

# Error Pattern of Whole Number Subtraction in the 3<sup>rd</sup> Graders of Elementary School

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**Abstract—**The objective of research was to find error pattern and cause in Whole Number counting operation in the 3<sup>rd</sup> graders of Elementary School. To reveal the pattern and cause of error in Whole Number subtraction operation in the 3<sup>rd</sup> graders of Elementary school, the author employed qualitative approach with action research design that was collaborative in nature. This research was taken place in the 3<sup>rd</sup> grade of Public Elementary School 2 of Panjer, Kebumen Sub District. The subject of research consisted of 40 (forty) 3<sup>rd</sup> graders of Elementary School. From the result of research, it could be found that: error pattern of whole number subtraction operation made by the 3<sup>rd</sup> graders of Elementary School included: (a) reading and writing number, (b) writing number symbol, (c) subtraction concept, (d) ones, tens, hundreds, thousands numbers are subtracted with ones number, (e) incomplete algorithm, (f) random error, (g) place value, (h) basic fact, (i) large number subtracted with small number, (j) error form of  $0 - a = 0$ , (k) error form of  $a - b = 0$ , if  $a < b$ , and (l) forgetting to subtract the figure after it has been moved.

**Keywords—**error pattern, subtraction, whole number

## I. INTRODUCTION

The learning of three basic competencies, reading, writing, and counting, still needs improvement in elementary school. Counting basic ability intended is contained in Mathematics subject. The principle of Mathematic learning is to start from the simpler to the more complex one, from the concrete to the abstract one, and from the closest to the broader environments. Corresponding to the curriculum prevailing in the 3<sup>rd</sup> graders of Elementary School, the main counting operation (summing, subtraction, multiplication, division) learning material has been given, that is, whole number. It means that the 3<sup>rd</sup> graders of Elementary School should master the main counting operation with the number size adjusted with the

grade level. However, the fact shows that there are still obstacles encountered by teachers and students in Mathematic teaching and learning. One of obstacles that the 3<sup>rd</sup> graders encounter in learning Mathematics is the difficulty of learning whole number subtraction operation. This learning difficulty can be seen from the error the students make during working on the whole number subtraction operation. To improve such error appropriately, teachers should diagnose the error to find out type and cause of error.

To reveal type, cause, and the way to correct the error, the 3<sup>rd</sup> graders of Elementary School make in whole number subtraction operation, this research focuses on diagnosing the students' error in the whole number subtraction operation in the 3<sup>rd</sup> graders of Elementary School. Meanwhile, the focus of research can be elaborated into the following problems: (1) what is the error pattern of whole number subtraction operation made by the 3<sup>rd</sup> graders of Elementary School? and (2) what are the causes of error in the whole number subtraction operation made by the 3<sup>rd</sup> graders of Elementary School? The objective of research is to get a clear and detailed explanation about pattern and cause of error in the whole number subtraction operation made by the 3<sup>rd</sup> graders of Elementary School.

## II. RELATED WORKS/LITERATURE REVIEW

Diagnosis is defined as a process conducted by teacher to detect and to determine the errors the students make in absorbing the lesson delivered by teachers, particularly in doing academic chores. Those errors can be error in receiving concept and principle, and in using algorithm, counting operation, and etc. Recalling the students' condition with different ability and the learning process conducted classically, there are still some errors made by the students. Those errors cannot be ignored as Mathematics builds on a very thigh essence. Inadequate understanding on one subject matter will result in difficulty in learning the next subject matter.

When teachers teach learning material in the form of concept or skill, students will learn the material. Some students will be able to learn the learning

material smoothly and some others found learning difficulty. Teachers find the students' difficulty and then diagnose the cause of such difficulty. Next, teachers analyze the learning difficulty from learning, student, and material aspect and help the students with learning difficulty solve their problem. It is conducted to prevent the students' difficulty from getting worse. The measures of diagnosing, analyzing learning difficulty, and giving help are called Remedial instruction or remediation.

### III. MATERIAL & METHODOLOGY

#### A. Data

This research was taken place in the 2<sup>nd</sup> Public Elementary School of Panjer (hereafter called, Sekolah Dasar Negeri 2 Panjer), Kelurahan Panjer, Kebumen Sub District, Kebumen Regency, Central Java Province. Sekolah Dasar Negeri 2 Panjer is located in a lower-middle social-economic class settlement. It can be seen from the data of students' parents' occupation, most of which are worker (labor). Nevertheless, parents' interest in enrolling their students in their children is very high.

