Resolving the Infeasibility of DEMATEL: Two Newly Proposed Solutions

Weijian Jin

School of Economics and Management, Nanjing Institute of Technology, No.1 Hongjing Avenue Jiangning Science Park, Nanjing, Jiangsu, PR. China

Abstract—DEMATEL method has been applied in numerous disciplines like airline safety, knowledge management, systems engineering, e-learning. However, there is a kind of infeasible problem that a normalized initial direct incidence matrix might not assemble to a null matrix in the process of gathering total relation matrix of DEMATEL. In order to solve this kind of infeasibility, the sums of all rows of normalized initial direct incidence matrix must be less than zero. This paper proposes the minimum multiplied solution and the vector normalization solution so that the total incidence matrix are obtained. Finally, two cases are illustrated and compared with the original DEMATEL method, the minimal additive solution, the minimal multiplied solution and the vector normalization solution. The results show that the two newly proposed solutions are feasible.

Keywords—DEMATEL; infeasibility; minimum multiplied solution; vector normalization solution

I. INTRODUCTION

The DEMATEL Method is an effective analytical method for building and analyzing direct or indirect relationship among complex factors [1-2]. It helps decision makers understand the relationship of structure and solve problem of complicated and intertwined system [3]. If decision makers focus on and improve these key or main factors obtained by DEMATEL, they can greatly promote the efficiency of management decisions[4-5]. Therefore, the method has been successfully applied in recent years. For example, Tzeng et al. (2005) solved the enterprise web sites problem[6]. Liou et al. (2008) built an effective safety management system by DEMATEL method[7].

Moreover, researchers combined original DEMATEL with other method to solve different decision making problems or extended fuzzy DEMATEL to make better decisions. For example, Wu & Lee (2007) carried out an effective fuzzy DEMATEL method to promote competency of global managers[5]. Liou et al. (2011) proposed fuzzy DEMATEL method in combination with fuzzy preference programming and ANP to build a model to select partners for choosing outsourcing providers[8]. Hsu & Liou (2013) discussed a hybrid method for combining DEMATEL and ANP method to form a provider decision model [9].

Prior studies focused on the application or extension of DEMATEL method. However, the DEMATEL method exists infeasibility itself. The averaging initial direct incidence matrix is often represented by values between zero and one. And thus the infinite-power of normalized initial direct incidence matrix might not be exist or not assemble to null matrix on the calculation of total incidence matrix [11]. Lee et al. (2013) keenly found this kind of drawback and proposed a solution which makes the sums of all columns of the direct relation matrix less than one to solve the infeasible problem. In this paper, we develop the minimal multiplied solution and the vector normalization solution. The two proposed solutions respectively guarantee the infinite normalized initial direct incidence matrix to assemble to null matrix and make the inverse matrix exist as well.

The rest of this paper is as follows. Section 2 puts forward the original DEMATEL method. Section 3 analyses infeasible of DEMATEL method. Section 4 introduces previous solution for this type of gathering the total incidence matrix. Section 5 develops two new solutions, the minimal multiplied solution and the vector normalization solution, to avoid such kind of infeasibility. Section 6 uses two cases to prove effectiveness of the two newly proposed solutions. Section 7 discusses the differences of the four solutions which include the original DEMATEL method, the minimum additive solution, and our two newly proposed solutions by using the two cases. Finally, conclusions are presented in section 8.

II. INFEASIBILITY OF DEMATEL

In real world, decision makers often invite several experts to construct the initial direct incidence matrixes of original DEMATEL. When they take a weighted average of the group initial direct incidence matrixes, the values of the averaging initial direct incidence matrix are between zero and one. However, sometimes $F = \lim(G+G^2+\dots+G^l)$ when $l \to \infty$ cannot convert to $G(1-G)^{-1}$ on the aggregating of the total incidence matrix. In other words, $\lim_{l\to\infty} (G+G^2+\dots+G^l)$ might not exist owing to $\lim_{l\to\infty} G^l \neq [0]_{n \le n}$, which cause infeasibility of DEMATEL.

If the results $G(1-G)^{-1}$ continue to be used, it would lead to deviation which will further affect misjudgment of decision makers.

To describe the infeasible problem, Let us discuss the following theorem and proof.

Theorem 1. If *G* is a normalized initial direct incidence matrix, the columns sum to unity or the columns sum less than one. then, $\lim_{l\to\infty} (G + G^2 + \dots + G^l) = G(I - G)^{-1}$ might or might not hold.

