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Econometric Models with Discrete Dependent Variable in Portfolio Analysis

Maria V. Dobrina
Department of Informational Technologies and Mathematical Methods in Economics
Voronezh State University
Voronezh, Russia
E-mail: nice.smirnova@yandex.ru

Yana A. Yurova
Department of Informational Technologies and Mathematical Methods in Economics
Voronezh State University
Voronezh, Russia
E-mail: ya.yurova@mail.ru

Galina V. Shurshikova
Department of Informational Technologies and Mathematical Methods in Economics
Voronezh State University
Voronezh, Russia
E-mail: gshurshikova@list.ru

***Abstract*—The authors substantiated the need for the use of econometric models with a discrete dependent variable for modeling investment decisions in the stock market. Sharpe's use of one-factor regression models in portfolio analysis allowed updating the theory of portfolio investment with new results. The authors created the risk structural idea and introduced the portfolio beta concept. Likewise the authors marked that in new econometric models the dependent variable is discrete. In many cases, it is convenient to represent the dynamics of the financial assets profitability in the discrete time series form. The authors justified this way the idea of applying these new models in portfolio investment tasks. The authors considered the possibility of using the binary choice econometric model for modeling the profitability of a market asset. Also the authors obtained the formulas for calculating the yield and variance of the asset on the basis of this model. The authors offered to use this derived formulas in the optimal portfolio of securities construction.**

Keywords—financial asset; the optimal portfolio; the profitability of the asset; econometric models; discrete dependent variable

I. INTRODUCTION

The relevance of the new approaches development to the formation of an effective securities portfolio has not decreased for several decades. There are a number of objective and subjective reasons that support this relevance and at the same time constrain the critical attitude to the modern theory of portfolio investment.

First of all, it should be noted that despite its theoretical significance and usefulness for the justification of practical solutions, the Markowitz model has not become a tool of financial management. Perhaps this is the reason that continues to stimulate the search for new approaches to the solution of an absolutely understandable task.

There is another issue the decision of which all suits and that is not accepted to criticize. It concerns the set of investment opportunities introduced by Markowitz, which is described by only two characteristics: profitability and risk. Operating with these two characteristics greatly simplifies the procedures of portfolio modeling, contributes to the implementation of a formal description for a tasks number, but at the same time forces to abandon the reproduction of the real multidimensionality of the stock market processes, also from some methods, the use of which would significantly expand portfolio analysis.

Below, as an alternative to Markowitz's optimization approach based on deterministic linear dependencies, it is proposed to use a probabilistic description of the processes interaction for financial assets in the stock market. The apparatus of regression equations with a discrete dependent variable used for this purpose significantly expands the possibilities and increases the level of simulation adequacy under uncertainty.

II. MAIN STAGES OF OPTIMIZATION APPROACH DEVELOPMENT

The first model of optimal portfolio was proposed by Markowitz [8] [9] [10]:

$$\mathbf{w}'\mathbf{\Sigma}\mathbf{w} \longrightarrow \min \quad (1)$$

$$\mathbf{w}'\mathbf{r} = \mu \quad (2)$$

$$\mathbf{w}'\mathbf{i} = 1 \quad (3)$$

Wherein, $\mathbf{\Sigma}$ — the covariance matrix of the relationships between assets; $\mathbf{w}$ — equity participation vector of assets in the portfolio; $\mathbf{i}$ — auxiliary vector of units.

The development of the efficient market modern theory, in fact, was started with this model. Recommendations that have been obtained through this model are widely used in the practice of justifying investment decisions. All investors recognized the need for risk diversification and successfully implemented it in practice. There was a convincing justification for the relationship between profitability and risk, which allowed investors to significantly improve the reliability of their decisions [7]. At the same time, as noted above, the model of optimal portfolio investment, despite its theoretical significance and usefulness for the justification of practical solutions, has not become such a tool as the Black-Scholes formula. Therefore, the search for a more effective version of this model continues [1].

Unfortunately, the results of these searches were only partially effective. But, even in these cases, almost all the results were important milestones in the development of the portfolio investment theory.

So Tobin [4], foreseeing in his model

$$\mathbf{w}'\mathbf{\Sigma}\mathbf{w} \longrightarrow \min \quad (4)$$

$$r_f w_0 + \mathbf{w}'\mathbf{r} = \mu \quad (5)$$

$$w_0 + \mathbf{w}'\mathbf{i} = 1 \quad (6)$$

the inclusion of a risk-free asset in the portfolio obtained a result called the separability theorem [2].