The data needed in this study was that of whole number subtraction operation error in the 3<sup>rd</sup> graders of SD Negeri 2 Panjer. The data was collected through test and analysis on the result of test and the result of interview about whole number subtraction operation. From the result of test, analysis and interview, data on error pattern and cause in whole number subtraction operation would be obtained.

#### B. Method

This research employed an Action Research type of qualitative approach. Action research aims to contribute to practical problems in problematic situation arising and to the objective of social science with collaboration in mutually approved ethical framework [7]. Thus, this action research aims to decide on or to give practical consideration in real situation, and its theory is validated through practice [7]. The form of action research employed in this study was the collaborative one. Collaborative action research was the research model involving teacher, developing staff, and experts in college or university aiming to improve the quality of learning practice, to develop education theory, and to prepare developing staff.

Data analysis employed in this research consisted of three activity plots occurring simultaneously: data reduction, data display, and conclusion drawing [10]. This activity was conducted continuously during and after data collection in field. Data validation in this study was carried out using triangulation technique. It was conducted through discussion and sharing opinion with teachers, peers and experts to draw an appropriate conclusion about the result of data analysis conducted by the author.

## IV. RESULTS AND DISCUSSION

#### A. Result

Considering the result of analysis on the result of test, observation, interview, and research action, it could be found that error pattern made by the 3<sup>rd</sup> graders of Elementary School in whole number subtraction operation includes: (1) reading and writing number, (2) writing symbol of number, (3) subtraction concept, (4) ones, tens, hundreds, and thousands digits subtracted by ones digit, (5) incomplete algorithm, (6) random error, (7) place value, (8) basic fact, (9) large number subtracted with small number, (10) error pattern of  $0 - a = 0$ , (11) error pattern of  $a - b = 0$ , if  $a < b$ , and (12) forgetting to subtract after the movement. Meanwhile, the causes of error are: (1) inadequate mastery of number and its symbol, (2) inadequate mastery of subtraction concept, (3) inadequate mastery of basic subtraction fact, (4) inadequate mastery of place value, (5) inadequate mastery of moving technique, (6) carelessness and imprecision, (7) not using estimated answer, and (8) writing less smoothly.

#### B. Discussion

Error pattern of reading and writing number with word (sentence) symbol made by students is due to their inadequate mastery of number symbol concept, or less clear writing. Meanwhile, the error pattern of writing a number's symbol made by students is due to their inadequate understanding on number symbol or place value concept. This fact can be seen from the students' work during writing a number's symbol using word symbol. In addition, students answer less smoothly the question about reading and writing number verbally during interview. It is in line with [1] stating that some students have special problems about language or in its relation to concept, presentation, and symbol. Meanwhile, the way of improving the error in reading and writing number symbol is to give the students the guidance about number and its symbol. The students are trained to utter and to write the symbol of a number presented by teacher both orally and in written manner. Additionally, to improve their understanding on number and its symbol, the students with teachers' guidance demonstrate the written/uttered number or the symbol of number presented using visual aid. The use of visual aid is considered as important as it is consistent with Elementary School students' intellectual development, at concrete operational level. The visual aid employed was 10-base block (ones cube). It is because the 10-base block is consistent with the material presented. It is also intended to make, the students familiar to the visual aid earlier, so that they will not feel strange with the visual aid in the next activity. Recalling that the 10-base block visual aid explains subtraction concept, basic fact of subtraction, place value, and technique of moving, this tool is used in this occasion.

Error pattern in whole number subtraction concept made by students is due to their less mastery of subtraction concept and their carelessness or imprecision during working on the problem. The students' inadequate mastery of subtraction concept can be seen when they were given problem  $\dots + 14 = 31$ . They count by means of trying out and working on it using summing operation. They determine a number, and then add it in order to produce 31. No student answer the problem by means of  $31 - 14 = \dots$ . Engelhardt [1] states that this case proves that the students understand poorly the definition of subtraction as the inverse of summation. Considering the reality in the class, teachers did not introduce the subtraction concept as the inverse of summation. Teachers only inculcate subtraction concept defined as “*taking*” or “*borrowing*”. To strengthen the mastery of subtraction concept, teachers should introduce the students with the subtraction concept through experience with various representations such as (1) taking, (2) comparing, (3) completing (inverse of summation), and (4) whole-part-part [9]. Students' carelessness and imprecision can be seen from their way of working on the problem  $\dots + 14 = 31$ . Because, the students are not introduced with the definition of subtraction as the inverse of summation, they solve the problem by trialing and counting using finger. This way is done because the students have poor summing skill. They do not use palm leaf rib or other visual aids. They only have 10 hand fingers, so that some fingers are counted more than once. It is at this time that they make error. Thus, the cause of students' error is imprecision or carelessness during counting using finger. The ways of correcting error due to inadequate mastery of subtraction concept are: (1) to define subtraction as “being taken”, (2) to subtract through “comparing” process, (3) to define subtraction as “the inverse of summation”, and (4) to determine one unknown part in whole number subtraction operation [6], [9]. In this activity, the elements of subtraction operation are also introduced. Because subtraction concept can be presented in some ways and it is presented in word problem, teachers should inculcate the conception on the problem, write it in mathematic sentence, and answer the question posed. In addition, teachers also guide the students to demonstrate the meaning of mathematic sentence existing in visual aid (10-base block). In this occasion, teachers also remind the students to count more carefully and precisely.

