At the receiver side, the detector and decoder exchange extrinsic information in an iterative way. Three cases are compared given spectral efficiency 0.5. Simulation results show that 0.8dB performance gain can be achieved compared with that of only optimization of LDPC code.**

Keywords—multiple access channel (MAC); power allocation; LDPC codes; Gaussian approximation (GA); degree profile

I. INTRODUCTION

The inherent requirements of 5G are ubiquitous massive low-power devices, high energy efficiency, flexible user loads, and low-complexity transmitters[1]. Interleave division multiple access (IDMA) is a relatively new technique that address these concerns due to its the spectral efficiency and low complexity, particularly at transmitter side[2]. The performance of IDMA schemes can be enhanced by optimized error correcting codes. Hence the code design for IDMA system needs to be studied[3][4].

Low-density parity-check (LDPC) codes are widely used to combat the detrimental effect of channel perturbations [5-8]. The optimization of degree distribution of codes is essential, due to the fact that optimized LDPC codes perform better both in direct coding and differential encoded LDPC coded systems[9], TWR systems[7], and LDPC coded orthogonal frequency division multiplexing(OFDM) [10] systems.

Meanwhile, multiple user information theory shows that the performance can be improved by means of unequal power allocation and code design. Wang P[11] studied power allocation issue for practically channel coded IDMA systems with MUD in MAC, and the performance can be enhanced by unequal power allocation was shown.

Ling J proposed LDPC-Coded IDMA system [12] due to the capability of IDMA to combat MUD and the ability of LDPC codes to approach Shannon capacity in various channels [13] with sum-product algorithm (SPA). We used irregular LDPC codes to LDPC-Coded IDMA system to further improve the performance, and showed via simulation that the performance can be enhanced [14]. However only equal power

case was considered in [14]. Besides we searched power profiles that can approach channel capacity based on mutual information criteria[15]. However [11] and [15] didn't give the optimized code. We will further enhance the performance of IDMA system by power allocation and optimization of codes.
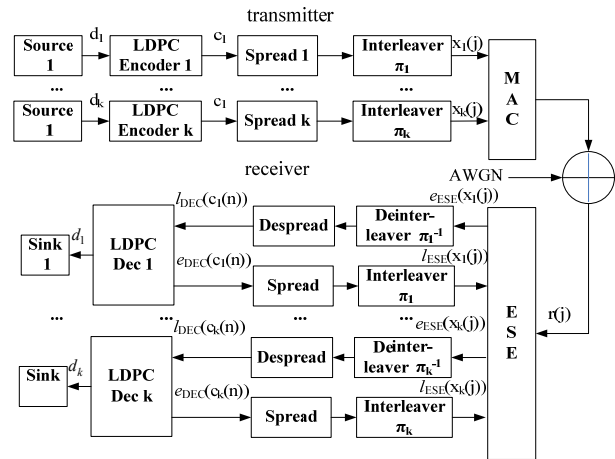
II. SYSTEM MODEL

FIGURE I. THE MODEL OF LDPC-CODED IDMA SYSTEM

A. Transmitter

The composition of LDPC coded IDMA system with power optimization at transmitter is shown on the upper part of Fig. 1. We consider MAC system, in which there is K users. The bit stream d_k from user k (k=1, 2, …, K) is first encoded by an encoder (ENCk), generating $\{c_k(i), 1 \le k \le K, 1 \le i \le I\}$, where I is the block length of bit stream. The encoded bit stream $\{c_k(i), 1 \le k \le K, 1 \le i \le I\}$ is then spread, interleaved, generating chip stream $x_k(i)$,, and then weighted by coefficient α_k and summed, then fed to MAC. We assume that binary phase-shift keying (BPSK) modulation is adopted and the equivalent discrete channel model is used. Then the received signal can be expressed as $y(j) = \sum_{k=1}^{K} \sqrt{P_k} h_k x_k(j) + n(j)$, where P_k is the transmitted power of user k, $\{n(j)\}$ are the sampled value of an AWGN process with distribution $N(0,\sigma^2)$ with $\sigma^2 = N_0/2$.