Proof 1. Assume $T = G + G^2 + \dots + G^l$, then

 $T - GT = (G + G^{2} + \dots + G^{l}) - G(G + G^{2} + \dots + G^{l}) = G - G^{l+1},$

 $T(I-G) = G(I-G^{l})$, it is obvious that I - G is diagonalizable, therefore, it is invertible matrix.

The equation $T(I-G) = G(I-G^{1})$ multiplies $(I-G)^{-1}$ in left and right side at the same time, and thus $T = G(I - G^{1})(I - G)^{-1}$ is $\lim G^l = [0]_{n \times n}$ Only when established. the equation $\lim_{l \to \infty} T = \lim_{l \to \infty} G(I - G^l)(I - G)^{-1} = G(I - G)^{-1}$ is obtained.

To make $\lim_{l \to \infty} G^l = [0]_{n \times n}$, therefore, we should take into consideration that $G_{ij} > 0$ and infinite G series strictly monotone decline as well and thus the equation $F = \lim(G + G^2 + \dots + G^l) = G(I - G)^{-1}$ is obtained. It is obviously that $G_{ij} > 0$ as the initial direct incidence matrix is subjective score matrix, therefore, we only prove that infinite G series strictly monotone decline.

Moreover, we assume G_k^l and $\max G_i^l$, where G_k^l denote the k - th row of l power of the matrix G and max G'_i denote the maximum value in G_k^i , $i = 1 \cdots k$.

We have

$$G_{ij}^{l+1} = \sum_{p=1}^{n} G_{ip}^{l} G_{pj} \le \sum_{p=1}^{n} \max G_{i}^{l} G_{pj} = \max G_{i}^{l} \sum_{p=1}^{n} G_{pj}$$

Only when $\sum_{p=1}^{n} G_{pj} < 1$.

 $\max G_i^l > \max G_i^{l+1}$, therefore, $G_{ij}^l > G_{ij}^{l+1}$, the *l* power of the matrix G strictly monotone decline when l tends to infinity. In other words, it will converge to null matrix when the power of the matrix G tends to infinity, that is $\lim_{l \to \infty} G^l = [0]_{n \times n}$

III. THE PROPOSED SOLUTIONS

Lee et al.(2013) proposed the minimal additive solution to solve the problem of the convergence of matrix. In this section, we put forward the minimal multiplied solution and the vector normalization solution, and the infinite normalized initial direct incidence matrix would also converge to a null matrix.

A. Solution 1. The Minimal Multiplied Solution

In order to ensure $\sum_{p=1}^{n} G_{pj} < 1$ as well, we use the following formula $S = \max(\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}) + \varepsilon + \varepsilon + \sum_{1 \le j \le n} \sum_{i=1}^{n} a_{ij})$ as the normalization of the initial direct incidence matrix. The ε is also a very smaller positive number, such as $\varepsilon = 10^{-5}$. We can also guarantee the infinite normalized initial direct incidence matrix to assemble to a null matrix and get the crisp values of total incidence matrix which can be compared and sorted.

The steps are as follows:

(1) The normalized initial direct incidence matrix G can be obtained.

$$S = \max(\max_{1 \le i \le n} \sum_{j=1}^{n} a_{ij}, (1+\varepsilon) * \max_{1 \le j \le n} \sum_{i=1}^{n} a_{ij})$$
(1)

$$G = A / S$$
 (2)

(2) The total incidence matrix F can be obtained according to formula (3).

$$F = \lim_{l \to \infty} (G + G^2 + \dots + G^l) = G(I - G)^{-1}$$
(3)

(3) We compute the value of effect group and cause group. R represent the sums of columns of the total incidence matrix F, C represent the sums of rows of the total incidence matrix and respectively. Then, the effect group R+C and the R-Ccause group are obtained respectively.

Example 1. The following averaging initial direct incidence matrix D is shown when the group decision-makers give the final results.

0	0	0.4	0.2	0	0.4	
0	0	0	0	0.4	0	
0.4	0	0	0.4	0	0.2	
0.3	0	0.3	0	0	0.4	
0	0.7	0	0	0	0	
0.3	0	0.3	0.4	0	0	
	0 0.4 0.3 0 0.3	0 0 0 0 0.4 0 0.3 0 0 0.7 0.3 0	$\begin{array}{ccccccc} 0 & 0 & 0.4 \\ 0 & 0 & 0 \\ 0.4 & 0 & 0 \\ 0.3 & 0 & 0.3 \\ 0 & 0.7 & 0 \\ 0.3 & 0 & 0.3 \end{array}$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	

We compute it by the minimal additive solution and our minimal multiplied solution respectively. The results are shown in table 1 and table 2.