The error pattern of ones, tens, and hundreds number is that the subtracted number is subtracted with ones number in the subtracting number when they are given subtraction problem involving the subtracted number consisting of two or more digits with subtracting number consisting of one digit. This error is because the students understand the subtraction concept, students do not use estimated answer, or students understand the meaning of number symbol poorly. The students' poor

understanding on the meaning of number symbol (multi digit) affects the subtraction operation [5]. The students understand poorly the elements of subtraction: subtracted number, subtracting number, and product of subtraction. Additionally, in answering the problem, the students do not employ estimated answer to be yielded. The students do not think that the problem given (stacked or vertical-format subtraction) can be changed into leveled or horizontal-format subtraction. The alternative way of correcting this type of error is to improve the mastery of subtraction concept and to improve the conception on number and its symbol. The students are guided to determine estimated answer to the problem worked on in order to get a correct answer.

The error pattern of incomplete algorithm is made by students when they work on subtraction problem using the correct procedure but the procedure is not done completely, or only some steps of correct procedure are done. This error is in line with Roberts [1] stating that the error the students make can be imperfect algorithm or incomplete algorithm. The students use incorrect working procedure. It is in line with [1] stating that the error is made because the students understand poorly the concepts of subtraction and value place, or they work on the problem insincerely. The way of correcting this error is to emphasize on correct reduction procedure corresponding to prevailing algorithm in whole number subtraction operation. To improve the students' understanding, visual aid is used in this activity to help the students abstracting the problem they work on.

Random error pattern is the one made by students by not creating a certain pattern. Even the students cannot explain what they have written. Roberts [1] mentions that this type of error is random response. This condition is reported by Ashlock [1] finding that from the result of interview with students, it can be seen that 3% of response is ignored because they cannot give the reason of the answer written. From the result of observation, it can be seen that some factors may cause the error: students concentrate poorly on the problem, the time allocated is limited, students master poorly the subtraction concept or technique of moving. The way of correcting this type of error is to guide the students to be more careful, precise, and to concentrate more during working on the problem. Teachers always remind the students about this in every learning process, so that it can be a habit.

Error pattern of place value is made by students when they are told to write a number's place value, particularly in the problem involving place value of thousands, hundreds, tens, and ones, not written orderly. This error is due to students understand poorly the concept of place value. This error is due to the students understanding poorly the relationship between place value and long form of a number, or

students reading imprecisely the problem given by teachers. To correct this type of error, teachers improve the conception on place value including ones, tens, hundreds, and thousands. Visual aid used is 10-base block consisting of ones (ones cube), tens (rod cube), hundreds (plate cube), and thousands (large cube arranged of 1000 ones cubes). In this activity, varying trainings are given. In addition, teachers guide the students to connect the concept of place value to the long form of a number. Then, teachers attribute place value concept and long form to horizontal-format subtraction operation.

Error pattern in the basic fact of whole number subtraction operation is made by students when they work on subtraction operation problem given. In fact, all students becoming the subject of research make this type of error with varying error locations. From the students' work and the result of interview, it can be found that generally the type of error in basic fact the students make is due to inadequate conception on basic fact of subtraction. It is in line with [1,12] explaining that the main cause of the error in the basic fact of whole number subtraction. The way of correcting this type of error is to emphasize more on the conception of basic fact concept in whole number subtraction. This activity is done by emphasizing on the definition of and the characteristics of basic fact, distinguishing basic fact from non-basic fact, and memorizing the basic fact of whole number subtraction. This activity of memorizing basic fact should be conducted since the first grade and in the 1<sup>st</sup> grade the students should be introduced with whole number subtraction operation.

Students make error pattern in large number subtracted with small number when they work on the subtraction problem involving technique of moving. This pattern of error is due to students understanding poorly the basic fact of subtraction and technique of moving [12], or using estimated answer inadequately during working on the problem. The students understand poorly the basic fact of subtraction, so that when they find the case aforementioned they consider that  $a - b = b - a$ . The way of correcting this type of error is to remind the students about basic fact. When they still find difficulty, teachers should help them using visual aid. Meanwhile, the way of correcting the error due to moving technique is to use long form or scratching technique.

