

Squaring and Hexagonizing Logical Relations Among Three Different Kinds of Deontic Modality “Obligatory”

A Model for Logically Formalized Axiomatic Epistemology Theory

Vladimir Lobovikov

Laboratory for Applied System-Research

Ural Federal University

Yekaterinburg, Russia

E-mail: vlobovikov@mail.ru

Abstract—In contrast and complement to the logical square and hexagon by G. Kalinowski modeling logic of norms, in this paper such logical square and hexagon of conceptual opposition are submitted which model logical interconnections among three different kinds of deontic modality “obligatory”. The qualitative differentiation among the three is in accordance with a logically formalized axiomatic epistemology theory. It is proved that the axiomatic theory is consistent and implies substantial difference between a-priori knowledge of obligatory-ness and a-posteriori one. The paper submits a formal inference (in that axiomatic theory) of the logic equivalence of modalities “necessary” and “obligatory” from the assumption that the relevant knowledge is a-priori one. Formal proving that theorem means the conditional equivalence of corresponding deontic and alethic modalities. This is a significant explication and correction of the modality views which many philosophers are used to. Squaring and hexagonizing logical relations among the three different kinds of deontic modality “obligatory” helps to grasp the novel idea effectively.

Keywords—logical; square; hexagon; alethic; deontic; modalities; obligatory

I. INTRODUCTION

Today plenty of substantially different not-traditional (non-quantificational) interpretations of the square and hexagon of opposition have been invented and studied. The alethic modality interpretation was created by P. Jacoby, A. Sesmat, and R. Blanché [1]. The deontic one was invented by G. Kalinowski [2]. Usually, the alethic-modal interpretation of the square and hexagon of opposition by P.

Jacoby, A. Sesmat, and R. Blanché is considered independently (separately) from the square and hexagon of opposition of normative concepts by G. Kalinowski.

Sometimes this separate consideration is justified by the well-known statement that “being” and “obligatory-ness” are separated by the logically unbridgeable gap: there are no (deductive) logical relations between alethic and deontic modalities in spite of the fact that there is a similarity (analogy) between them [3].

However, this statement is not quite precise. There is for instance the famous principle by I. Kant: rational obligatory-ness implies possibility [4]. Moreover, many lawyers and moralists believe that rational obligation implies possibility to violate the obligation. In other words, rational making obligatory implies existence of moral (legal) freedom, namely, a possibility of moral choice. If action is alethically necessary or impossible, then there is no freedom of moral (legal) choice. Consequently, obligatory-ness implies alethic contingency. Owing to the above-said it is possible to combine Jacoby-Sesmat-Blanché hexagon of alethic modalities with Kalinowski hexagon of deontic ones. This option of logical combining is graphically represented by the following picture (“Fig. 1”).

In “Fig. 1” and hereafter in this paper the symbols \neg , \wedge , \vee , \leftrightarrow , and \rightarrow stand for classical logic operations “negation”, “conjunction”, “(not-exclusive) disjunction”, “equivalence”, and “implication”, respectively. The symbols: \square , \diamond , and ∇ stand for “alethically necessary”, “alethically possible”, and “alethically contingent”, respectively. The symbol $O\alpha$ stands for deontic modality “it is obligatory that α ”.