B. Receiver

The receiver is shown below transmitter in Fig.1. It is consisted of an elementary signal estimator (ESE), K interleaver/de-interleaver, spreader/de-spreader, and K a posteriori probability (APP) decoder (DEC). De-interleaver,

and de-spreader perform the inverse function of interleaver, and spreader, respectively. The ESE performs the function of detection. The DEC performs standard maximum a posteriori probability (MAP) decoding in iterative way. Bit extrinsic information is transferred in the iterative decoding process.

III. PROBLEM FORMULATION

The transmitter part of LDPC coded IDMA system can be seen as a way of multilevel coding (MLC) scheme if the bit stream of a user is regarded as a layer of MLC [16]. Therefore the detection and decoding method of MLC still applies to IDMA system. Fig.2 shows the mapping of K input data streams to channel input. The mapping module $\Sigma(x)$ maps a bi-polar vector $(x_1, x_2, ..., x_K)$ to $x \in R$, where $\{x_k = \pm 1, 1 \le k \le K\}$. A discrete AWGN channel model is used, and the channel noise is denoted by n, channel output by y.
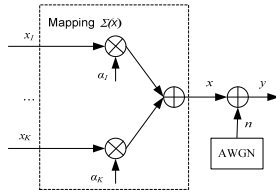

FIGURE II. MLC SCHEME WITH POWER ALLOCATION

The optimization of MLC modulation system includes two steps: the first is optimization of user power, the second is optimization of codes [17]. For the first step the target function is mutual information, mathematically expressed as

$$\max\left(I_{\Sigma(x)}(SNR)\right) = \max\left(E\left(\log_2 \frac{P_{Y/X}(y/x)}{P_Y(y)}\right)\right)$$

$$s.t.\begin{cases} x = \sum_{k=1}^{K} \alpha_k x_k \\ P_k = \alpha_k^2 \\ SNR_k = \frac{P_k}{\sigma^2} \\ SNR = \sum_{k=0}^{K-1} SNR_k \end{cases} \quad (1)$$

Where $X_t \in R$ and $Y_t \in R$ are the channel input and output at time t respectively, and $N_i \in R$ is the noise sample at time t, $N_i \sim N(0, \sigma^2)$. P_k is the transmitted power of user k. SNR is signal-noise-ratio. We assume that $\sigma^2 = 1$ without loss of generality. Therefore, SNR_k will be changed by changing α_k.

The mutual information I_Σ of the given signal mapper Σ is defined as the mutual information $I(X_1, X_2, ..., X_K ; Y)$. It is evident that $I_\Sigma = I(X_1, X_2, ..., X_K; Y) = I(X; Y) = E\left(\log_2 \frac{P_{Y/X}(y/x)}{P_Y(y)}\right)$, where $P_{Y/X}(y/x) = \frac{1}{\sqrt{2p}} \exp\left(-\left(y - x\right)^2 / 2\right)$ is the channel transition probability density function (PDF) and $P_Y(y) = \sum_x P_X(x) P_{Y/X}(y/x)$.

We apply differential evolution (DE) [18] for seeking optimized power profile. Next we will take full advantage of the power profile and design practical LDPC codes.

Due to the optimization of mutual information is equivalent to maximization of standard noise deviation σ given bit energy, the problem can be mathematically described as:

$$\min\left(SNR_k\right)$$

$$s.t.\begin{cases} P_{e_k} \le P_{th} \\ SNR_k = \frac{P_k}{\sigma^2} \end{cases} \quad k \in \{1, 2, ..., K\} \quad (2)$$

Where SNR_k is the required SNR of user k to communicate with arbitrarily small bit error rate (BER), P_{th} is the preset BER threshold, and Pe_k is the BER of user k.

Next we optimize LDPC code degree profiles for MAC based on the optimized power profile.

