A New Interval-number DEMATEL Method Based on 2-Tuple

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Abstract—**It is used for solving complicated problem groups by the DEMATEL method. However, it has not dealt with the relationship between components of a system in some specific complex problems. Therefore, a new interval-number DEMATEL method based on 2-tuple is proposed. Firstly, we construct an interval-number initial direct-relation matrix by comparing factors of a system. Secondly, we map the interval-number initial direct incidence matrix into a binary set's direct-relation matrix and transform into a 2-tuple matrix. Thirdly, we transform the 2-tuple matrix into a normalized real number matrix. Fourth, the influence degree, the effect degree, the center degree and the reason degree are obtained by the DEAMTEL decision-making process. Finally, using a case to identify the influence factors of knowledge transfer in IT outsourcing problem as an example, we compare the results of exist method ith the results of our method, and show that the results are the same.**

Keywords—interval number; 2-tuple; DEMATEL; decision making

I. INTRODUCTION

Fontela&Gabus (1976) put forward the DEMATEL method to compare interdependent factors in network structure in the 1970s[1]. The basic principle of the method is to make a comprehensive arrangement of problems through a questionnaire survey or expert interview so as to clarify the nature of interrelated problems. Since it was proposed, the method has been widely recognized by people and become one of the main decision-making tools. Especially in the 21st century, it has been widely applied to analyze a complex decision-making problem in different fields. Due to the fuzziness of human cognition and the complexity of judgment on objective things, the method embodied different fuzzy evaluation information has been studied in recent year. One of which, interval-number DEMATEL method, also gets the attention of some scholars[2-5].

Using interval algorithm and the possible degree of sorting, Gao Peiran (2012) proposed an algorithm of interval-number DEMATEL method. The algorithm takes the possibility degree to obtain the decision-making results. Gao further studied the influence factors of knowledge transfer in IT outsourcing by this kind of DEMATEL method [6]. However, this kind of DEMATEL method made the decision result unreasonable because of calculating the center degree and reasons degree by splitting upper and lower limit of the interval number. Moreover, Gao's method is relatively complex and loss of information to some extent because of employing the possible degree algorithm, which leads to the approximating decisive result. Therefore, it has the specific limitations.

Herrera et al. proposed a 2-tuple analysis method in 2000[9]. The method mainly transformed the fuzzy evaluation information into 2-tuple evaluation information, which effectively solved the loss of information in the decision-making process and it was widely used in different fields. Fan et al. (2008) depict the interaction between risk factors by language evaluation information[10]. They firstly use 2-tuple to obtain evaluation information, and then realize the mutual transformation between 2-tuple and real number. Then traditional DEMATEL method was used to obtain the decision-making result. In view of obtaining cooperative research and development risk factor, Suo et al.(2008) firstly assembly language judgment information by 2-tuple. Then they get the ordering and classification of cooperative research and development risk factors by traditional DEMATEL decision-making method [11].

Since fuzzy numbers are equivalent to the corresponding 2-tuple set[12], this paper introduces a new interval-number DEMATEL method based on 2-tuple and makes it more accurate for the decision-making result. The rest of this paper is as follows. The second parts briefly introduces some relevant concepts and basic knowledge. The third parts present a new interval-number DEMATEL decision method based on 2-tuple. The fourth parts use a knowledge transfer influencing factor problem in IT outsourcing to verify science and rationality of the new method. The fifth parts give a summary.

II. PRELIMINARIES

In this section, some concepts and knowledge are briefly illustrated.

① Interval number [11]

Let $\tilde{a}$ be $\tilde{a}=(a^L,a^U)$, $0\le a^L\le a^U\le 1$,we call it as interval number.

Let $\tilde{b}=(b^L,b^U)$ be a interval number, $0\le b^L\le b^U\le 1$, the linear operation rule of the interval number is:

$$\tilde{a}+\tilde{b}=(a^L+b^L,a^U+b^U)$$
$$\tilde{a}*\tilde{b}=(a^Lb^L,a^Ub^U)$$
$$1/\tilde{a}=(1/a^U,1/a^L)$$
$$\lambda\tilde{a}=(\lambda a^L,\lambda a^U)$$

②2-tuple [9][12]

2-tuple information is based on the concept of symbolic transfer. It is a binary group representation (s_k,η_k) of the evaluation value given to a target. The meanings of elements s_k and η_k are as follows: the term s_k is the *kth* language element in the predefined language evaluation set S , $S=(s_0,s_1,...s_l)$. S satisfies the following properties.