Students make error pattern of  $0 - a = 0$  when they work on subtraction problem in which the subtracted number involved number 0, while the subtracting number is not 0. To classify this error into the error of  $a - b = 0$ , if  $a < b$ . From the result of observation, it can be found that this pattern of error is due to the students understanding poorly the basic fact of subtraction or technique of moving in subtraction process. Because of the poor mastery of basic fact and moving technique, during working on the problem of  $0 - a$ , the students answer 0 directly. The students

understand poorly that  $0 - 0$  is not equal to  $a - 0$ . Additionally, the students do not think that in subtraction, when a number cannot be subtracted, any number should be moved. Thus, the students understand poorly the technique of moving in subtraction operation [1,5,12]. The way of correcting this error is to guide the students to remember the basic fact of subtraction, to remind the students about the technique of moving with long or scratching method.

The students make error pattern of  $a - b = 0$ , if  $a < b$  when they complete subtraction operation with number at certain place of the subtracted number smaller than the number located at the sample place in subtracting number. [12] stated that this type of case is subtraction involving the technique of moving. From the result of observation, it can be found that this error pattern is due to the students understanding poorly the basic fact of subtraction or the technique of moving in subtraction operation. Because the students master poorly the basic fact or the moving technique, when they deal with such the case, they write 0 directly as the product. The ways of correcting this error are to remind the students about basic fact of subtraction and to guide technique of moving in whole number subtraction operation.

The students make error pattern of forgetting to subtract after the movement when they complete the subtraction operation involving technique of moving. This type of error is the one made mostly by students. All students as the subject of research make error of forgetting to subtract after the movement. Considering the students' work and the result of interview, it can be found that the error the students make is due to their inadequate mastery of moving technique in subtraction operation [5,12]. The way of correcting this error is to remind the students about the long form of a number, and then to connect the long form to stacked (vertical-format) subtraction operation with long form. Because the ultimate goal of vertical-form subtraction operation is the short form, in this activity the teachers should focus more on subtraction operation using short way by involving technique of moving. Teachers can use two ways to explain technique of moving: (1) long form, and (2) scratching technique. From the result of observation, it can be found that the error made most frequently by students related to technique of moving is subtraction operation with the subtracted number involving more than one 0 (zero) number. For example, 1003-7. For that reason, teachers should give the students more practice with varying numbers. During working on the problem, students' carefulness and precision should be built and emphasized on, recalling that some causes of error are carelessness and imprecision.

Considering the discussion above, it can be concluded that for the 3<sup>rd</sup> graders of Elementary School (8-9 years old) with parents' lower-middle social-economic background still make many errors in

whole number subtraction operation. The errors the students make can be classified into 12 patterns: (1) reading and writing number, (2) writing number symbol, (3) subtraction concept, (4) ones, tens, hundreds, thousands numbers are subtracted with ones number, (5) incomplete algorithm, (6) random error, (7) place value, (8) basic fact, (9) large number subtracted with small number, (10) error form of  $0 - a = 0$ , (11) error form of  $a - b = 0$ , if  $a < b$ , and (12) forgetting to subtract the figure after it has been moved. Meanwhile the cause of error is that the students master inadequately: (1) number and its symbol, (2) subtraction concept, (3) basic fact of subtraction, (4) place value, (5) technique of moving, (6) carelessness/imprecision, (7) estimated answer, and (8) unclear writing. The students making the errors have parents with lower-middle education, social, and economic conditions.

#### V. CONCLUSION

Considering the result of data analysis, the following conclusions can be drawn regarding the error pattern and its cause in whole number subtraction in the 3<sup>rd</sup> graders of elementary school:

- 1) The errors the students make can be classified into 12 patterns: (1) reading and writing number, (2) writing number symbol, (3) subtraction concept, (4) ones, tens, hundreds, thousands numbers are subtracted with ones number, (5) incomplete algorithm, (6) random error, (7) place value, (8) basic fact, (9) large number subtracted with small number, (10) error form of  $0 - a = 0$ , (11) error form of  $a - b = 0$ , if  $a < b$ , and (12) forgetting to subtract the figure after it has been moved.
- 2) The cause of error is that the students master inadequately: (1) number and its symbol, (2) subtraction concept, (3) basic fact of subtraction, (4) place value, (5) technique of moving, (6) carelessness/imprecision, (7) estimated answer, and (8) unclear writing.

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