

# Modeling and Analysis of Warehousing Business Process Based on Petri Net

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**Abstract**—Taking the finished product warehouse of precision casting enterprise as the research object, the traditional warehouse business process is investigated and analyzed, and the warehouse business process model based on Petri net is established. The reachability tree method is used to analyze the network model. Taking the outbound business as an example, Markov chain is established. Through the two performance indicators of warehouse busy rate and change utilization rate, the key links of the business are found out and improved, providing the basis for improving the efficiency of the warehousing business.

**Keywords**—Petri net modeling; reachable tree; Markov chain; process optimization

## I. INTRODUCTION

For an enterprise, how to realize the efficient management of storage is an important issue. Warehouse not only provides storage space for raw materials and finished products, but also creates utility for them. The dependence of the enterprise on warehouse enables the enterprise to provide more convenient delivery services for customers and realize the rapid acquisition of products whenever and wherever. However, due to poor management, there is a lot of waste in the storage link, which is often neglected but needs urgent improvement. Therefore, taking warehousing activities as the research object, strengthening the management of warehousing workflow, improving the efficiency of goods entering and leaving the warehouse and reducing waste are important methods to reduce costs and improve delivery efficiency.

In recent years, the research direction of warehousing business process mainly includes the allocation of cargo space, modeling and simulation, or adopting technical means to improve the level of warehousing management. Hou Zhong and others discussed the allocation of cargo space for auto parts according to the product characteristics of auto parts. By establishing a model and solving it with genetic algorithm, they improved the storage efficiency for enterprises [1]. Qian and Lai proposed an innovative method to improve the genetic algorithm, using the adaptive double boundary constraint strategy to improve the search efficiency, and applying this improved algorithm to the storage

allocation problem in storage management, greatly improving the storage efficiency and speed. Finally, the conclusion that the adaptive double boundary constraint genetic algorithm, can effectively solve the optimization problem of storage allocation. [2] Faber N believed that warehouse management system (WMS) is a powerful tool to improve the efficiency of warehouse business, and a perfect information system can save enterprise costs [3]. Huang took a company's storage process as the research object, through the establishment of a Flexsim model, analyzed the bottleneck links in the model, and put forward improvement suggestions, finally achieving the goal of system improvement [4].

Like many non-productive processes, warehousing business processes have the characteristics of loose and flexible structure. The sequence of tasks in the process is complex, and sometimes several tasks can be carried out simultaneously. According to these characteristics of storage business process, this paper established a Petri net model of storage business and applies related tools to analyze the performance. Through sorting out the business logic of the enterprise warehouse, draw the business flow chart, then map the flow chart to the workflow model, and further apply the reachability tree and Markov chain to analyze the model. The key business process-product outbound business is optimized. Finally, conclusions and optimization suggestions are given based on the method of industrial engineering.

## II. OVERVIEW OF PETRI NET THEORY

### A. Petri Nets

Petri net originated from the description of signal transmission, so it is suitable for describing the system characterized by resource flow. It is composed of  $S_$  element representing state and  $T_$  element representing change. Therefore,  $T_$  element causes the resource flow in  $S_$  element. The flow relationship between the two elements is expressed by  $F$ , and  $S$  and  $T$  are respectively called the place set and transition set [5].

The triple  $N=(S, T; F)$  is a directed network, and the sufficient and necessary conditions are:

- 1)  $S \cap T = \emptyset$ ;
- 2)  $S \cup T \neq \emptyset$ ;
- 3)  $F \subseteq S \times T \cup T \times S$  (" $\times$ " is Cartesian product);
- 4)  $\text{dom}(F) \cup \text{cod}(F) = S \cup T$ , within  
 $\text{dom}(F) = \{x \mid \exists y: (x, y) \in F\}$   
 $\text{cod}(F) = \{y \mid \exists x: (x, y) \in F\}$

They are the domain of definition and range of  $F$  respectively.

The intersection of the place and transition sets is empty, indicating that the place and transition are two different elements. The union of the place set and the transition set cannot be empty, indicating that there must be at least one element in the network. There is a kind of resource stored in each place, and the place can only have direct flow relation with changes, indicating the flow of resources. Isolated place are places that are not related to any changes in resource flow. Similarly, changes that do not cause resource flow are also isolated, and the last condition indicates that there cannot be isolated elements in the network.

### B. Workflow Model

The concept of workflow net (WF\_net) began in the early 1990s and was proposed by Dutch scholar Aalst. He also defined the concepts of activity and boundedness to evaluate the safety of workflow net.

WF\_net is used to describe the control flow in the workflow. Changes represent the tasks in the workflow and are usually represented by rectangles or thick bars. Dependencies  $F$  between tasks are represented by the connections of the places, circles or ellipses are used to represent places, and the number of black dots in the places represents the number of resources, which is called Token. In fact, the tasks are not only executed sequentially, but also several tasks may be executed simultaneously, or only one or more of several tasks may be selected for implementation. Therefore, special symbols are used to represent such business rules, as shown in "Fig. 1".