A. Messages Update

The message PDF of different nodes on factor graph ought to be known in order to obtain optimized degree profiles [19]. We use Gaussian approximation [20] as a tool in tracking the PDF of nodes. Assuming the same channel symmetric condition[20], then the PDF of messages of different nodes are Gaussian and consistent Gaussian, and the variance of message is twice that of expectation. The subscript of variable denotes user index, and superscript denotes iteration number index. All messages are expressed as log-likelihood ratio (LLR) form.

1) Messages update formula from VN to CN

Denote a VN from user k by $v_k(i, n)$, which means it is connected with i CNs and n CFNs. The message means of $v_k(i, n)$ at the l-th iteration is denoted by $mv_k^{(l)}(i, n)$. we can easily get $mv_k^{(l)}(i, n)$ from the extrinsic information update rule, $mv_k^{(l)}(i, n) = n \cdot mu0_k^{(l-1)} + (i-1) \cdot mu_k^{(l-1)}$, where $mu0_k^{(l-1)}$ is the message means from CFN to VN at $(l-1)$-th iteration user k, and $mu_k^{(l-1)}$ denotes the message means from CN to VN of user k at the $(l-1)$th iteration. The messages means from VN to CN at the l th iteration of user k can be got by weighting $mv_k^{(l)}(i, n)$ with coefficients and summing them

$$mv_k^{(l)} = \sum_{i=2}^{dv} \lambda_{k_i} mv_k^{(l)}(i, n) \quad (3)$$

Where (λ_k, ρ_k) denotes the degree profile of user k. The messages from VN to CN have the following density:

$$f_{v_k}^{(l)} = N\left(mv_k^{(l)}, 2 \cdot mv_k^{(l)}\right)$$

Assuming all "1" is transmitted, the mean error rate P_e can be calculated by

$$P_e = \frac{1}{\sqrt{4\pi m_v}} \int_{-\infty}^{0} e^{-\frac{(u-m_v)^2}{4m_v}} \, du \qquad (4)$$

2) Messages update formula from CN to VN

Messages update from CN to VN is similar to that of [20].

$$mu_k^{(l)} = \sum_{j=2}^{dc} \rho_{k_j} \phi^{-1} \left\{ 1 - \left[1 - \sum_{i=2}^{dv} \lambda_{k_i} \phi\left(mv_k^{(l)}(i,n)\right) \right]^{j-1} \right\} \qquad (5)$$

The messages from CN to VN of user k have the following

density: $f_{u_k}^{(l)} = N\left(mu_k^{(l)}, 2 \cdot mu_k^{(l)}\right)$.

3) Messages update formula from VN to CFN

CFN receive messages from all the VN connecting to it except the one that receives the message. At the l-th iteration, the mean message from $v_k(i, n)$ to CFN is denoted by $mv0_k(l)(i, n)$, then we can obtain $mv0_k(l)(i, n)$ based on extrinsic rule:

$$mv0_k^{(l)}(i,n) = (n-1)\cdot mu0_k^{(l-1)}(i,n) + i\cdot mu_k^{(l-1)}(i,n).$$

Weighting $mv0_k(l)(i, n)$ with coefficients λ_k and summing them, the message means from VN to CFN can be obtained:

$$mv0_k^{(l)} = \sum_{i=2}^{dv} \lambda_{k_i} mv0_k^{(l)}(i,n) \qquad (6)$$

At the l-th iteration, the message from VN to CFN has the

following mixture density: $f_{v0_k}^{(l)} = N\left(mv0_k^{(l)}, 2 \cdot mv0_k^{(l)}\right)$

4) Messages update formula from CFN to VN

Denote by $x_k(j)$, $y(j)$ the j-th chip of user k and channel output respectively,

then $\quad y(j) = \sum_{k=1}^{K} h_k x_k(j) + n(j), j = 1,2,\ldots,J$.

Assuming user k is the target user , then $y(j) = h_k x_k(j) + x_k(j)$,

where ξ_k (j) is the sum of interference from other users and noise with respective to user k. If the user number K is large, ξ_k (j) can be approximated as Gaussian distribution .