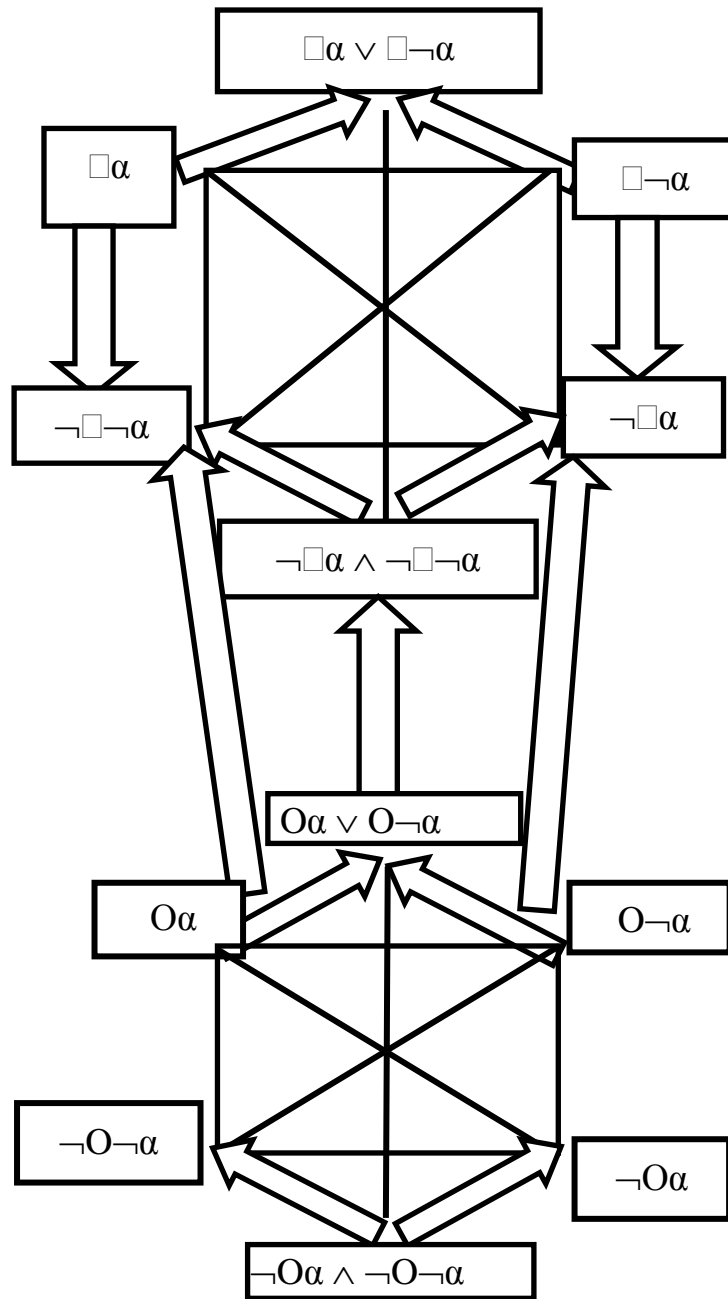


Fig. 1. Visual representation of logical subordination between the deontic hexagon and the alethic one.

Owing to “Fig. 1” one can see that from $O\alpha$, $O\neg\alpha$, and $(O\alpha \vee O\neg\alpha)$ it follows logically that $(\neg\Box\alpha \wedge \neg\Box\neg\alpha)$, $\forall\alpha$, $(\Diamond\alpha \wedge \Diamond\neg\alpha)$. From this it follows logically that if $O\alpha$, then α is alethically contingent, consequently, $\neg\Box\alpha$, $(O\alpha \leftrightarrow \neg\Box\alpha)$, $\neg(O\alpha \leftrightarrow \Box\alpha)$. It seems that such technically simple corollaries have no connections with nontrivial philosophical problems. But I do not think so. I consider that the indicated corollaries are necessarily connected with an old and knotty philosophical problem which is really interesting and quite nontrivial, namely, with the problem of a-priori-ism-versus-empiricism. The problem has been discussed during many centuries but there are such aspects of the problem which are not clear even today.

For instance, it is not quite clear whether it is possible to create a logically consistent philosophical theory representing a synthesis of the a-priori-ism and the empiricism. This nontrivial problem is considered below.

II. A-PRIORI-ISM AND EMPIRISM SYNTHESIZED IN ONE CONCEPTUAL SCHEME VISUALIZED BY THE SQUARE AND HEXAGON OF OPPOSITION

In addition to the classical epistemic modality $K\alpha$ (agent knows that α) let us consider two nonclassical ones, namely, $A\alpha$ (agent a-priori knows that α) and $E\alpha$ (agent a-posteriori knows that α). The system of logical relations among $K\alpha$,

$A\alpha$, and $E\alpha$ is modeled by the following square and hexagon of opposition.

