# Regression-matrix Model and Its Application in Socio-economic Foresight* 

Valery Davnis<br>Voronezh State University<br>Voronezh, Russia<br>E-mail: vdavnis@mail.ru

Tatiana Pankova<br>Voronezh State University<br>Voronezh, Russia<br>E-mail: pankova@rgph.vsu.ru

Elizaveta Shulgina<br>Voronezh State University<br>Voronezh, Russia<br>E-mail: elizaveta_aleksandrovna.96@mail.ru


#### Abstract

This paper proposes a regression-matrix model for predicting the socio-economic development of regional systems. The ability to build up such a model provides for the classification of indicators into groups depending on their socio-economic purpose. The structured data presentation has led to the pre-requisite for a graded construction of models that ensure a proactive balance of the dynamics at two levels: cluster-type and system-based. According to the assumption, group balance should ensure the consistency of the expected growth of each indicator with the group dynamics of the corresponding block. The systemic balance is provided by the introduction of group dynamics indicators in the form of principal components, in turn, the regression-matrix approach is applied.


Keywords-matrix predictor; autoregression models; multidimensionality; systemic balance; socio-economic foresight

## I. Introduction

In the modern world, the role of forecasting in solving economic problems has increased significantly. The foresight has become a major reference point in decision-making for both commercial and state government agencies of management and control.

If we consider the approaches used for making forecasts of the socio-economic development of systems, it is possible to see that the proposed toolkit is quite diverse. Primarily these are well-known econometric models [1]. Various neural network models are popular today [2]. Also, do not forget about the possibility of using genetic algorithms [3].

At the same time, the rather complex interaction of economic processes, taking into account their multidimensional nature, is the main problem in developing the apparatus for forecasting socio-economic development.

The problem of multidimensionality of data makes it

[^0]necessary to search for alternative solutions that could meet modern requirements for the reflection of real processes, the main of which is the system consistency of multidimensional forecast calculations that have a meaningful meaning. Consequently, it is impossible to deny the fact that today the relevance of developing models of a new type continues to grow.

It is assumed that one of the solutions to this problem can be considered the combination of the econometric component with the algorithmic capabilities of multidimensional forecasting. For these purposes, the authors propose to build up a combined model based on a matrix predictor, in which the econometric approach will be built into the multidimensional forecasting procedure [4]. The issue of consistency of multidimensional data in this case is solved by the system balance capabilities, which is achieved in the matrix predictor based on the use of indirect growth rates.

## II. Basic Ideas for Building a Combined Model

Before proceeding to the description of the model itself, it is first proposed to consider the basic principles of its construction.

Schematically, the idea of building a model can be represented in the following form (see "Fig. 1"):


Fig. 1. Schematic diagram of system-balanced forecast calculations.

The logic of building the model presented in "Fig. 1" should be clarified.

Firstly, due to the fact that a sufficiently large number of indicators are predicted, it seems unlikely to build a single model that adequately reflects all the diversity of indicators. Therefore, for such a number of indicators it is proposed to structure the data by dividing them into special blocks, taking into account the substantial meaning.

Secondly, for each block, it is necessary to build up a model reproducing the dynamics of each individual indicator in a system-balanced calculation of the forecast estimates of all indicators of the block. In addition, the group dynamics of each block must be balanced with the group dynamics of other blocks, for which it is proposed to use the specially introduced "group dynamics indicators".

Next, the indicators use a forecasting procedure that takes into account systemic balance, similar to the one with which the indicators of each block are predicted.

Finally, balanced indicators are built into the model system and the "block approach with separated variables" is applied to the extended (due to group dynamics indicators) composition of the block [4].

The above sequence of actions builds the logic of the implementation of the main provisions of the construction of the combined model, which will be described in detail later.

## III. System-balanced Forecast Calculations

Developing the proposed approach based on combining the matrix predictor with the econometric component, let us pay special attention to the idea of a consistent modification of the matrix predictor, where the idea of turning the matrix predictor into a regression-matrix one takes a special place [5].

Considering in this case the futility of building multifactor models, we will use the autoregressive principle to describe individual trends. This is due to the fact that autoregressive models are fairly simple to use and are a reliable tool for predictive calculations, and also take into account the fact that the economy of the future grows from the past.