①The set is ordered: $i\geq j$, if and only $S_i\geq S_j$; ②There is a negation operator:: if $j=g-i$, there are $Neg(S_i)=S_j$, $g+1$ indicating the number of elements in the language evaluation set S . ③There is maximization and minimization operator: if $S_i\geq S_j$, there are $\max(S_i,S_j)=S_i$, $\min(S_i,S_j)=S_j$ 。

η_k is the symbolic transfer value, which satisfies the condition $\eta_k\in[-0.5,0.5)$, and it represents the deviation between the evaluation result and S_k .

Next, 2-tuple are defined in detail and the related operation rules are given.

Definition 1 let $s_k\in S$ be a language element. The 2-tuple is obtained by the following functions θ .

$$\theta: S\rightarrow S\times[-0.5,0.5)$$
$$\theta(S_i)=(S_i,0),S_i\in S \tag{1}$$

Definition 2 let $\beta=[0,l]$ be real number assembled for language evaluation set S ,where l is the numbers of the language evaluation set S ,and thus β is get by the following function Δ.

$$\Delta:[0,l]\rightarrow S\times[-0.5,0.5)$$
$$\Delta(\beta)=\begin{cases} S_k, k=round(\beta) \\ \eta_k=\beta-k,\eta_k\in[-0.5,0.5) \end{cases} \tag{2}$$

Where round is rounding operator.

Definition 3 (s_k,η_k) is a 2-tuple in which s_k is the *kth* element, $\eta_k\in[-0.5,0.5)$. There has been an inverse function Δ^{-1} that makes it convert to the corresponding value $\beta=[0,l]$.

$$\Delta^{-1}: S\rightarrow S\times[-0.5,0.5)\rightarrow[0,l]$$
$$\Delta^{-1}:(S_k,\eta_k)=k+\eta_k=\beta \tag{3}$$

Definition 4 let (s_1,η_1) (s_2,η_2) , $\cdots$, (s_n,η_n) be a set of 2-tuple, then the arithmetic average operator is

$$(\bar{S},\bar{\eta})=\Delta\left[\frac{1}{n}\sum_{i=1}^{n}\Delta^{-1}(S_j,\eta_j)\right],\bar{S}\in S,\bar{\eta}\in[-0.5,0.5) \tag{4}$$

Definition 5 let (s_1,η_1) (s_2,η_2) , $\cdots$, (s_n,η_n) be a set of 2-tuple information, $\omega=(\omega_j,\beta_j)$ are a 2-tuple weight vector, and the weighted arithmetic average operator is

$$(\bar{S},\bar{\eta})=\Delta\left[\frac{\sum_{i=1}^{n}\Delta^{-1}(\omega_j,\beta_j)\times\Delta^{-1}(S_j,\eta_j)}{\sum_{i=1}^{n}\Delta^{-1}(\omega_j,\beta_j)}\right] \tag{5}$$
$$\bar{S}\in S,\bar{\eta}\in[-0.5,0.5)$$

(3) The mutual transformation between interval number and 2-tuple [12]

If the language evaluation set is $S=(s_0,s_2,\dots,s_{l-1})$, where l is the numbers of the language evaluation set. Then, the triangular fuzzy number of s_i is expressed as

$$\mu_{s_k}(\vartheta)=\begin{cases} a_0^L=0 \\ a_i^L=\dfrac{\vartheta-1}{l-1} & 1\leq\vartheta\leq l-1 \\ a_i^M=\dfrac{\vartheta}{l-1} & 0\leq\vartheta\leq l-1 \\ a_i^U=\dfrac{\vartheta+1}{l-1} & 1\leq\vartheta\leq l-2 \\ a_{i-1}^U=1 \end{cases} \tag{6}$$

(4) The transformation between interval number and 2-tuple

Definition 6 let $\tilde{a}=(a^L,a^U)$ be an interval number, $S=(s_0,s_2,\dots,s_{l-1})$ be a language evaluation set, $\tau(\tilde{a})=(s_k,\alpha_k)|k\in(0,1,\dots,l-1)$ be a 2-tuple set. Then, the interval number $\tilde{a}$ will convert into binary set $\tau(\tilde{a})$ through the following map, $\mu_{\tilde{a}}(\vartheta)$ are the membership functions of the interval number $\tilde{a}$, $\mu_{s_k}(\vartheta)$ are the membership functions of language evaluation set S .

$$\tau:\tilde{a}\rightarrow\tau(\tilde{a})$$
$$\tau(\tilde{a})=\{(s_k,\alpha_k)|k\in(0,1,\dots,l-1)\} \tag{7}$$

Where

$$\alpha_k=\max_{\vartheta}\min\{\mu_{\tilde{a}}(\vartheta),\mu_{s_k}(\vartheta)\}$$

Such as the following example.