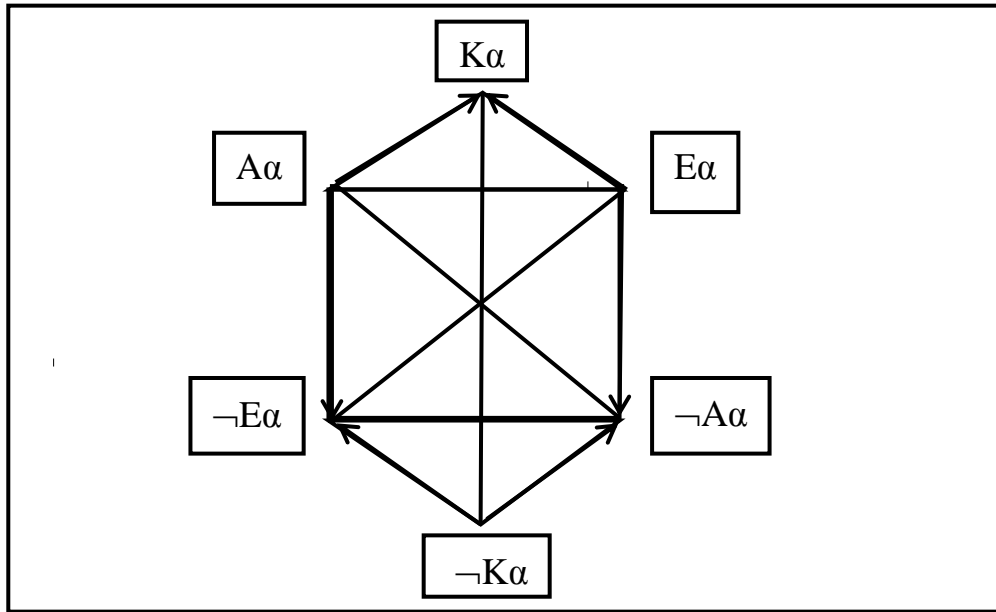


Fig. 2. Synthesizing a-priori-ism and empiricism in one conceptual scheme of general philosophical epistemology.

Here the symbols $K\alpha$, $A\alpha$, and $E\alpha$ stand for epistemic modalities “agent knows that α ”, “agent a-priori knows that α ”, “agent a-posteriori knows that α ”, respectively. The “Fig. 2” shows that the relation between $A\alpha$ and $E\alpha$ is not the contradictoriness but the contrariety. However, it is necessary somehow to define precisely the newly-introduced epistemic modalities. Below meanings of the symbols $A\alpha$ and $E\alpha$ are defined precisely (although not directly) by the system of axioms of the logically formalized axiomatic epistemology system Ξ .

III. PRECISE DEFINING THE FORMAL AXIOMATIC THEORY Ξ

In axiomatic-epistemology-system Ξ , symbols α and β (belonging to meta-language) stand for any formulae of Ξ . Additional formulae of Ξ are obtained by the following rule: if α is a formula of Ξ then $\Psi\alpha$ is a formula of Ξ as well. The symbol Ψ belonging to meta-language stands for any element of the set of modalities $\{\square, K, A, E, F, T, P, Z, G, O, S, B, U, Y\}$. Symbol \square stands for the alethic modality “necessary”. Symbols $K, A, E, F, T, P, Z, G, O, S, B, U, Y$, respectively, stand for modalities “agent knows that...”, “agent a-priori knows that...”, “agent a-posteriori knows that...”, “agent believes that...”, “it is true that...”, “it is provable that...”, “there is an algorithm (a machine could be constructed) for deciding that...”, “it is (morally) good that...”, “it is obligatory that...”, “under some conditions in some space-and-time a person (immediately or by means of some tools) sensually perceives (has sensual verification) that...”, “it is beautiful that ...”, “it is useful that ...”, “it is pleasant that ...”. Meanings of the mentioned symbols are defined by the following schemes of own-axioms of epistemology-system Ξ which axioms are added to the axioms and inference-rules of classical propositional logic.

Schemes of axioms and inference-rules of the classical propositional logic are applicable to all formulae of Ξ (including the additional ones).