It should be noted that the combined model based on the matrix predictor is in fact the integration into a single system of models. Therefore, specification refinement begins with consideration of each individual model of this system, represented in its initial form by an autoregressive first-order equation

$$
\begin{equation*}
\mathrm{x}_{\mathrm{ti}}=\alpha_{\mathrm{i}}+\beta_{\mathrm{i}} \mathrm{x}_{\mathrm{t}-\mathrm{i}}+\varepsilon_{\mathrm{ti}} \tag{1}
\end{equation*}
$$

where $\mathrm{i}=\overline{1, \mathrm{n}} \quad$ is the number of indicators corresponding to the number of models;
$\alpha_{i}, \beta_{i}$ is estimated coefficients of the autoregressive equation of each i indicator;
$\varepsilon_{\mathrm{ti}}$ is random component characterizing that part of the change $\mathrm{X}_{\mathrm{ti}}$, which is not explained by the corresponding changes in the past.

In order to provide the possibility of constructing a matrix predictor, it is necessary to present the lagging variable of the autoregressive model (1) in the following form:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{t}-1 \mathrm{i}}=\mathrm{x}_{\mathrm{t}-2 \mathrm{i}}+\left(\mathrm{x}_{\mathrm{t}-1 \mathrm{i}}-\mathrm{x}_{\mathrm{t}-2 \mathrm{i}}\right) \tag{2}
\end{equation*}
$$

Wherein, $\mathrm{i}=\overline{1, \mathrm{n}}$.
With this view it is possible to build up a matrix of indirect growth rates, ensuring consistency in the dynamics of all the forecasted indicators.

After the lagging variable in the form of the expression (2) is substituted into the autoregression equation (1) the next step is to estimate the model using the method of least squares. Consequently, we obtain

$$
\begin{equation*}
\mathrm{x}_{\mathrm{ti}}=\hat{\alpha}_{\mathrm{i}}+\hat{\beta}_{\mathrm{i}} \mathrm{x}_{\mathrm{t}-2 \mathrm{i}}+\hat{\mathrm{d}}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{t}-1 \mathrm{i}}-\mathrm{x}_{\mathrm{t}-2 \mathrm{i}}\right) \tag{3}
\end{equation*}
$$

Wherein, $\mathrm{i}=\overline{1, \mathrm{n}}$.
The equation that was obtained by converting as a result of substitution (2) into expression (1) is rather not an autoregressive model, but a multifactorial regression, in which the factors are the value of the lagging variable and the increase in the indicator over the past period.

Then the following question arises: what observations should be used as factors (the lagging variable and the increase in the indicator over the past period) when calculating the predicted values of the model.

It is assumed that there are several options for solving this problem, one of which is the usage of recent observations. The application of recent observations is based on the fact that more recent data better reflect current trends, and these trends are more likely to be carried forward.

Returning to the mathematical expression (3), you should pay attention to the gains. The increase in the indicator in the framework of the econometric approach is adjusted by estimating the regression coefficient, so we obtain the following:

$$
\begin{equation*}
\Delta^{\mathrm{d}_{\mathrm{i}}} \mathrm{X}_{\mathrm{t}-\mathrm{i}}=\hat{\mathrm{d}}_{\mathrm{i}}\left(\mathrm{x}_{\mathrm{t}-1 \mathrm{i}}-\mathrm{x}_{\mathrm{t}-2 \mathrm{i}}\right) \tag{4}
\end{equation*}
$$

Wherein, $\mathrm{i}=\overline{1, \mathrm{n}}$.
Assuming that the increment of each indicator is formed under the influence of all other indicators, and on the formation of growth all indicators have a uniform effect, it is possible to consider $\Delta^{\mathrm{d}_{\mathrm{i}}} \mathrm{X}_{\mathrm{t}-\mathrm{li}}$ as

$$
\begin{equation*}
\Delta^{\mathrm{d}_{\mathrm{i}}} \mathrm{x}_{\mathrm{t}-\mathrm{li}}=\frac{1}{\mathrm{n}-1} \sum_{\mathrm{j} \neq \mathrm{i}} \frac{\Delta^{\mathrm{d}_{\mathrm{i}}} \mathrm{x}_{\mathrm{t}-\mathrm{li}}}{\mathrm{x}_{\mathrm{tj}}} \mathrm{x}_{\mathrm{tj}} \tag{5}
\end{equation*}
$$