The next model, which introduced new elements into the theory of portfolio investment, is a model that takes into account the investor's attitude to risk through the parameter τ

$$\tau\mathbf{w}'\mathbf{r} - \mathbf{w}'\mathbf{\Sigma}\mathbf{w} \longrightarrow \min \quad (7)$$

$$\mathbf{w}'\mathbf{i} = 1 \quad (8)$$

With the help of this model, the result was obtained, providing for the formation of two components portfolio:

$$\mathbf{w} = \mathbf{w}_{\min} + \tau\mathbf{w}_c \quad (9)$$

The first component is a portfolio with a minimum risk of all effective portfolios, and the second component is a self-financing portfolio, providing for the purchase of some assets through the sale of others, in order to obtain the maximum possible yield [5]. The resulting portfolio is a linear combination of these components, with the share of the self-financing portfolio generating the maximum return being regulated by a parameter reflecting the investor's risk aversion.

The considered models are the basis of the modern theory in portfolio investment. But the use of these models in the practice of portfolio investment does not provide reliable results, as, in fact, they are used to build portfolios of missed opportunities. These models can be successful only if all the patterns of the past are repeated in the future [3]. The probability of such a situation is almost zero. At the same time, in such a situation, there is a desire to use an approach in which the actual values of the past are replaced by predictive estimates of the future in the construction of the model. To do this, first of all, we need a model that adequately describes the mechanism of forming the profitability of a financial asset in the stock market. Convenient for these purposes was the econometric version of the Lintner-Sharp model [6]:

$$r_{it} = \alpha_i + \beta_i r_{It} + \varepsilon_{it} \quad (10)$$

where r_{it} — the yield of the i-th asset at time t; r_{It} — the profitability of the index at time t; α_i, β_i — coefficients of the i-th asset for the regression equation; ε_{it} — a random component of the regression equation for the i-th asset.

W. Sharp was the first to apply this single-factor regression model to modeling portfolio solutions. Its diagonal portfolio investment model is written as follows

$$\mathbf{w}'_{n+1}\mathbf{\Sigma}_d\mathbf{w}_{n+1} \longrightarrow \min \quad (11)$$

$$\mathbf{w}'_{n+1}\boldsymbol{\alpha} = \mu \quad (12)$$

$$\mathbf{w}'\mathbf{i} = 1 \quad (13)$$

$$\mathbf{w}'\boldsymbol{\beta} = 1 \quad (14)$$

$\mathbf{\Sigma}_d$ — diagonal matrix with the residual variances elements $\sigma^2_{\varepsilon_i}$ on the diagonal and the variances of the market index σ^2_I at the end of the diagonal; $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ — the vectors of regression model coefficients; $\mathbf{w}_{n+1}$ — vector, the first n components of which define the structure of the portfolio, and the latter is the "portfolio beta", which, in accordance with the last limitation of the model, is defined as the products sum of the portfolio weight coefficients with financial assets beta coefficients included in the portfolio and can be written as [7]:

$$w_{n+1} = \sum_{i=1}^{n} w_i\beta_i \quad (15)$$

W. Sharp with his model has significantly expanded the possibilities of portfolio analysis. First of all, the concept of "portfolio beta", which essentially allows you to assess the preference of investments in the market assets portfolio compared to investments in a risk-free asset. If, by analogy with the equation of the Lintner-Sharpe

$$r_i = r_f + \beta_i(r_I - r_f) \quad (16)$$

write the equation for the portfolio, then get the expression:

$$r_p = r_f + (\sum_{i=1}^{n} w_i\beta_i)(r_I - r_f) \quad (17)$$

From which it follows that the risk premium for investments in the portfolio is proportional to the "portfolio beta" [2].

The value of the "portfolio beta", being a weighted value, is less than the maximum value of this parameter, but more than the minimum value. As a rule, "portfolio beta" demonstrates higher stability than the beta of any asset included in the portfolio. It follows from this fact that investing in a portfolio is preferable to investing in an individual asset.

In addition, the risk in the Sharpe model received a structured representation. It identified a systematic component that characterizes the risk of the market in which the assets included in the portfolio are traded and a diversified component that characterizes the own risk of the portfolio assets [1]. This representation of risk makes clear the opportunities that are available in risk management and that are most often used in solving practical problems.

Summarizing the above, it can be stated that the development of the portfolio investment theory was based on two factors: the modification factor of the model representation in the framework of the optimization approach and the factor of data information description used to build models of portfolio investment. Moreover, as shown by the model of W. Sharpe, information description plays no less important role than making changes to the model.