TABLE I. THE MINIMAL ADDITIVE SOLUTION

	V1	V2	V3	V4	V5	V6
effect group	199999.9999	2.3055	199999.9999	199999.9999	2.3055	199999.9999
cause group	0	- 0.4166	0	0	0.4166	0

TABLE II. THE MINIMAL MULTIPLIED SOLUTION

	V1	V2	V3	V4	V5	V6
effect group	199999.9999	2.3055	199999.9999	199999.9999	2.3055	199999.9999
cause group	0	- 0.4166	0	0	0.4166	0

The results for the effect group and the cause group which are sorted are the same as the results obtained from the minimal additive solution. Therefore, it shows that the minimal multiplied solution here is feasible for the example 1.

B. Solution 2. The Vector Normalization Solution

Consider that the following vector normalization solution. Let $A = (a_{ij})_{n \times n}$ be an averaging initial direct incidence matrix and $G = (G_{ij})_{(n \times n)}$ be a normalized initial direct incidence matrix.

$$G_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{n} (\sum_{j=1}^{n} a_{ij})^2}$$

Theorem 2. Let $G_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{n} (\sum_{j=1}^{n} a_{ij})^2}$ be the mode of vector normalization, we can obtain $\sum_{j=1}^{n} G_{jj} < 1$ except the special situation that one column of the averaging initial direct incidence matrix have the scores and the other columns are zero. The results are $\sum_{p=1}^{n} G_{pj} = [0,0,...,1,...,0,0]_{1 \times n}$ in the special situation. This special

situation makes no sense for decision-makers.

Proof 2. Let a_{ik} be the *k*-th column of the averaging initial direct incidence matrix A. Suppose a_{ii} have the nonzero values in j column and the other columns are zero.

We have
$$\sqrt{\sum_{i=1}^{n} (\sum_{j=1}^{n} a_{ij})^2} = suma_{ik}$$

After using the equation $G_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{n} (\sum_{j=1}^{n} a_{ij})^2} = a_{ij} / suma_{ik}$,

Owing to a_{ii} have the nonzero values and the other columns of the matrix A are zero, it is obvious that $\sum_{n=1}^{n} G_{pj} = [0, 0, ..., 1, ..., 0, 0]_{1 \times n}.$

Moreover, if two columns or above of the averaging initial direct incidence matrix A have $a_{ij} \neq 0$, we can easily prove

Example 3 the normalized initial direct incidence matrix of example 1 by using the vector normalization solution is shown as

	0	0	0.1855	0.0927	0	0.1855
	0	0	0	0	0.1855	0
C	0.1855	0	0	0.1855	0	0.0927
G=	0.1391	0	0.1391	0	0	0.1855
	0	0.3246	0	0	0	0
	0.1391	0	0.1391	0.1855	0	0
Oł	oviou	sly tł	hat $\sum_{p=1}^{6}$	$G_{pj} < 1$		_

Thus, the steps are shown in the following.

(1) According to formula (4), the normalized initial direct incidence matrix G is obtained.

$$G_{ij} = a_{ij} / \sqrt{\sum_{i=1}^{n} (\sum_{j=1}^{n} a_{ij})^2}$$
(4)

(2) F are acquired according to formula (5).

$$= \lim_{l \to \infty} (G + G^2 + \dots + G^l) = G(I - G)^{-1}$$
(5)

(3) The value of effect group and cause group are computed. *R* represent the sums of columns of the total incidence matrix *F*, *C* represent the sums of rows of the total incidence matrix and respectively. Then, R + C and R - C that are called the effect and cause group are obtained. We calculate the effect group and cause group from example 1. The results are shown in table 3.

TABLE III. VECTOR NORMALIZATION SOLUTION

	V1	V2	V3	V4	V5	V6
effect group	1.7295	0.6709	1.7295	1.7295	0.6709	1.7295
cause group	0	-0.1480	0	0	-0.1480	0

The sorted result is the same as the minimal additive solution and the minimal multiplied solution. It shows that the solution is feasible.

IV. CASES

The two following cases from the study of Lee et. al. (2013) have been used proving feasibility of the minimal additive solution. We also use the two cases to verify the feasibility of the new two solutions.