Wherein, $\mathrm{i}=\overline{1, \mathrm{n}}$.
At the same time,

$$
\begin{equation*}
\frac{\Delta^{\mathrm{d}_{\mathrm{i}}} \mathrm{x}_{\mathrm{t}-\mathrm{i}}}{\mathrm{x}_{\mathrm{tj}}}=\mathrm{v}_{\mathrm{ij}}^{\mathrm{d}_{\mathrm{i}}} \tag{6}
\end{equation*}
$$

Wherein, $\mathrm{i}, \mathrm{j}=\overline{1, \mathrm{n}}$.
The simulated indicator from equation (3), taking into account (5) and (6), is converted as follows:

$$
\begin{equation*}
\mathrm{x}_{\mathrm{ti}}=\hat{\alpha}_{\mathrm{i}}+\hat{\beta}_{\mathrm{i}} \mathrm{x}_{\mathrm{t}-2 \mathrm{i}}+\frac{1}{\mathrm{n}-1} \sum_{\mathrm{j} \neq \mathrm{i}} \mathrm{v}_{\mathrm{ij}}^{\mathrm{d}_{\mathrm{i}}} \mathrm{x}_{\mathrm{tj}} \tag{7}
\end{equation*}
$$

Wherein, $\mathrm{i}=\overline{1, \mathrm{n}}$.
Entering the notation

$$
\begin{aligned}
& \mathbf{X}_{\mathbf{t}}=\left(\begin{array}{c}
\mathrm{x}_{\mathrm{t} 1} \\
\mathrm{x}_{\mathrm{t} 2} \\
\vdots \\
\mathrm{x}_{\mathrm{tn}}
\end{array}\right) \\
& \hat{\mathbf{A}}+\hat{\mathbf{B}} \mathbf{X}_{\mathbf{t - 2}}=\left(\begin{array}{c}
\hat{\alpha}_{1}+\hat{\beta}_{1} \mathrm{x}_{\mathrm{t}-21} \\
\hat{\alpha}_{2}+\hat{\beta}_{2} \mathrm{x}_{\mathrm{t}-22} \\
\vdots \\
\hat{\alpha}_{\mathrm{n}}+\hat{\beta}_{\mathrm{n}} \mathrm{x}_{\mathrm{t}-2 \mathrm{n}}
\end{array}\right) \\
& \mathbf{V}_{\mathrm{t}-\mathbf{1}}=\frac{1}{\mathrm{n}-1}\left(\begin{array}{cccc}
0 & \mathrm{v}_{12}^{\mathrm{d}_{1}} & \cdots & \mathrm{v}_{1 \mathrm{n}}^{\mathrm{d}_{1}} \\
\mathrm{v}_{21}^{\mathrm{d}_{2}} & 0 & \cdots & \mathrm{v}_{2 \mathrm{n}} \\
\vdots & \vdots & \cdots & \vdots \\
\mathrm{~d}_{2} \\
\mathrm{v}_{\mathrm{n} 1}^{\mathrm{d}_{\mathrm{n}}} & \mathrm{v}_{\mathrm{n} 2}^{\mathrm{d}_{\mathrm{n}}} & \cdots & 0
\end{array}\right)
\end{aligned}
$$

we obtain the system of models (7) in the vector form

$$
\begin{equation*}
\mathbf{X}_{\mathbf{t}}=\hat{\mathbf{A}}+\hat{\mathbf{B}} \mathbf{X}_{\mathbf{t}-\mathbf{2}}+\mathbf{V}_{\mathbf{t}-\mathbf{1}} \mathbf{X}_{\mathbf{t}} \tag{8}
\end{equation*}
$$

The solution of the obtained system (8) can be written as follows:

$$
\begin{equation*}
\mathbf{X}_{\mathbf{t}}=\left(\mathbf{I}-\mathbf{V}_{\mathbf{t}-\mathbf{1}}\right)^{-\mathbf{1}}\left(\hat{\mathbf{A}}+\hat{\mathbf{B}} \mathbf{X}_{\mathbf{t}-\mathbf{2}}\right) \tag{9}
\end{equation*}
$$