$\xi_k(j) \square N\left(0, \sigma^2 + \sum_{k'\neq k}\left(1 - \tanh^2 \frac{mv0_{k'}^l(i,n)}{2}\right)\right)$ from central limit

theorem. CFN performs ESE operation, and output LLR message of a given chip of all the users. Having received input from the channel and K a posteriori information from DEC, CFN generate extrinsic information about chip [2]:

$$mu0^{(l)} = 2h_k \alpha_k \frac{y(k) - E\left(y(k)\right) + h_k E\left(x(k)\right)}{\text{var}\left(y(k)\right) - |h_k|^2 \alpha_k^2 \text{var}\left(x(k)\right)}$$

$$\approx \frac{2\alpha_k |h_k|^2}{\sigma^2 + \sum_{k'\neq k} |h_{k'}|^2 \alpha_k^2 \sum_{i=2}^{dv} \lambda_{k'} \eta_{n_i} \varphi\left(mv0_k^{(l-1)}(i,n)\right)} \qquad (7)$$

Where $\varphi(x)$ is defined as the following for computing $mu0^{(l)}$

efficiently: $\varphi(x) = \frac{1}{\sqrt{4\pi x}} \int_{-\infty}^{+\infty} \left(1 - \tanh^2\left(\frac{u}{2}\right)\right) \exp\left(-\frac{(u-x)^2}{4x}\right) du$

For simplicity we assume that all h_k are 1.Therefore we can start the iterative process by using (5), (2),(3) and (4).

B. Optimization of Degree Profiles by Different Evolution

Differential evolution was used in the search process. After initialization procedure which sets initial value of mv, mu and $mv0$ to 0, iterative decoding and detection is carried out. In this iterative process, LLR messages are passed between VN,CN and CFN. Specifically, one complete iteration starts by computing $mu0^{(l)}$ with (7), followed by computing $mv^{(l)}$ with (3), then computing $mu^{(l)}$ with (5), in the end computing $mv0^{(l)}$ with (6). After a predefined number of iteration, the mean residual bit error rate can be calculated with (4). The acceptability of the degree can be determined by comparing the residual BER with the threshold value. Record it if the degree is acceptable, otherwise discard it.

Given user number K and noise level N_0, the problem of LDPC degree profile optimization for MAC belongs to that of constraint optimization problem, mathematically expressed as

$$\min \ P_e$$
$$s.t. \begin{cases} \sum_{i\in\Phi} \lambda_i = 1 \\ \sum_{i=2}^{d_c} \frac{\rho_i}{i} \Big/ \sum_{i=2}^{d_v} \frac{\lambda_i}{i} = 1 - R \end{cases} \qquad (8)$$

Where R is the code rate, and ϕ is a finite set containing several degree types that need to be optimized. For ease of search with little performance loss, only a limited number of degree types are considered from ϕ, with an aim to find near optimal allocations for each degree type. Based on the equal error probability criteria of different layer in MLC schemes [21] we set the code rate in the following way. The higher the optimized power is, the higher the code rate will be. Note that in search of degree profiles, we keep the sum code rate of all users fixed. We find an optimal degree sequence that minimizes BER.

IV. SIMULATION RESULT

We fix the user number to 2 and compare three cases. The first is (3, 6) regular code with equal power. The second is optimization of LDPC code with every user's power equal, and the third is optimization of every user's LDPC code with power profile obtained by the method in section 3. In the first and second cases the code rate is 0.5, and in the third case the sum

of code rate is set to 1. The spread factor (SF) is set to the user number in all cases, therefore the spectrum efficiency of the given system is fixed to 0.5 in this way. In the iterative degree profile searching process, the iteration number 200 is enough for convergence. We obtained degree profiles for the latter two cases by using proposed method. The degree profiles are illustrated in Table I, Table II and Table III respectively. The code rate of user 1 and 2 is 0.56 and 0.44 respectively given the optimized power profile in the third case based on the rules in [21].