Axiom scheme AX-1: $A\alpha \rightarrow (\square\beta \rightarrow \beta)$.

Axiom scheme AX-2: $A\alpha \rightarrow (\square(\alpha \rightarrow \beta) \rightarrow (\square\alpha \rightarrow \square\beta))$.

Axiom scheme AX-3: $A\alpha \leftrightarrow (K\alpha \ \& \ (\square\alpha \ \& \ \square\neg S\alpha \ \& \ \square(\beta \leftrightarrow \Omega\beta)))$.

Axiom scheme AX-4: $E\alpha \leftrightarrow (K\alpha \ \& \ (\neg\square\alpha \ \vee \ \neg\square\neg S\alpha \ \vee \ \neg\square(\beta \leftrightarrow \Omega\beta)))$.

In AX-3 and AX-4, the symbol Ω (belonging to the meta-language) stands for any element of the set $\{\square, K, F, T, P, Z, G, O, B, U, Y\}$. Elements of this set are called “perfection-modalities” or simply “perfections”.

The square and hexagon modelling logical relations among the three different philosophical-epistemology-concepts ($K\alpha$, $A\alpha$, and $E\alpha$) are represented, for instance, in [5]. The axiomatic epistemology system Ξ is represented, for instance, in [6].

IV. A FORMAL PROOF OF THEOREM-SCHEME ($A\alpha \rightarrow (O\alpha \leftrightarrow \square A)$) IN Ξ

The following succession of schemes of formulae is a scheme of proofs of/for ($A\alpha \rightarrow (O\alpha \leftrightarrow \square A)$) in Ξ .

1. $A\alpha \leftrightarrow (K\alpha \ \& \ (\square\alpha \ \& \ \square\neg S\alpha \ \& \ \square(\beta \leftrightarrow \Omega\beta)))$: axiom scheme AX-3.
2. $A\alpha \rightarrow (K\alpha \ \& \ (\square\alpha \ \& \ \square\neg S\alpha \ \& \ \square(\beta \leftrightarrow \Omega\beta)))$: from 1 by the rule of elimination of \leftrightarrow .
3. $A\alpha$: assumption.

4. $(K\alpha \ \& \ \Box\alpha \ \& \ \Box\neg S\alpha \ \& \ \Box(\beta \leftrightarrow \Omega\beta))$: from 2 and 3 by modus ponens.
5. $\Box(\beta \leftrightarrow \Omega\beta)$: from 4 by the rule of elimination of $\&$.
6. $(\beta \leftrightarrow \Omega\beta)$: from 5 by the rule of elimination of \Box .
7. $(\alpha \leftrightarrow \Box\alpha)$: from 6 by substituting $(\alpha$ for β , and \Box for $\Omega)$.
8. $(\alpha \leftrightarrow O\alpha)$: from 6 by substituting $(\alpha$ for β , and O for $\Omega)$.
9. $(O\alpha \leftrightarrow \alpha)$: from 7 by commutativity of \leftrightarrow .
10. $(O\alpha \leftrightarrow \Box\alpha)$: from 9 and 7 by transitivity of \leftrightarrow .
11. $A\alpha \mid\text{---} (O\alpha \leftrightarrow \Box\alpha)$: by 1—10.
12. $\mid\text{---} A\alpha \rightarrow (O\alpha \leftrightarrow \Box\alpha)$: from 11 by the rule of introduction of \rightarrow .

Here you are.

This formally proved theorem seems to be in logic contradiction with the above fig. 1 and its moral-legal-philosophy context. It seems that the theory Ξ contradicts to the deontic logic created and developed by G.H. von Wright and G. Kalinowski in the middle of XX century. But in my opinion, it only seems so. The classical deontic logic of Kalinowski-Wright modeled by the fig.1 represents that specific particular case of Ξ (the case of empiricism) in relation to which the formula-scheme T2: $((K\alpha \ \& \ (O\alpha \leftrightarrow \neg\Box\alpha)) \rightarrow E\alpha)$ is provable in Ξ . However, T2 does not

contradict logically to the formula-scheme T1: $(A\alpha \rightarrow (K\alpha \ \& \ (O\alpha \leftrightarrow \Box\alpha)))$ which is also provable in Ξ but represents another specific particular case of Ξ (namely, the case of a-priori-ism).