The forecast is obtained by recalculating the elements of the matrix $\mathbf{V}_{\text {through }} \mathbf{X}_{\mathbf{t}}$ and $\mathbf{X}_{\mathbf{t - 1}}$, and in (9) instead $\mathbf{X}_{\mathbf{t - 2}}$ it is possible to substitute $\mathbf{X}_{\mathbf{t} \mathbf{- 1}}$, namely

$$
\begin{equation*}
\mathbf{X}_{\mathbf{t}+\mathbf{1}}=\left(\mathbf{I}-\mathbf{V}_{\mathbf{t}}\right)^{\mathbf{- 1}}\left(\hat{\mathbf{A}}+\hat{\mathbf{B}} \mathbf{X}_{\mathbf{t}-\mathbf{1}}\right) \tag{10}
\end{equation*}
$$

Thus, we obtain a regression-matrix model, which is a peculiar system of interrelated regression models, the construction of which was based on the first-order autoregression. The matrix predictor in this model allows for systemic balance through indirect growth rates.

It is worth noting that the regression-matrix model can also be built on the basis of second or third order autoregression, but the considered variant is simpler and more visual and does not require a significant amount of data for its application, while higher order autoregressive models are built up using more measurements.

## IV. Group Dynamics Indicators

The authors considered one of the approaches to ensure that multidimensional predictive calculations are carried out, taking into account the systemic balance of dynamics. Meanwhile, it is worth noting that the forecasting of socioeconomic development is carried out on the basis of a significant number of indicators. Given the multidimensionality of data, it is unlikely to build an adequate model using all real indicators. Therefore, for such a number of indicators, the data should be structured by dividing them into special blocks, taking into account the meaningful meaning.

In this case, the model used to build the forecasts should provide a mechanism to realize systemic balance inside the blocks (block indicators should be balanced among themselves according to their dynamics) and between blocks (blocks should be balanced with each other by group dynamics).

If the systemic balance of indicators inside the blocks is carried out using indirect growth rates used to build the matrix predictor, it is proposed to perform systemic balance between the blocks using the first principal component, built for the indicators of each block and having, as a rule, a "synergistic effect". In this case, by virtue of the properties of the principal components, we will call them "indicators of group dynamics".

At the same time, it should be understood that it is not always possible to fully describe the dynamics of a block using only the first principal component; you may need to build two or even three principal components.

In this paper, such a task was not considered, and systemic balance is carried out only with the help of the first principal component. However, in the future, it is planned to use the second (and possibly the third) principal component for a more complete description of group dynamics.

Considering the fact above in general aspect the first principal component of certain $g$ block $(g=1,2, \ldots)$ can be represented as

$$
\begin{equation*}
\mathrm{u}^{(\mathrm{g})}=\gamma_{1}^{(\mathrm{g})} \mathrm{x}_{1}^{(\mathrm{g})}+\gamma_{2}^{(\mathrm{g})} \mathrm{x}_{2}^{(\mathrm{g})}+\ldots+\gamma_{\mathrm{p}}^{(\mathrm{g})} \mathrm{x}_{\mathrm{p}}^{(\mathrm{g})} \tag{11}
\end{equation*}
$$

Wherein, $\left(\gamma_{1}, \gamma_{2}, \cdots, \gamma_{p}\right)$ - p-measuring column vector.

Now, on the basis of (11), logically distributing all the available indicators in blocks (taking into account the content component), the first principal component should be constructed for each block. Since the distribution of indicators by blocks implies their rearrangement, each indicator in the block will be assigned a new index, and then we get the following notation:

$$
\left\{\begin{array}{c}
u^{(1)}=\gamma_{1}^{(1)} \mathrm{x}_{1}^{(1)}+\gamma_{2}^{(1)} \mathrm{x}_{2}^{(1)}+\ldots+\gamma_{\mathrm{n}}^{(1)} \mathrm{x}_{\mathrm{n}}^{(1)}  \tag{12}\\
\mathrm{u}^{(2)}=\gamma_{1}^{(2)} \mathrm{x}_{1}^{(2)}+\gamma_{2}^{(2)} \mathrm{x}_{2}^{(2)}+\ldots+\gamma_{\mathrm{n}}^{(2)} \mathrm{x}_{\mathrm{n}}^{(2)} \\
\vdots \\
\mathrm{u}^{(\mathrm{k})}=\gamma_{1}^{(\mathrm{k})} \mathrm{x}_{1}^{(\mathrm{k})}+\gamma_{2}^{(\mathrm{k})} \mathrm{x}_{2}^{(\mathrm{k})}+\ldots+\gamma_{\mathrm{n}}^{(\mathrm{k})} \mathrm{x}_{\mathrm{n}}^{(\mathrm{k})}
\end{array}\right.
$$

Since in fact whole of $\mathrm{u}^{(\mathrm{g})}(\mathrm{g}=\overline{1, \mathrm{k}})$ from the system (12) represent the vector of values, which reflects the group dynamics by years, it is possible to represent each first principal component in the following form:

$$
\begin{equation*}
\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}=\alpha^{(\mathrm{g})}+\beta^{(\mathrm{g})} \mathrm{u}_{\mathrm{t}-1}^{(\mathrm{g})}+\varepsilon_{\mathrm{t}}^{(\mathrm{g})} \tag{13}
\end{equation*}
$$

Wherein, $\mathrm{u}^{(\mathrm{g})}=\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}$ (where $\left.\mathrm{g}=\overline{1, \mathrm{k}}\right)$.
Expression (13) is similar to equation (1) in its essence; the only difference is that now instead of the indicator, the group dynamics indicator is used.

Such a representation (13) will make it possible to include the group dynamics indicator $\mathrm{u}^{(\mathrm{g})}$ in the system of indicators in each block.

To do this, as in the case of indicators, the lagging variables must be represented in the form of expression (2), and then substituted them in such as in the original equation. After that, an assessment is carried out, by analogy with (3), with the result of

$$
\begin{equation*}
\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}=\hat{\alpha}^{(\mathrm{g})}+\hat{\beta}^{(\mathrm{g})} \mathrm{u}_{\mathrm{t}-2}^{(\mathrm{g})}+\hat{\mathrm{d}}^{(\mathrm{g})}\left(\mathrm{u}_{\mathrm{t}-1}^{(\mathrm{g})}-\mathrm{u}_{\mathrm{t}-2}^{(\mathrm{g})}\right) \tag{14}
\end{equation*}
$$

Wherein, $\mathrm{g}=\overline{1, \mathrm{k}}$.
Now we substitute each $\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}$ in its block. Then all the blocks can be represented as follows:

$$
\begin{gathered}
\mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}=\hat{\alpha}_{1}^{(\mathrm{g})}+\hat{\beta}_{1}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-21}^{(\mathrm{g})}+\hat{\mathrm{d}}_{1}^{(\mathrm{g})}\left(\mathrm{x}_{\mathrm{t}-11}^{(\mathrm{g})}-\mathrm{x}_{\mathrm{t}-21}^{(\mathrm{g})}\right) \\
\vdots \\
\mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}=\hat{\alpha}_{\mathrm{n}}^{(\mathrm{g})}+\hat{\beta}_{\mathrm{n}}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-2 \mathrm{n}}^{(\mathrm{g})}+\hat{\mathrm{d}}_{\mathrm{n}}^{(\mathrm{g})}\left(\mathrm{x}_{\mathrm{t}-\mathrm{n}}^{(\mathrm{g})}-\mathrm{x}_{\mathrm{t}-\mathrm{n}}^{(\mathrm{g})}\right) \\
\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}=\hat{\alpha}^{(\mathrm{g})}+\hat{\beta}^{(\mathrm{g})} \mathrm{u}_{\mathrm{t}-2}^{(\mathrm{g})}+\hat{\mathrm{d}}^{(\mathrm{g})}\left(\mathrm{u}_{\mathrm{t}-1}^{(\mathrm{g})}-\mathrm{u}_{\mathrm{t}-2}^{(\mathrm{g})}\right)
\end{gathered}
$$

Wherein, $\mathrm{g}=\overline{1, \mathrm{k}}$.
Calculations with indicators are carried out by analogy with indicators. However, when forming a matrix of indirect
growth rates, one interesting feature arises, which is that all variables are divided into two groups, and the matrix of indirect growth rates is divided into four parts. This approach is called the "matrix approach with divided changes." We propose to consider its role in our model in more detail.