TABLE I. 2-USER DEGREE PROFILE WITH UNEQUAL POWER(USER 1)

$\lambda(x)$	$0.18068+0.43828 \cdot x^2+0.11404 \cdot x^3+0.15685 \cdot x^4$ $+0.03061 \cdot x^5+0.07954 \cdot x^{19}$
$\rho(x)$	$0.413221 \cdot x^5+0.58679 \cdot x^6$

TABLE II. 2-USER DEGREE PROFILE WITH UNEQUAL POWER(USER 1)

$\lambda_1(x)$	$0.19118+0.53401 \cdot x^2+0.00825 \cdot x^3+0.05229 \cdot x^4+0.19362 \cdot x^5+0.02065 \cdot x^{19}$
$\rho_1(x)$	$0.870381 \cdot x^6+0.129619 \cdot x^7$

TABLE III. 2-USER DEGREE PROFILE WITH UNEQUAL POWER(USER 2)

$\lambda_2(x)$	$0.25716+0.45180 \cdot x^2+0.00707 \cdot x^3+0.03141 \cdot x^4$ $+0.18376 \cdot x^5+0.06879 \cdot x^{19}$
$\rho_2(x)$	$0.397777 \cdot x^4+0.602223 \cdot x^5$

We conducted power optimization at SNR 2dB. The power profile obtained by the method in section III is $(\alpha1,\alpha2) = (0.891934, 0.888452)$ in the case of unequal power allocation. The modulation scheme adopted is BPSK. By using the optimized degree profiles in Table I, Table II, and Table II, check matrix are constructed randomly, and simulations are. In all these cases the code length is 4096, the interleavers are generated independently and randomly, therefore the uniqueness is ensured. The outer and inner iteration number is 60 and 10 respectively. Fig. 3 shows simulation results of 2-user LDPC Coded IDMA system of the three cases.
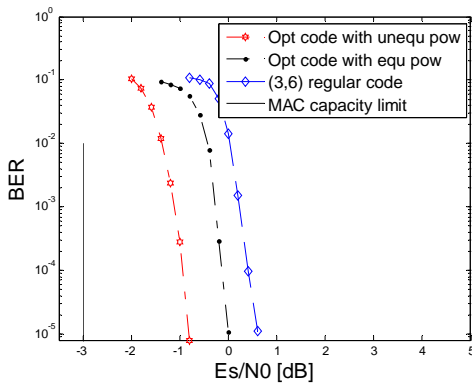

FIGURE III. PERFORMANCE COMPARISON AT SPECTRAL EFFICIENCY 0.5

Firstly the BER performance is improved about 0.6dB at BER 10^{-5} by means of optimizing of LDPC code degree profile in equal power cases. Secondly we will compare the performance of LDPC codes optimization cases, which include optimization of LDPC codes with equal power and with unequal power. The performance can be improved about 0.8dB at BER 10^{-5} by means of optimizing of LDPC code with optimized power profile. The performance gain is achieved with no overall power increase at transmitter and no computational complexity increase at receiver. Therefore optimized irregular LDPC codes should be used with unequal power allocation to different users to improve the performance with the same complexity. One point must be mentioned is that we only considered the case of two users. When the user number is increased, we can divide the users into groups based on their power profile and optimize corresponding LDPC codes. The grouping technique and optimization methods remain to be studied.

V. CONCLUSIONS

LDPC coded IDMA system under AWGN multiple access channel (MAC) is studied by means of power and code optimization. Interleavers are applied as signature to distinguish signal from different users. We treat MAC as a special case of multiple level coding(MLC) and obtain power profiles by differential evolution based on maximization of mutual information. In the receiver the detector and decoder exchange extrinsic information in an iterative way. Message update process of different kinds of nodes is derived, and the message distribution is approximated by Gaussian approximation. Near optimal LDPC code degree distributions are obtained in iterative way. Simulation results show that the performance of the system can be enhanced by means of optimization of power and LDPC code. Future works involve cases of fading MAC channel and higher order modulation.

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