III. New Approaches to the Modeling of the Assets Dynamics

The above-mentioned dependence of portfolio investment decisions on the information description of the initial data focuses on improving the adequacy of this description. And not only in order to build a more efficient portfolio, but also to expand the opportunities for analysis of market processes. This need has now become apparent. It is known that simultaneously with the development of portfolio theory, investment strategies were developed, the possibility of practical implementation of which largely depends on the results of market analysis. And the possibilities of analysis, as it is easy to understand from the logic of our reasoning, directly depend on the methods and models of describing market processes.

As you know, the theory of portfolio investment uses two models to describe market processes. In fact, the model of the observed values:

$$r_{it} = \bar{r}_i + (r_{it} - \bar{r}_i) \quad (18)$$

and the regression model (10), the expectation of which is represented as:

$$\mathrm{E}(r_{it}) = \alpha_i + \beta_i x_{it} \quad (19)$$

At a certain stage, the capabilities of these models suited both academic and applied science [5]. However, more and more questions have accumulated and continue to accumulate, the answers to which are almost impossible to obtain using only these models.

In addition, the most painful question remains unanswered. Why such a wonderful theory of portfolio investment does not have a full-scale application? There may be a few General recommendations. One of these recommendations is probably the recommendation to use the beta coefficients of the regression model (6) when choosing the most promising assets for investment. For all assets, these coefficients are calculated daily on the New-York stock exchange and investors are able to use them in their practice. This fact significantly reinforces the thesis of the need to improve the adequacy of the model description for market processes. In addition, the practical use success of the well-known Black-Scholes formula makes you want to understand the reasons for this success and, if possible, to find out the features, apply them in portfolio investment. A feature, in our opinion, is that this formula retains the probabilistic nature of market processes.

It can not be denied that in the theory of portfolio investment with understanding relate to the fact that the calculations are carried out in an environment of random processes. But at transition to mathematical expectation which is necessary for correctness in similar situations, all uncertainty of casual processes turns into the stiffened constants reflecting results of the past and having, as a rule, extremely low probability of repetition in the future [4]. The logic of the reasoning leads to an unambiguous conclusion, the meaning of which is the need to consider a new model, which otherwise describes the probabilistic nature of financial markets.

Incidentally, this is facilitated not only the need to solve the problem of the insufficient adequacy level for the models used in the modeling of market processes, but the emergence of a new apparatus of econometric modeling. This refers to regression models with a discrete dependent variable, one of which, in our opinion, could be used in the Cox-Ross-Rubinstein model [4]. Acting on the analogy of how W. Sharpe expressed the dependence of the asset yield on the level of average yield in the stock market, we write a nonlinear model of this dependence, using a binary choice model [6].

Usually in the binary choice model the simulated variable takes two values 0 and 1 [2]. For our purposes, since the yield of an asset can be both positive and negative, it is necessary to assume that the simulated variable can also take both negative and positive values [1]. Thus, in the most general form, the pattern that follows the dynamics of the asset yield can be represented in a fairly simple form:

$$r_{it} = d_i x_{it} \quad (20)$$

Wherein, x_{it} is a random variable that takes values according to the following rule

$$x_{it} = \begin{cases} \text{took a value} +1, \text{if the yield was positive} \\ \text{took a value} -1, \text{if the yield was negative.} \end{cases}$$

Since for use in the calculations of yield, represented as a random variable, you need to know its mathematical expectation, for this, first of all, identify the conditional distribution in the form of a regression equation with a dichotomous dependent variable, which in general can be written in the following form:

$$\mathrm{P}(x_{it} = 1 / r_{It}) = \mathrm{F}(b_0 + b_1 r_{It}) \quad (21)$$

This ratio provides for the calculation by means of F function the probability of an event $x_{it} = 1$ consisting in the growth of the asset yield at a time depending on the state of the stock market described by the yield of the index at the same time [13].

In general, the F function must have all the properties of the distribution function. Therefore, probabilistic distributions of random variables are most often used as this function. If, for example, a normal probability distribution function is used

$$F(b_0 + b_1 r) = \int_{-\infty}^{b_0+b_1 r} e^{-\frac{z^2}{2}} dz \quad (22)$$

the regression dependence obtained in this case is called the probit model [5]. In the same cases when the logistic function is selected for modeling:

$$F(b_0 + b_1 r) = \frac{1}{1 + e^{b_0 + b_1 r}} \quad (23)$$

the constructed regression equation is called the logit model.

The logit model is much more convenient than the probit model when performing analysis and various analytical transformations [7]. Therefore, further presentation will be devoted to the study of the practical use possibilities for the logit model in the tasks of portfolio investment.