Example 1 let $\tilde{a}=(0.2,0.5)$ be the interval number and $l=5$ be the number of language evaluation sets, applying (6) and (7) , the binary set $\tau(\tilde{a})$ is obtained as follow.

$$\tau(\tilde{a})=(s_0,0.2),(s_1,1),(s_2,1),(s_3,0),(s_4,0).$$

Accordingly, if $\tilde{a}=(0.2,0.4)$, $l=9$.

In the similar manner,

$$\tau(\tilde{a})=(s_0,0.4),(s_1,1),(s_2,1),$$
$$(s_3,0.2),(s_4,0),(s_5,0),(s_6,0),(s_7,0),(s_8,0).$$

Definition 7 Let $\tau(\tilde{a})=\{(s_k,\alpha_k)|k\in(0,1,\dots,l-1)\}$ be the 2-tuple set of interval number $\tilde{a}$, the real number β is obtained by mapping function ξ , and then 2-tuple (s_k,η_k) is acquired.

$$\xi:F(S)\rightarrow[0,l-1]$$
$$\xi(\tau(\tilde{a}))=\xi(F(S))=\xi((s_j,\alpha_j)|j=0,1,\dots,l-1))$$
$$=\sum_{j=0}^{l-1}(j*\alpha_j)/\sum_{j=0}^{l-1}(\alpha_j)=\beta \tag{8}$$

In the example 1, the interval number $\tilde{a}=(0.2,0.5)$, using (8), $\beta=1.36$ is obtained, and the corresponding 2-tuple value is $(s_1,0.36)$. Accordingly, the interval number $\tilde{a}=(0.2,0.4)$, $\tau(\tilde{a})=2.38$ is obtained, and the corresponding 2-tuple value is $(s_2,0.38)$. Literature [12] has proved that the above transformation is effective and reasonable.

III. The proposed DEMATEL method

Based on the above analysis, we propose a new interval-number DEMATEL decision method based on 2-tuple. The decision-making steps are as follows.

In order to identify the key factors, decision makers build the interval-number matrix by comparing factors. The interval-number initial direct incidence matrix is constructed.

Step 1: The initial direct incidence matrix $A(\tilde{a})=[a_{ij}^L,a_{ij}^U]_{n\times n}$ is constructed after experts' discussion. The value is selected in $(0,0)\leq(a_{ij}^L,a_{ij}^U)\leq(1,1)$. The closer to (0,0) the value is, the weaker the factor's relationship is. Similarly, the closer to (1,1) the value is, the stronger the factor's relationship is.

Step 2: Using the formula (5), the initial direct incidence matrix $A(\tilde{a})$ is mapped into a binary set's direct-relation matrix $A(\tau(\tilde{a}))$, where the number of language evaluation sets l can be selected as one of the following situation.

①In accordance with matching the traditional five-scale relationship with DEMATEL method, we can set the corresponding numbers of language evaluation set $l=5$.

②Dytczak (2013) proposed that the scale in DEMATEL method can use any positive integer from 0 to infinity[14]. For example, $l=9$.

Step 3: the binary set's direct-associated matrix is mapped to a 2-tuple matrix $(s_{ij},\eta_{ij})_{n\times n}$ by formula (6).

Step 4: the normalized real number matrix $G=[g_{ij}]_{n\times n}$ is obtained by the following formula.

$$g_{ij}=\frac{\Delta^{-1}\left(S_{ij},\eta_{ij}\right)}{\max\limits_{1\le i\le n}\sum\limits_{i=1}^{n}\Delta^{-1}\left(S_{ij},\eta_{ij}\right)},i,j=1,2,...,n \tag{9}$$

Step 5: the comprehensive influence-relation matrix T is obtained, where "I" is the unit matrix and"q"is the power of the normalized real number matrix.

$$T=\lim_{q\to\infty}(G+G^2+\cdots+G^q)=G(I-G)^{-1} \tag{10}$$

Step 6: Formula (11) and (12) are used to calculate the influence degree R and effect degree D.

$$R_i=\sum_{j=1}^{n}T_{ij} \tag{11}$$

$$D_j=\sum_{i=1}^{n}T_{ij} \tag{12}$$

Step 7: we calculate the center degree and the reason degree using formula (13) and (14).

$$P_i=R_i+D_i \tag{13}$$

$$H_i=R_i-D_i \tag{14}$$

The greater the center degree P_i is, the more important the corresponding factor i is. The decision maker should pay more attention to the factor and the factor might be a key factor. The reason degree H_i can be divided into two categories, the positive values are the reason factor, which might be regarded as the key factors. The negative values can be classified as outcome factors and generally are not considered as key factors.