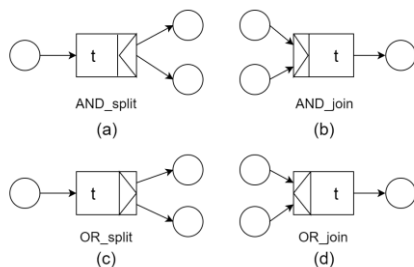


Fig. 1. Symbols of a common structure.

AND\_split means that the tasks of the two output arcs conform to parallel rule, and each output place obtains a token; AND\_join means that all tasks connected to the input arc are completed before the transition is executed, that is, each input place should contain a token. OR\_split indicates

that only one of the tasks connected to the output arc can be executed, so only one output place can obtain token; OR\_join means that as long as the task of the input arc is completed, that is, there is at least one input place obtain token, the transition will be executed. Among them, OR\_split and OR\_join do not conform to the transition rules, but are actually formed by merging the two transitions.

### III. WAREHOUSING BUSINESS PROCESS

The warehousing business process studied in this paper is mainly the product loading and unloading process in the finished product warehouse. The products in the warehouse come from the quality inspection or final inspection workshop of the enterprise. The complete operation contents include five items: receiving and inspecting goods, warehousing, packing and storing goods, loading and shipping, and inventory checking. There is no strict operation sequence. In the actual operation process, several tasks may need to be carried out at the same time.

#### A. Warehousing Business Process

The business process of product warehousing is shown in "Fig. 2". Firstly, the workshop shall fill in the product warehousing ticket in triplicate according to the imported products, which shall be signed by the warehousing personnel to take effect. One copy shall be kept by the workshop, one copy shall be submitted to the production statistics, and one copy shall be submitted to the warehouse keeper. The products that pass the inspection shall be transported directly from the workshop to the finished product warehouse by forklift in the form of pallets. The warehouse keeper shall check the name and quantity of the products according to the warehousing ticket at the first time of product warehousing. If the products are in conformity, they shall be stored normally. If they are not in conformity, the warehouse keeper shall inform the quality inspection department or the final inspection department to modify the warehousing ticket or change the products.

In the unloading process, it should be noted that it is necessary to identify whether the products need to be delivered on the same day. For the products delivered on the same day, the products can be packed and stocked according to the quantity requirements, and the remaining products can be put into storage. After the warehouse keeper checks that the receipt and the product are correct, the product should be put on the shelf reasonably according to the material, and the account should be posted in time to ensure that the account is consistent with the material.

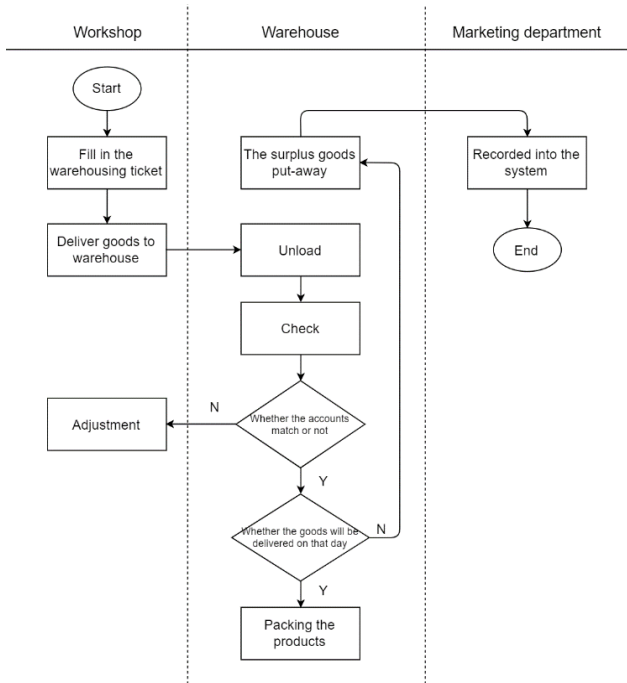


Fig. 2. Product warehousing process.

**B. Outbound Business Process**

In the process of product delivery, the warehouse keeper shall pack and stock the products according to the weekly plan or the temporary delivery plan, select the appropriate packing method, and fill in the Packing Details. After packing, the packing quantity and the remaining quantity in the warehouse shall be checked against the quantity in the account. Before delivery, the warehouse management personnel shall fill in the "Goods Issue Acceptance Form", and the relevant salesman shall check and sign it, take away the "Packing Details" and issue an issue document. The warehouse keeper shall contact the third party logistics for timely delivery. When delivering goods, the warehouse keeper shall fill in the "Exit Permit" and correctly mark the number and weight of goods, which shall be checked by the doorman. The product outbound business process is shown in "Fig. 3".