IV. Return and Risk of the Market Asset

Assuming that a binary choice model is constructed, and for each one, the probability of expected positive returns can be calculated

$$P_{it} = \frac{1}{1 + e^{b_0 + b_1 r_{It}}} \quad (24)$$

then write the mathematical expectation of the asset yield

$$E(r_{it} / r_{It}) = d_i[P_{it} - (1 - P_{it})] = d_i(2P_{it} - 1) \quad (25)$$

The expressions (24) and (10) give an identical explanation for the possible decrease in the yield of the asset. According to these models, a decrease in the yield of an asset follows a decrease in the average yield on the market. But, if in accordance with (10) it always happens, from (25) should only decrease the probability of positive income. Perhaps the model (25) more accurately reflects the market reality of possible yield fluctuations. Returns on some assets do not always follow market returns. These are known facts.

The next comparison of these models concerns their sensitivity to market changes. In the Sharpe model, the sensitivity characterizes the beta coefficient, which is a constant value equal to the first derivative of the yield:

$$[\alpha_i + \beta_i r_I]` = \beta_i \quad (26)$$

In proportion to this coefficient, changes in the yield of the corresponding asset occur when the average yield of the market changes. It is recommended to focus on when forming a portfolio, but do not always warn that a high beta value is preferable only in cases where the market is growing.

Differentiating (25) we get an idea of the sensitivity for the asset return, the patterns of change which takes into account the probabilistic nature of the stock market. Indeed, after differentiation we obtain the expression:

$$[d_i(2P_{it} - 1)]` = 2d_i P_{it}(1 - P_{it})b_i \quad (27)$$

in which there is a probability.

As in (26), the sensitivity is proportional to the regression coefficient, but taking into account the level of uncertainty, which is described by the probability density of the logistic distribution law. The higher the probability density is, the higher the sensitivity level is [6]. As it is easy to understand from the expression (27) the highest level of sensitivity is achieved when $P_i = 0{,}5$.

Determine the variance in the case where the yield of a market asset is determined by the expression (20). Acting in accordance with the classical definition of variance, we derive the formula for its calculation, for which we carry out a series of transformations of the mathematical expectation for deviations squares sum:

$$\sigma_i^2 = E[(d_i x_i - E(d_i x_i))^2] = E[d_i^2(x_i - 2P_i + 1)^2] =$$
$$= E[d_i^2(x_i^2 + 4P_i^2 + 1 - 4P_i x_i + 2x_i - 4P_i)]$$

Given that the expectation of a random variable x_i is determined by the expression $E(x_i) = 2P_i - 1$, after simple transformations we obtain:

$$\sigma_i^2 = d_i^2(-4P_i^2 + 4P_i) = 4d_i^2 P_i(1 - P_i) \quad (28)$$

Thus, in the variance of profitability, as in the first derivative, there is the same reflection mechanism of the process probabilistic nature for profitability formation using the density of distribution. Moreover, both characteristics (sensitivity and dispersion) reach their maximum value at $P_i = 0{,}5$. This is a natural result, since, in fact, it is a bifurcation point, in which the achieved level of profitability remains unchanged with the highest level of dispersion. In addition, the high level of variance according to derivative (27) is well correlated with the high level of reaction for the asset yield in the event of changes in the market.

In fact, if at the point the probability of growth is equal to the probability of a decrease in the asset yield, then these are clear signs of a bifurcation state in which, as evidenced by (25), the yield does not change, and in accordance with (28) there is the highest variance. In addition, at this point there is the highest level of uncertainty:

$$H_i = -P_i \log_2 P_i - (1 - P_i)\log_2(1 - P_i) \quad (29),$$

since the calculated by (29) entropy has the highest value.

Summarizing the above, it should be noted that the proposed approach to the description of market processes allows obtaining explicitly additional information about the nature of market processes, which is certainly useful to justify investment decisions.

V. Conclusion

After W. Sharpe, who suggested using econometric models to solve portfolio investment issues, there were not so many studies that would provide for further development of the direction focused on the model presentation of data used in the justification of portfolio decisions. Perhaps the most notable proposal concerned the construction of a Sharpe model based on adaptive regression equations. At the same time, econometrics has been supplemented with new models that have not yet received a registration in the tasks of portfolio investment. The research results of the questions connected with an approaches variety to modeling of portfolio decisions given in article have brought to need of application of new econometric models with a discrete dependent variable. It is shown that the reflection of market processes using these models more fully reflects the stochastic nature of the market, and, consequently, their application should improve the efficiency of investment decisions.

There is one more question which is not accepted to criticize. It concerns the set of investment opportunities introduced by Markowitz, which is described by only two characteristics: profitability and risk. Operating with these two characteristics greatly simplifies the procedures of portfolio analysis, contributes to the simplified formalization of tasks number, but at the same time forces to abandon the reproduction of the real multidimensionality of stock market processes, from some methods and techniques, the use of which would significantly expand portfolio analysis.

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