V. THREE DIFFERENT KINDS OF “OBLIGATORY” SYNTHESIZED IN ONE CONCEPTUAL SCHEME VISUALIZED BY THE SQUARE AND HEXAGON OF OPPOSITION

I think that the mentioned feeling of logic contradiction may be eliminated by the following nontrivial novelty. In contrast and in complement to the well-known classical deontic logic theorizing created and developed by G.H. von Wright and G. Kalinowski in the middle of XX century, let us introduce and define two different kinds of nonclassical deontic modality “obligatory”, namely, $O^1\alpha$ and $O^2\alpha$, by the below equivalences.

- 1: $O^1\alpha \leftrightarrow (O\alpha \ \& \ \Box\alpha)$. And, consequently, $O^1\alpha \leftrightarrow (O\alpha \ \& \ (O\alpha \leftrightarrow \Box\alpha))$.
- 2: $O^2\alpha \leftrightarrow (O\alpha \ \& \ \neg\Box\alpha)$. And, consequently, $O^2\alpha \leftrightarrow (O\alpha \ \& \ \neg(O\alpha \leftrightarrow \Box\alpha))$, and $O^2\alpha \leftrightarrow (O\alpha \ \& \ (O\alpha \leftrightarrow \neg\Box\alpha))$.
- 3: $O\alpha \leftrightarrow (O^1\alpha \ \vee \ O^2\alpha)$.

If the equivalences are accepted then the logic connections among the deontic modalities $O\alpha$, $O^1\alpha$, $O^2\alpha$ are modeled by the below square and hexagon of opposition.

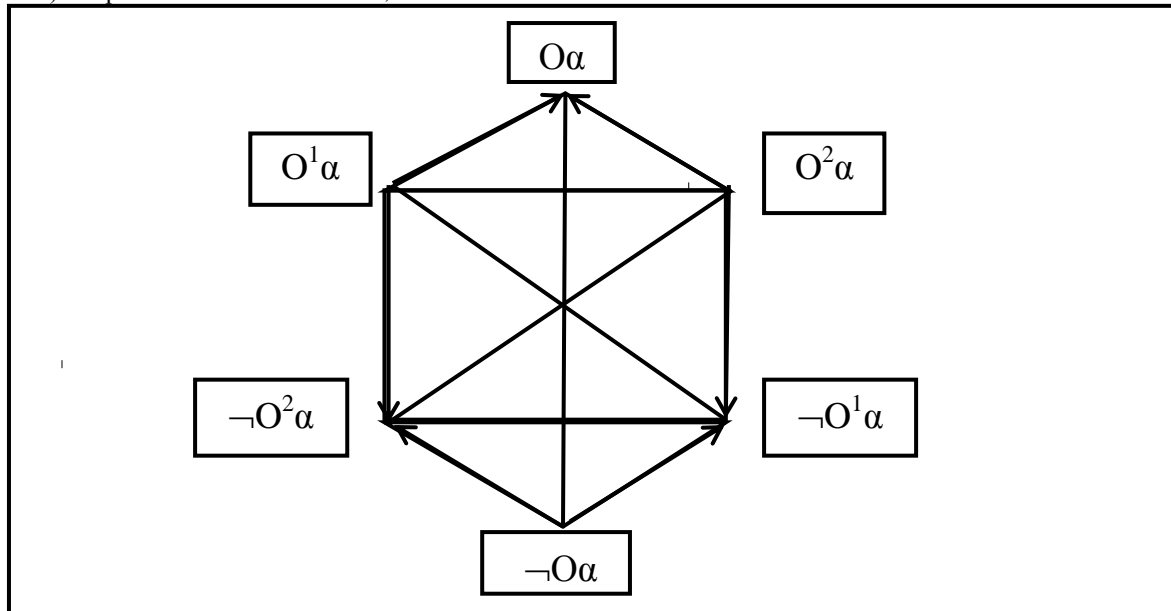


Fig. 3. Hexagon of three different kinds of deontic modality “obligatory”.