To begin with, we are to turn to the previously expressed idea that the increments in the framework of the econometric approach are corrected by estimating the regression coefficient, that is, by analogy with expression (4).

$$
\begin{align*}
& {\left[\begin{array}{c}
\mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}=\hat{\alpha}_{1}^{(\mathrm{g})}+\hat{\beta}_{1}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-21}^{(\mathrm{g})}+\frac{1}{(\mathrm{n}+1)-1}\left(0 \cdot \mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}+\ldots+\frac{\Delta^{\mathrm{d}_{1}^{(\mathrm{g})}} \mathrm{X}_{\mathrm{t}-11}^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}} \cdot \mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}+\frac{\Delta^{\mathrm{d}_{1}^{(\mathrm{g})}} \mathrm{x}_{\mathrm{t}-11}^{(\mathrm{g})}}{u_{\mathrm{t}}^{(\mathrm{g})}} \cdot \mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}\right) \\
\vdots
\end{array}\right.} \\
& \left\{\begin{array}{l}
x_{t n}^{(g)}=\hat{\alpha}_{n}^{(\mathrm{g})}+\hat{\beta}_{\mathrm{n}}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-2 \mathrm{n}}^{(\mathrm{g})}+\frac{1}{(\mathrm{n}+1)-1}\left(\frac{\Delta^{\mathrm{d}_{\mathrm{n}}^{(\mathrm{g})}} \mathrm{x}_{\mathrm{t}-1 \mathrm{n}}^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}} \cdot \mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}+\ldots+0 \cdot \mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}+\frac{\Delta^{\mathrm{d}_{\mathrm{n}}^{(\mathrm{g})}} \mathrm{x}_{\mathrm{t}-1 \mathrm{n}}^{(\mathrm{g})}}{\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}} \cdot{u_{\mathrm{t}}^{(\mathrm{g})}}_{(\mathrm{g}}\right) \\
\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}=\hat{\alpha}^{(\mathrm{g})}+\hat{\beta}^{(\mathrm{g})} \mathrm{u}_{\mathrm{t}-2}^{(\mathrm{g})}+\frac{1}{(\mathrm{n}+1)-1}\left(\frac{\Delta^{\mathrm{d}^{(\mathrm{g})}} \mathrm{u}_{\mathrm{t}-1}^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}} \cdot \mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}+\ldots+\frac{\Delta^{\mathrm{d}^{(\mathrm{g})}} \mathrm{u}_{\mathrm{t}-1}^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}} \cdot \mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}+0 \cdot \mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}\right)
\end{array}\right. \tag{15}
\end{align*}
$$

Next, we will use the aforementioned "matrix approach with separated changes" in order to construct a matrix of

In accordance with the approach used, it is necessary to separate the variables as follows (see "Fig. 2").
indirect growth rates and find predicted values. To do this, we will present the system of models (15) in a matrix form:

In this case, unlike the situation considered earlier, the increment of each indicator is formed under the influence of all other indicators and the group dynamics indicator, while it is still considered that all indicators (including the indicator) have a uniform effect on the formation of growth [6].

Then every $g$ block is converted as follows:

$$
\mathbf{V}_{\mathbf{1 1}}=\frac{1}{(\mathrm{n}+1)-1}\left(\begin{array}{ccc}
0 & \cdots & \frac{\Delta^{\mathrm{d}_{1}^{(\mathrm{g})}} \mathrm{x}_{\mathrm{t}-11}^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{tn}}^{(\mathrm{g})}} \\
\vdots & \vdots & \vdots \\
\frac{\Delta^{\left.\mathrm{d}_{\mathrm{n}}^{(\mathrm{g}}\right)} \mathrm{x}_{\mathrm{t}-\ln }^{(\mathrm{g})}}{\mathrm{x}_{\mathrm{t} 1}^{(\mathrm{g})}} & \cdots & 0
\end{array}\right)
$$

and variable group dynamics growth

$$
\mathbf{V}_{12}=\frac{1}{(\mathrm{n}+1)-1}\left(\begin{array}{c}
\frac{\Delta^{\mathrm{d}_{1}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-11}^{(\mathrm{g})}}}{\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}} \\
\vdots \\
\frac{\Delta^{\mathrm{d}_{\mathrm{n}}^{(\mathrm{g})} \mathrm{x}_{\mathrm{t}-1 \mathrm{n}}^{(\mathrm{g})}}}{\mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}}
\end{array}\right)
$$

In the clarifying equation, by analogy, we obtain the matrix $\mathbf{V}_{21}$ and $\mathrm{V}_{22}=(0)$.