IV. CASE ILLUSTRATION

To improve core competitiveness and reduce the risk of knowledge transfer, identifying the influence factors of knowledge transfer in IT outsourcing problem in Nanjing is used as a case illustration by the proposed method [6]. The key factors would be chosen from the various factors in the process of the IT outsourcing. There are eight various factors as follows. R1. learning culture, R2. transfer subject, R3. relationship quality, R4. investment degree, R5. knowledge characteristics, R6. policies and regulations, and R7. project characteristics, R8. transfer channel. The decision makers invited 5 experts to evaluate the interrelationship of influencing factors of knowledge transfer in IT outsourcing enterprises. After weighted average, the initial direct incidence matrix, evaluation information is interval number, is shown in table 1.

TABLE I. INITIAL DIRECT INCIDENCE MATRIX

	R1	R2	R3	R4	R5	R6	R7	R8
R1	(0,0)	(0.1,0.2)	(0,0.2)	(0,0)	(0.4,0.7)	(0.3,0.8)	(0.4,0.6)	(0.2,0.4)
R2	(0.1,0.4)	(0,0)	(0.2,0.7)	(0.1,0.4)	(0,0.2)	(0.3,0.6)	(0.6,1)	(0.3,0.5)
R3	(0.2,0.4)	(0.5,0.9)	(0,0)	(0.1,0.4)	(0.1,0.2)	(0.6,1)	(0.6,0.8)	(0.4,0.8)
R4	(0,0)	(0.4,0.7)	(0.5,0.7)	(0,0)	(0.2,04)	(0.5,0.8)	(0.4,0.8)	(0.3,0.7)
R5	(0.1,0.2)	(0.1,0.3)	(0.2,0.5)	(0,0)	(0,0)	(0.2,0.3)	(0.2,0.4)	(0.3,0.5)
R6	(0.1,0.2)	(0.4,0.8)	(0.3,0.6)	(0.2,0.3)	(0.2,0.4)	(0,0)	(0.6,1)	(0.4,0.9)
R7	(0.2,0.3)	(0.3,0.5)	(0.2,0.6)	(0,0.2)	(0.1,0.2)	(0.3,0.6)	(0,0)	(0.2,0.5)
R8	(0.2,0.4)	(0.1,0.2)	(0.1,0.4)	(0.1,0.3)	(0.2,0.3)	(0.1,0.4)	(0.3,0.6)	(0,0)

According to the proposed decision method, the key factors would be identified as follows.

(1) The initial direct incidence matrix transformed into an 2-tuple initial direct incidence matrix by step 2 and step 3 in which we set $l=5$. As is shown in table 2.

TABLE II. 2-TUPLE INITIAL DIRECT RELATION MATRIX

	R1	R2	R3	R4
R1	$(s_0,0)$	$(s_1,0.43)$	$(s_0,0.44)$	$(s_0,0)$
R2	$(s_1,0)$	$(s_0,0)$	$(s_2,-0.2)$	$(s_1,0)$
R3	$(s_1,0.22)$	$(s_3,-0.15)$	$(s_0,0)$	$(s_1,0)$
R4	$(s_0,0)$	$(s_2,0.18)$	$(s_2,0.44)$	$(s_0,0)$
R5	$(s_1,0.43)$	$(s_1,-0.22)$	$(s_1,0.36)$	$(s_0,0)$
R6	$(s_1,0.43)$	$(s_2,0.38)$	$(s_2,-0.18)$	$(s_1,0)$
R7	$(s_1,0)$	$(s_2,-0.44)$	$(s_2,-0.38)$	$(s_0,0.44)$
R8	$(s_1,0.22)$	$(s_1,0.43)$	$(s_1,0)$	$(s_1,-0.22)$