The “Fig. 3” visually demonstrates (just shows) that the axiomatic epistemology system in question is quite consistent in spite of the above-mentioned alleged contradiction between the deontic logic of Wright-Kalinowski and the axiomatic epistemology system [5], [6]. The classical modal deontic logic of Wright-Kalinowski is quite adequate in relation to $O^2\alpha$ but not adequate in relation

to $O^1\alpha$. For empiricist-minded philosophers, moralists, and lawyers, the “Fig.1” is necessary and sufficient but for a-priori-ism-minded ones it is not so. The later believe, that within their paradigm, $(O\alpha \leftrightarrow \Box\alpha)$ is true. The notorious contradiction-problem existed during centuries owing to absence of precise definition of the border-line between own domains of the two paradigms.

The axiomatic epistemology system Ξ precisely defines the border-line between the two not-empty paradigm-domains. The two are logically combined in one conceptual scheme modelled by relevant square and hexagon of opposition showing convincingly that the relation between a-priori-ism and empiricism is the contrariety one [5], [6]. Those who believe that a-priori-knowledge does not exist, believe in monopoly position of the above fig.1. If one recognizes existence of a-priori-knowledge, then to save unity of the ones consciousness it is necessary to find (discover) or invent (construct) a solid basis for synthesizing the two knowledge-kinds consistently. (Otherwise the one shall have a split of consciousness and doubling of personality.)

VI. A PROOF OF CONSISTENCY OF THE FORMAL AXIOMATIC THEORY Ξ

Now let us consider Ξ in relation to its consistency. Let the meta-language symbols α and β be substituted by the object-language symbol q . Also let the meta-language symbol Ω be substituted by the object-language symbol O (It is obligatory that). In this particular case the system of axiom-schemes of Ξ is represented by the following axioms, respectively.

Axiom AX-1*: $Aq \rightarrow (\Box q \rightarrow q)$.

Axiom AX-2*: $Aq \rightarrow (\Box(q \rightarrow q) \rightarrow (\Box q \rightarrow \Box q))$.

Axiom AX-3*: $Aq \leftrightarrow (Kq \ \& \ (\Box q \ \& \ \Box \neg Sq \ \& \ \Box(q \leftrightarrow Oq)))$.

Axiom AX-4*: $Eq \leftrightarrow (Kq \ \& \ (\neg \Box q \vee \neg \Box \neg Sq \vee \neg \Box(q \leftrightarrow Oq)))$.

In relation to the consistency of Ξ , let us consider the following function called “interpretation \odot ”. The interpretation-function \odot is defined as follows. (It is implied here that ω and π stand for any formulae belonging to Ξ .)

$\odot \neg \omega = \neg \odot \omega$ for any formulae ω .

$\odot(\omega \oplus \pi) = (\odot \omega \oplus \odot \pi)$ for any formulae ω and π , and for any classical-logic binary-connective \oplus .

$\odot q = \text{false}$.

$\odot Aq = \text{false}$.

$\odot Kq = \text{true}$.

$\odot Eq = \text{true}$.

$\odot \Box q = \text{true}$.

$\odot \Box \neg Sq = \text{true}$.

$\odot \Box(q \rightarrow q) = \text{true}$.

$\odot Oq = \text{true}$.

$\odot \Box(q \leftrightarrow Oq) = \text{false}$.

In the interpretation \odot , the formula $\Box(q \leftrightarrow Oq)$ is evaluated as false according to “the Guillotine of Hume” [7]. Also, substantial philosophical-content-analysis of that

formula making up solid grounds for affirming falsity of $\Box(q \leftrightarrow Oq)$ is given in [8].

In the interpretation \odot , all the axioms of Ξ are true, consequently, Ξ has a model, hence Ξ is consistent. Moreover, in the interpretation \odot , the formulae $\neg(Kq \rightarrow q)$, $\neg(Eq \rightarrow q)$, and $\neg(\Box q \rightarrow q)$ are true as well. Consequently, $(Kq \rightarrow q)$, $(Eq \rightarrow q)$, and $(\Box q \rightarrow q)$ are not provable in Ξ .