Thus, we can convey two separate equations:

$$
\begin{align*}
& \mathbf{X}_{\mathrm{t}}^{(\mathbf{g})}=\hat{\mathbf{A}}^{(\mathrm{g})}+\hat{\mathbf{B}}^{(\mathrm{g})} \mathbf{X}_{\mathrm{t}-\mathbf{2}}^{(\mathbf{g})}+\mathbf{V}_{11} \mathbf{X}_{\mathbf{t}}^{(\mathbf{g})}+\mathbf{V}_{\mathbf{1 2}} u_{t}^{(\mathrm{g})}  \tag{16}\\
& \mathrm{u}_{\mathrm{t}}^{(\mathrm{g})}=\hat{\alpha}^{(\mathrm{g})}+\hat{\beta}^{(\mathrm{g})} \mathbf{u}_{\mathrm{t}-2}^{(\mathrm{g})}+\mathbf{V}_{\mathbf{2 1}} \mathbf{X}_{\mathbf{t}}^{(\mathbf{g})}+\mathrm{V}_{22} \mathrm{u}_{\mathrm{t}}^{(\mathrm{g})} \tag{17}
\end{align*}
$$

Wherein, $g=\overline{1, \mathrm{k}}$.
In fact, expression (17) is not a vector, but a certain number characterizing the group dynamics, and is of value rather in theoretical terms. In practice, only (16) is used for forecasting.

At the beginning is the solution of the system (16), as a result we gain

$$
\begin{align*}
& \mathbf{X}_{\mathrm{t}}^{(\mathrm{g})}=\left(\mathbf{I}-\mathbf{V}_{11}\right)^{-1}\left(\hat{\mathbf{A}}^{(\mathrm{g})}+\hat{\mathbf{B}}^{(\mathrm{g})} \mathbf{X}_{\mathrm{t}-2}^{(\mathrm{g})}+\mathbf{V}_{12} u_{t}^{(\mathrm{g})}\right)  \tag{18}\\
& \text { Wherein, } \mathrm{g}=\overline{1, \mathrm{k}}
\end{align*}
$$

Then, it is necessary to recalculate the increments of the matrix $\mathbf{V}_{\mathbf{1 1}}$ and $\mathbf{V}_{\mathbf{1 2}}$ through $\mathbf{X}_{\mathbf{t}}$ and $\mathbf{X}_{\mathbf{t - 1}}$, and in (18) instead of $\mathbf{X}_{t-\mathbf{2}}$ substitute $\mathbf{X}_{t-\mathbf{1}}$ and instead of $u_{t}$ we are to use $\mathrm{u}_{\mathrm{t}+1}$, that is

$$
\begin{align*}
& \mathbf{X}_{\mathbf{t}+\mathbf{1}}^{(\mathbf{g})}=\left(\mathbf{I}-\mathbf{V}_{\mathbf{1 1 f o r}}\right)^{-\mathbf{1}}\left(\hat{\mathbf{A}}^{(\mathbf{g})}+\hat{\mathbf{B}}^{(\mathbf{g})} \mathbf{X}_{\mathbf{t - 1}}^{(\mathbf{g})}+\mathbf{V}_{\mathbf{1 2 f o r} .} \mathrm{u}_{\mathrm{t}+1}^{(\mathrm{g})}\right)  \tag{19}\\
& \text { Wherein, } \mathrm{g}=\overline{1, \mathrm{k}}
\end{align*}
$$

Here there is some difficulty in that the required predictive value of the group dynamics indicator is missing $\mathrm{u}_{\mathrm{t}+1}$.