A. Case 1

Shieh et. al. (2010) used original DEMATEL method to ascertain the key factors in hospital quality of service[12]. Seven major factors V1,V2,V3,V4,V5,V6 and V7are chosen to evaluate the importance of hospital management. The averaging initial direct incidence matrix A of twenty-one managerial personnel's scores is obtained and shown as follows:

	0	1.5789	2.0526	1.7895	2.2632	2	1.3158
	1.5263	0	2.0526	2.3684	2.3684	2.0526	1,6316
	1.9474	1.9474	0	2.0526	2.4211	2.5263	1.9474
A =	1.3684	2.2632	2.1053	0	2.2105	2.2632	1.5789
	1.8421	2	2.2105	1.7895	0	2.2105	1.4737
	2.0526	1.8421	2.1579	1.8421	2.2105	0	1.6842
	1.0526	1.7368	1.8421	1,6316	1.5263	1.7368	0

The result for effect group and cause group are obtained by using the original DEMATEL method, the minimal additive solution, the minimal multiplied solution and the vector normalization solution, and they are shown in table 4, 5, 6 and 7 respectively.

TABLE IV. THE ORIGINAL DEMATEL METHOD

	V1	V2	V3	V4	V5	V6	V7
effect group	16.537	18.2381	19.4625	18.1635	19.0332	19.0456	15.3
cause group	0.7913	0.4372	0.2636	0.2573	-0.9766	-0.6993	- 0.0734

TABLE V. THE RESULTS FOR MINIMAL ADDITIVE SOLUTION

	V1	V2	V3	V4	V5	V6	V7
effect group	16.5369	18.238	19.4623	18.1633	19.0331	19.0455	15.2999
cause group	0.7913	0.4372	0.2636	0.2573	-0.9766	-0.6993	-0.0734

TABLE VI. THE RESULTS FOR MINIMAL MULTIPLIED SOLUTION

	V1	V2	V3	V4	V5	V6	V 7
effect group	16.5354	18.2363	19.4605	18.1617	19.0313	19.0437	15.2985
cause group	0.7912	0.4371	0.2635	0.2573	-0.9765	-0.6992	-0.0734

TABLE VII. THE RESULTS FOR VECTOR NORMALIZATION SOLUTION

	V1	V2	V3	V4	V5	V6	V7
effect group	20.5215	22.6253	24.139	22.5328	23.6093	23.6242	18.988
cause group	0.9784	0.5412	0.3256	0.3193	-1.208	-0.8657	- 0.0908

The sorted of effect group and cause group are shown in table 8. It is obvious that the results are same.

TABLE VIII. THE SORTED RESULTS

	effect group	cause group
The original DEMATEL method	V3≻V6≻V5≻V2≻V4≻V1≻V7	<i>V</i> 1≻ <i>V</i> 2≻ <i>V</i> 3≻ <i>V</i> 4≻ <i>V</i> 7≻ <i>V</i> 6≻ <i>V</i> 5
the minimal additive solution	V3≻V6≻V5≻V2≻V4≻V1≻V7	<i>V</i> 1≻ <i>V</i> 2≻ <i>V</i> 3≻ <i>V</i> 4≻ <i>V</i> 7≻ <i>V</i> 6≻ <i>V</i> 5
The minimal multiplied solution	V3≻V6≻V5≻V2≻V4≻V1≻V7	<i>V</i> 1≻ <i>V</i> 2≻ <i>V</i> 3≻ <i>V</i> 4≻ <i>V</i> 7≻ <i>V</i> 6≻ <i>V</i> 5
the vector normalization solution	V3≻V6≻V5≻V2≻V4≻V1≻V7	V1≻V2≻V3≻V4≻V7≻V6≻V5

B. Case 2

Taiwan's innovational supporting system of higher education has seven major criteria which are academic research (V1), administrative process (V2), faculty and staff (V3), market development (V4), organizational structure (V5), organizational culture (V6), leadership style (V7) [13]. Sixty-six educational experts used DEMATEL method to score these seven factors, and the mean of initial direct incidence matrix A is obtained by the averaging sixty-six matrices, which is shown as follows.