VII. CONCLUSION

Two logically different interpretations may be given to Leibniz genius intuition of fundamental unity of corresponding Aristotelian (alethic) and juridical (deontic) modalities [9]. The first one is similarity (analogy) between $\Box q$ and Oq [3]. The second one is equivalence of $\Box q$ and Oq . The similarity (analogy) idea has been created and elaborated within the empiricism paradigm. The equivalence idea has been created and developed within the a-priori-ism paradigm. Hitherto there was a nontrivial problem of precise defining limits of adequateness-domains of the two paradigms.

In this paper the border-line between not-empty adequateness-domains of the paradigms of empiricism and a-priori-ism is defined precisely by the logically formalized axiomatic epistemology theory Ξ . A rigorous definition of Ξ is given. The formal axiomatic epistemology theory Ξ is consistent. A proof of consistency of Ξ is submitted. Also (in Ξ) the theorem $(A\alpha \rightarrow (O\alpha \leftrightarrow \Box \alpha))$ is formally proved. This means the conditional equivalence of corresponding deontic and alethic modalities: $(O\alpha \leftrightarrow \Box \alpha)$ if agent a-priori-knows that α . The analytical discourse about epistemology and its logically formalized axiomatizing is supported effectively by geometric models, namely, by the logical square and hexagon of conceptual opposition among the three kinds of knowledge $(K\alpha, A\alpha, E\alpha)$. Squaring and hexagonizing is used also for visual modeling logical relations among the three substantially different kinds of deontic modality “obligatory”. Some corollaries of this article change some modality tenets philosophers are used to.

REFERENCES

- [1] R. Blanché. Intellectual Structures. Paris, Vrin, 1966, 151 p.
- [2] G. Kalinowski. Logic of norms. Paris: University Press of France, 1972. 218 p.
- [3] G.H. von. Wright. Philosophical Papers. Vol. I: Practical Reason. Oxford: Basil Blackwell, 1983, 214 p.
- [4] I. Kant “The Critique of Pure Reason. Fundamental Principles of the Metaphysics of Morals. The Critique of Practical Reason. Preface and Introduction to the Metaphysical Elements of Ethics. General Introduction to the Metaphysics of Morals. The Science of Right. The Critique of Judgement,” in: Mortimer J. Adler (Ed.). Great Books of the Western World. V. 39: Kant. Chicago; London; Madrid: Encyclopedia Britannica, Inc. 1994, 613 p.
- [5] V.O. Lobovikov “An Axiomatization of Philosophical Epistemology,” Journal of Tomsk State University. Series: Philosophy, Sociology, Political Studies. 2016. Vol. 36. No 4, pp. 69-78. DOI: 10.17223/1998863X/36/7
- [6] V.O. Lobovikov. “Moving from the Opposition of Normal and Not-Normal Modal Logics to Universal Logic: Synthesizing T, S4, Tr,

Verum and Falsum systems by the Square and Hexagon,” In: Jean-Yves Beziau, Arthur Buchsbaum, and Christophe Rey (eds). *Handbook of the 6th World Congress and School on Universal Logic in Vichy, France (June 16-26, 2018)*. Vichy, France: Université Clermont Auvergne, 2018, pp. 449-450. UNILOG’2018 <https://www.uni-log.org/Vichy2018>

- [7] D. Hume “A Treatise of Human Nature being an Attempt to Introduce the Experimental Method of Reasoning into Moral Subjects.” Ed. T.H. Green and T.H. Grose. *The Philosophical Works of David Hume in Four Volumes. Vol. II*. London: Longmans, Green, and Co., 1874, pp. 1-374.
- [8] M.J. Adler. *Six Great Ideas*. New York: Simon & Schuster Inc., 1997, 256 p.
- [9] G.W. Leibniz. “Elementa Juris Naturalis [Elements of the Natural Law]”, in: G.W. Leibniz. *Philosophische Schriften. Erster Band [Philosophical writings. First volume] (1663-1672)*. Berlin: Akademie-Verlag, 1971. S. 466, 481.