However, it can be quite easily resolved, since in fact the value $\mathrm{u}_{\mathrm{t}+1}$ the value can be predicted using a regressionmatrix model by analogy with the process of predicting indicators (1) - (10) with the only difference that instead of indexes, the indicator of each block is used. Now having passed the same stages, it is possible to substitute the received $\mathrm{u}_{\mathrm{t}+1}$ in expression (19) and get a foresight of target indicators.

Consequently, the forecasting procedure was fully considered, taking into account the operation of balance, which consists in the system balance of indicators among themselves on their own dynamics and blocks with each other on group dynamics.

The approach considered in this paper has already been used in practice to test predict the development of a particular real socio-economic system. Actually, as such a local system was the city district Voronezh. Its foresight was performed for 29 key indicators, which were divided into eight blocks in accordance with the content meaning.

According to the results of calculations, indicators of all eight blocks were obtained, the actual values of which are presented in "Table I".

TABLE I. Predicted Assessment of Indicators by Blocks (in Actual Values)

| Block 1 | 1042.51 | -0.80 | 8.64 | 72.69 | x | x |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Block 2 | 4.84 | 152.38 | 9.41 | x | x | x |
| Block 3 | 29457.47 | 28.26 | 0.19 | 96.24 | 95.37 | x |
| Block 4 | 130.74 | -106.01 | 72.98 | x | x | x |
| Block 5 | 5.68 | 1592.01 | 402.90 | x | x | x |
| Block 6 | 267842.13 | 3.63 | 15.46 | 78282.54 | 579.93 | 43.89 |
| Block 7 | 104.30 | 91.04 | x | x | x | x |
| Block 8 | 17819.30 | 16944.15 | 17143.36 | x | x | x |

Block 1 - "Demography", block 2 - "Employment and incomes", block 3 - "Housing, engineering infrastructure", block 4 - "Transport infrastructure", block 5 - "Health factors", block 6 - "Production, its material factors", block 7 - "Environmental factors", block 8 - "Financial result".

Thus, a foresight was obtained to describe the development of such a local socio-economic system as the city district of Voronezh. The presented foresight was built up for 29 key indicators. At the same time, it should be said that the available data allow us to construct foresights for 35 indicators, however, in the considered work, the aim was to
demonstrate the methodology more visually and clearly, therefore indicators that for one reason or another are difficult to attribute to any block were not considered.

## V. CONCLUSION

In conclusion, it should be said that the combined model based on the matrix predictor is quite easy to understand, the results obtained with its help are easily interpreted.

The undoubted advantage of this model is that it satisfies modern requirements for the reflection of complex multidimensional processes, while ensuring a systemic balance of indicators (the preservation of a certain mutual proportionality in the dynamics of development). At the same time, this model takes into account the division of a large array of data into blocks in accordance with the content meaning, since it uses the first principal components to provide a mechanism that ensures the system balance of the blocks with each other according to group dynamics.

In addition, the combined model has good potential for development and modification. First of all, this model has the prerequisites for the use of an adaptive mechanism in it, which, with the help of targeted ("target") installations, can form new trends in the predicted trajectory. This ensures the formation of the forecast image of the socio-economic development of the subject, the advantage of which is that with its help it is possible to conduct a study of various options for the future.

Meanwhile, in the process of building up a combined model a lot of questions still remain, the search for answers to which is planned to be continued in further research.

Among such issues the possibility of building a model based on autoregression of the second and third order, as well as the question of the choice of factors in forecasting, which most fully reflect the prevailing trends in indicators. At the same time, the usage of exponential smoothing is considered which is another simplest form of adaptation, that can be easily applied to the combined model. In practice, the use of exponentially smoothed variables in the forecasting process has not yet been considered due to fairly short time series, but this idea has grounds for more detailed consideration in the future.

In addition, group dynamics was predicted only with the help of the first principal component, which in practice, in some cases, does not fully describe the behavior of the indicators inside the block. This necessitates the usage of the second principal component, which is planned to be included in the next stage of testing the combined model.

Despite the remarks made above, the combined model based on the matrix predictor has obvious prospects for its application in practical foresight calculations characterizing the development of socio-economic systems. At the same time, the necessity to develop such a model is primarily related to its potential, which is particularly significant determining the description's conversion of the social and economic processes into digital area.

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