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Computation of Atomic Time Scale

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***Abstract*—This paper gives two conclusions on the computation of atomic time scale. One is that if the noise of an atomic time scale is expressed as the linear sum of five power law spectra noises of atomic clocks, the noise type of atomic time is same as that of an atomic clock. Another is that if the noise of an atomic time is expressed as the linear sum of five power law spectra noises of atomic clocks, the coefficients of power law spectra of the atomic time must obey the an inequation. The results are the criterion to evaluate atomic time, and they are also the theoretic foundation to find optimized atomic time algorithm.**

Keywords-atomic time; algorithm; noise; atomic clock

I. INTRODUCTION

An atomic clock is comprised of five noise types named power law spectra model, then, dose the noise of atomic time belong to the five noise types? If a transformation is used, the numerical characteristic of the stochastic process will generally take place change, in this case, the noise type of atomic time may not be the five noise types. Generally speaking, we hope that the noise type of atomic time doesn't change, because the five power law spectra noises have already analyzed perfectly, and formed a theoretical framework, while it is difficult to analyze other noise type in theory. On the other hand, it is more essential that if to add new noise source, it is very disadvantageous for accuracy and stability of atomic time, even they may be worse those of an atomic clock, and it is to be contrary to our aim of establishment atomic time.

II. INVARIABILITY OF NOISE TYPES OF ATOMIC TIME

The following theorem gives an important conclusion on atomic time algorithm.

Theorem 1: Atomic Time is denoted as T and its noise process is denoted as X. If the X can be expressed as the linear sum of five power law spectra noises of n atomic clocks, that is

$$X(t) = \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C(\alpha,i)\cdot n(\alpha,i,t)$$

where $\sum_{i=1}^{N} C(\alpha,i) = 1$; $n(\alpha,i,t)$ (α= -2,-1,0,1,2) is the five type noises of atomic clock, then, the noise type of atomic time is same as that of atomic clock.

In fact,

$$R_{XX}(\tau) = EX(t)X(t+\tau)$$

$$= \sum_{\alpha,\beta=-2}^{2}\sum_{i,j=1}^{N} C(\alpha,i)\cdot C(\beta,j)\cdot En(\alpha,i,t)n(\beta,j,t+\tau)$$

Based on the independence of noises, have

$$En(\alpha,i.t)n(\beta,j,t+\tau) = 0 \quad (\alpha\neq\beta \text{ or } i\neq j)$$

then,

$$R_{XX}(\tau) = \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C^2(\alpha,i)\cdot En(\alpha,i,t)n(\alpha,i,t+\tau)$$

$$= \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C^2(\alpha,i)\cdot R_{nn}(i,t)$$

$$S_{XX}(f) = \int_{-\infty}^{\infty} R_{XX}(\tau)\exp(-j2\pi f\tau)dt$$

$$= \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C^2(\alpha,i)\int_{-\infty}^{\infty} R_{XX}(\tau)\exp(-j2\pi f\tau)dt$$

$$= \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C^2(\alpha,i)\frac{1}{(2\pi)^2}h_\alpha(i)f^{\alpha-2}$$

$$= \frac{1}{(2\pi)^2}\sum_{\alpha=-2}^{2}[\sum_{i=1}^{N} C^2(\alpha,i)h_\alpha(i)]f^{\alpha-2}$$

Let, $h_\alpha(s) = \sum_{i=1}^{N} C^2(\alpha,i)h_\alpha(i)$

then,

$$S_{XX}(f) = \frac{1}{(2\pi)^2}\sum_{a=-2}^{2} h_a(s)f^{a-2} \qquad (1)$$

The (1) is just the power law spectra noise model of atomic clock, $h_a(s)(a=-2,-1,0,1,2)$ is the coefficients of power law spectra.

The theorem indicates that if the X can be expressed as the linear sum of five power law spectra noises of n atomic clocks, the noise type of atomic time is same as that of an atomic clock. Nevertheless, conversely, if the noise type of atomic time is same as that of an atomic clock, dose the X be definitely expressed as the linear sum of five power law spectra noises of n atomic clocks?

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III. THE EXTREMITY OF IMPROVEMENT OF ATOMIC TIME

The purpose of study atomic time algorithm is to decrease the coefficients of power law spectra noises and to increase the stability of atomic time. Thus, whether there is an extremity of improvement of atomic time, the following theorem gives an answer on the problem.