	R5	R6	R7	R8
R1	$(s_2,0.18)$	$(s_2,0.2)$	$(s_2,0)$	$(s_1,0.22)$
R2	$(s_0,0.44)$	$(s_2,-0.18)$	$(s_3,0.15)$	$(s_2,-0.44)$
R3	$(s_1,0.43)$	$(s_3,0.14)$	$(s_3,-0.22)$	$(s_2,0.38)$
R4	$(s_1,0.22)$	$(s_3,-0.36)$	$(s_2,0.38)$	$(s_2,0)$
R5	$(s_0,0)$	$(s_1,0)$	$(s_1,0.22)$	$(s_2,-0.44)$
R6	$(s_1,0.22)$	$(s_0,0)$	$(s_3,0.15)$	$(s_3,-0.4)$
R7	$(s_1,0.43)$	$(s_2,-0.18)$	$(s_0,0)$	$(s_1,0.36)$
R8	$(s_1,0)$	$(s_1,0)$	$(s_2,-0.18)$	$(s_0,0)$

(2) Decision Results

We use step 4, step 5, step 6 and step 7 to obtain the influence degree, the effect degree, the center degree and the reason degree .As is shown in table 3 and table 4.

TABLE III. INFLUENCE DEGREE AND EFFECT DEGREE

	R1	R2	R3	R4	R5	R6	R7	R8
Influence degree	2.1202	2.8379	3.5898	3.4215	1.7077	3.2241	2.2731	1.9306
Effect degree	1.6429	2.8784	2.7291	1.2616	1.7889	3.3707	4.1746	3.2588

TABLE IV. CENTER DEGREE AND REASON DEGREE

	R1	R2	R3	R4	R5	R6	R7	R8
Center degree	3.7632	5.7163	6.319	4.6831	3.4967	6.5948	6.4477	5.1895
Reason degree	0.4773	-0.0405	0.8607	2.1599	-0.0812	-0.1465	-1.9015	-1.3282

(3) Sort

The sort of center degree is $R6 \succ R3 \succ R7 \succ R2 \succ R8 \succ R4 \succ R1 \succ R5$, and the sort of reason degree is $R4 \succ R1 \succ R3 \succ R2 \succ R6 \succ R5 \succ R8 \succ R7$, which is completely consistent with the sorting results of literature[8].It verifies the rationality of the proposed method.

(4) To verify the science of the method, we set the number of language evaluation sets $l = 9$. The corresponding influence degree, effect degree, center degree and reason degree is shown in table 6 and table 7.

TABLE V. INFLUENCE DEGREE AND EFFECT DEGREE

	R1	R2	R3	R4	R5	R6	R7	R8
Influence degree	2.105	2.8119	3.567	3.383	1.7122	3.2024	2.2492	1.9202
Effect degree	1.6307	2.8788	2.6871	1.2501	1.7809	3.3292	4.1526	3.2417

TABLE VI. CENTER DEGREE AND REASON DEGREE

	R1	R2	R3	R4	R5	R6	R7	R8
Center degree	3.7357	5.6907	6.2541	4.6331	3.493	6.5316	6.4019	5.1618
Reason degree	0.4743	-0.0669	0.88	2.133	-0.0687	-0.1268	-1.9034	-1.3215

The sort of the center degree is $R6 \succ R3 \succ R7 \succ R2 \succ R8 \succ R4 \succ R1 \succ R5$, and the reason degree is $R4 \succ R1 \succ R3 \succ R2 \succ R6 \succ R5 \succ R8 \succ R7$, which is exactly the same as the result of $l = 5$.

V. ANALYSIS OF THE NEW METHOD

It can be seen that the method has following advantages from the above case. ① It is scientific. The number of language evaluation sets l can set dynamically while we are transforming the initial direct incidence matrix. The example shows that the sort of $l = 5$ is the same as $l = 9$ .However, other literature [6-8] are not realized. ② It is more reasonable. The interval number is taken as the whole to make a decision, which solves the irrational splitting problem of upper and lower limits of the interval number.③ It reduced information loss. Different from the other method, the method transforms information as 2-tuple.

VI. THE CONCLUSION

In this paper, we have developed a kind of decision-making in DEMATEL method based on 2-tuple. It transforms the initial direct incidence matrix into a 2-tuple initial direct incidence matrix, in which it not only sets the scale dynamically that make full use of the advantage of 2-tuple, but also avoids information loss in language transforming process. Meanwhile, it solves the irrational problem of upper and lower limits of the interval number. Compared with the case of literature, the new method is scientific and rational. The new DEMATEL method is more practical and has wide application space.

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