	0	0.12	1.35	1.62	0.27	0.33	0.03
	1.24	0	2.33	0.57	1.13	0.06	0.71
	3.91	3.76	0	2.97	1.19	0.23	0.04
A =	3.29	0.24	0.26	0	0.3	1.75	1.22
	1.07	2.93	3.35	1.1	0	3.63	1.32
	3.01	1.25	2.63	2.77	1.29	0	1.1
	2.98	3.03	3.42	2.2	3.78	3.89	0

The result for effect group and cause group are obtained by using the original DEMATEL method, the minimal additive solution, the minimal multiplied solution and the vector normalization solution, and they are shown in table 9, table 10, table 11 and table12 respectively.

TABLE IX. THE ORIGINAL DEMATEL METHOD

	V1	V2	V3	V4	V5	V6	V7
effect group	1.9532	1.7409	2.3189	1.8667	2.1273	2.0623	2.3775
cause group	- 1.2125	- 0.4646	- 0.2445	- 0.4667	0.6387	0.2728	1.4768

TABLE X. THE RESULTS FOR MINIMAL ADDITIVE SOLUTION

	V1	V2	V3	V4	V5	V6	V7
effect group	1.9532	1.7409	2.3189	1.8667	2.1273	2.0623	2.3775
cause group	- 1.2125	- 0.4646	- 0.2445	- 0.4667	0.6387	0.2728	1.4768

TABLE XI. THE RESULTS FOR MINIMAL MULTIPLIED SOLUTION

	V1	V2	V3	V4	V5	V6	V7
effect group	1.9531	1.7408	2.3188	1.8667	2.1273	2.0622	2.3775
cause group	- 1.2125	- 0.4646	- 0.2445	- 0.4667	0.6387	0.2728	1.4768

TABLE XII. THE RESULTS FOR VECTOR NORMALIZATION SOLUTION

	V1	V2	V3	V 4	V5	V6	V 7
effect group	0.9113	0.8175	1.1146	0.8691	1.0003	0.9894	1.1241
cause group	-0.562	- 0.2288	- 0.0963	- 0.2113	0.2847	0.1193	0.6943

We use those solution to compare and sort Taiwan's higher educational innovational supporting system. The results are shown in table 13. Our proposed two solutions are same to the minimal additive solution.

TABLE XIII. THE SORTED RESULTS OF FOUR SOLUTIONS

	effect group	cause group
The original DEMATEL method	<i>V</i> 7≻ <i>V</i> 3≻ <i>V</i> 5≻ <i>V</i> 6≻ <i>V</i> 1≻ <i>V</i> 4≻ <i>V</i> 2	V7≻V5≻V6≻V3≻V2≻V4≻V1
the minimal additive solution	<i>V</i> 7≻ <i>V</i> 3≻ <i>V</i> 5≻ <i>V</i> 6≻ <i>V</i> 1≻ <i>V</i> 4≻ <i>V</i> 2	V7≻V5≻V6≻V3≻V2≻V4≻V1
The minimal multiplied solution	<i>V</i> 7≻ <i>V</i> 3≻ <i>V</i> 5≻ <i>V</i> 6≻ <i>V</i> 1≻ <i>V</i> 4≻ <i>V</i> 2	V7≻V5≻V6≻V3≻V2≻V4≻V1
the vector normalization solution	<i>V</i> 7≻ <i>V</i> 3≻ <i>V</i> 5≻ <i>V</i> 6≻ <i>V</i> 1≻ <i>V</i> 4≻ <i>V</i> 2	V7≻V5≻V6≻V3≻V2≻V4≻V1

C. Discussion

To sum up, the results of case 1 and case 2 are shown in table 14. The decision-result between original DEMATEL method and our solutions is completely same. In other words, the results are effective and feasible. The two newly proposed solutions give a better and more choosing approach.

TABLE XIV. SUMMARY OF THE EXAMPLES

	The original DEMATEL method	The minimal multiplied solution	the minimal additive solution	the vector normalization solution
Case 1	The same	The same	The same	The same
Case 2	The same	The same	The same	The same

V. CONCLUSION

In this paper we put forward the minimal multiplied solution and the vector normalization solution to avoid this kind of infeasible problem. Our two proposed solutions are compared with the original DEMATEL method and the minimal multiplied solution by two cases. The results are identical. Moreover, due to the limitation of space, we also verify the sorted results with the other cases of study of Lee et al. (2013) and get the same order. The results show the two proposed solutions are scientific and feasible. The solutions have good practical value as well. We are confident the result for various example would obtain the similar conclusions. In addition, we hope that the other solutions are continually explored and give decision makers more interesting surprise in the further.

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