Theorem 2: Atomic time is denoted as T and its noise is denoted as X. If the X can be expressed as the linear sum of five noise types of n atomic clocks, that is

$$X(t) = \sum_{\alpha=-2}^{2}\sum_{i=1}^{N} C(\alpha,i)\cdot n(\alpha,i,t)$$

where $\sum_{i=1}^{N} C(\alpha,i) = 1$; $n(\alpha,i,t)$ ($\alpha =$ -2,-1,0,1,2) is the five type noises of atomic clock, then, the coefficients of noises of the atomic time $h_a(s)(a = -2,-1,0,1,2)$ must obey the following inequation

$$h_a(s) \geq \frac{1}{\sum_{i=1}^{N} 1/h_a(i)} \qquad a = -2,-1,0,1,2$$

In fact, in Theorem 1, it has been proved that the coefficients of noises of the atomic time are

$$h_\alpha(s) = \sum_{i=1}^{N} C^2(\alpha,i) h_\alpha(i)$$

$$\sum_{i=1}^{N} C(\alpha,i) = 1, \ \alpha = -2,-1,0,1,2$$

Next, we will firstly prove that there exists the minimum of $h_\alpha(s)$, after that, to solve out the minimum.

Let,

$$F = \sum_{i=1}^{N} C_i^2 h_i + \lambda(1 - \sum_{i=1}^{N} C_i)$$

where $C(\alpha,i) = C_i$,

then,

$$D_i = \begin{vmatrix} \frac{\partial^2 F}{\partial C_1^2} & \frac{\partial^2 F}{\partial C_1 \partial C_2} & \cdots & \frac{\partial^2 F}{\partial C_1 \partial C_i} \\ \frac{\partial^2 F}{\partial C_2 \partial C_1} & \frac{\partial^2 F}{\partial C_2^2} & \cdots & \frac{\partial^2 F}{\partial C_2 \partial C_i} \\ \cdots & & & \\ \frac{\partial^2 F}{\partial C_i \partial C_1} & \frac{\partial^2 F}{\partial C_i \partial C_2} & \cdots & \frac{\partial^2 F}{\partial C_i^2} \end{vmatrix}$$

$$= \begin{vmatrix} 2h_1 & 0 & \cdots & 0 \\ 0 & 2h_2 & \cdots & 0 \\ & & & \\ 0 & 0 & \cdots & 2h_i \end{vmatrix}$$

$$= 2i \prod_{k=1}^{i} h_k \qquad i = 1,2,\ldots,N$$

Due to $h_i > 0$, $D_i > 0$ $\quad i = 1,2,\ldots,N$

therefore, the F is a strongly convex function, so there exists the minimum of $h_\alpha(s)$.

Let,

$$\begin{cases} \frac{\partial F}{\partial \lambda} = 0 \\ \frac{\partial F}{\partial C_i} = 0 \end{cases} \qquad i = 1, 2, \ldots, N \qquad (2)$$

Solving the system of equations (2), obtain the minimum,

$$c_i = \frac{1/h_i}{\sum_{j=1}^{N} 1/h_j}$$

$i = 1, 2, \ldots, N$

In this case, having,

$$h_\alpha^*(s) = \frac{1}{\sum_{i=1}^{N} 1/h_a(i)} \qquad a = -2,-1,0,1,2,$$

Since $h_a^*(s)$ is the minimum of $h_a(s)$,

$$h_a(s) \geq \frac{1}{\sum_{i=1}^{N} 1/h_a(i)} \qquad a = -2,-1,0,1,2 \qquad (3)$$

IV. DISCUSSION

Theorem 2 indicates that the improvement of atomic time based on any algorithm is limited. When equal signs in (3) are true to all α, the noise of atomic time reaches the least, the stability of atomic time is best, thus the algorithm is optimal. In practice, an algorithm will make one or several equal signs in (3) be true, thus, an improvement to atomic time via different algorithm may be very difference. Some algorithms are effective to some noises, while other algorithms are effective to other noises. In any case, the coefficients of noises are always greater than or equals $h_a(s)$. If there are two algorithms, they are all optimal, then, the coefficients of noises obtained from the two algorithms must be same, the equals sign in (3) for any α are true. So the Theorem 2 is a criterion to evaluate atomic time, and it is also a theoretic foundation to find optimized atomic time algorithm.

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