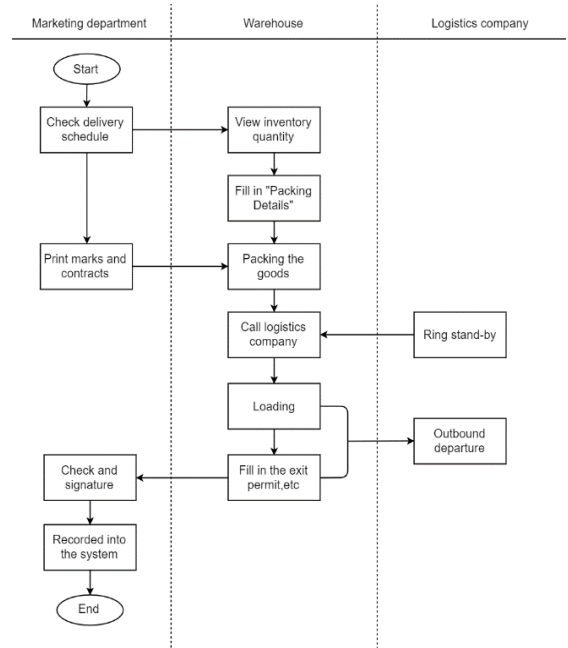


Fig. 3. Outbound business process.

**IV. PETRI NET MODELING AND ANALYSIS**

**A. Business Process Petri Net Modeling**

Combined with the product input/output business flow chart, the logic between work tasks is combed and mapped to Petri net model. "Fig. 4" depicts the business flow of product receipt and the whole business flow includes 6 changes and 7 places. From the perspective of warehouse management, T0 represents that the product has been transported from the workshop to the warehouse. T1 is the task of checking the product according to the receipt document. In fact, T1 will be triggered by two situations. These two situations are the new product receipt or the re-receipt of the receipt document and the product after the workshop has adjusted. T3 is the "account adjustment" task. There are two "judgments" in the whole process, T2 and T4 respectively, and the judgment result can only be one of the two. Therefore, the OR\_split structure is adopted to ensure that only one output place obtains a token, and finally, only one token in the final place (S7).

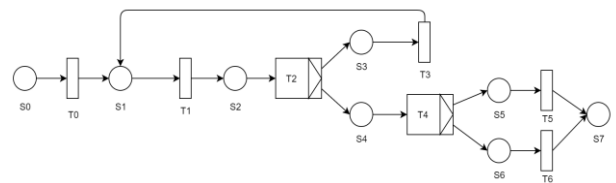


Fig. 4. Petri net model of product warehousing business.

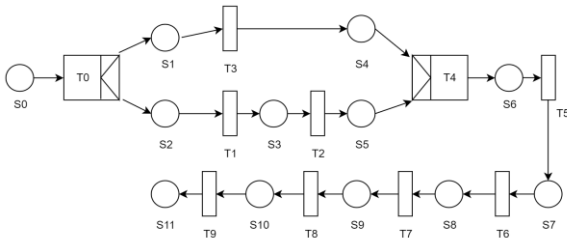


Fig. 5. Petri net model of product outbound business.

As shown in "Fig. 5", the Petri net model of product outbound business includes 10 transitions, i.e. T\_ elements. Starting from T0 "check delivery plan", T3 and T1, T2 are parallel tasks. T0 adopts AND\_split structure, which means that the marketing department personnel print shipping marks, contract tasks and warehouse keeper check inventory, and fill in the form tasks simultaneously. T4 can only be started after T1, T2, T3 are completed. Therefore, transition T4 in the model is represented by AND\_join structure. Token represents the end of the process when it arrives at the final place (S11) from the starting place (S0). See Table 1 for the description of the corresponding relationship between the remaining transitions and the process tasks. (see "Table I")

TABLE I. DESCRIPTION OF CORRESPONDENCE BETWEEN T ELEMENT AND PROCESS

The warehousing business model	Corresponding process	The outbound business model	Corresponding process
T0	Products delivered from workshop	T0	Check delivery schedule
T1	Inventory check	T1	View inventory quantity
T2	Judging whether the accounts match or not	T2	Fill in " Packing Details"
T3	Account adjustment	T3	Print shipping marks and contracts
T4	Judge whether the goods will be delivered on that day.	T4	Packing
T5	Products are stored on shelves.	T5	Call logistics company
T6	Product packing and stock up	T6	Loading
		T7	Fill in the exit permit, etc.
		T8	Check signature
		T9	Outbound departure

B. Reachable Tree Analysis of Petri Net Model

Petri net is an increasingly complete modeling method, which includes a variety of analysis techniques, including reachable identification sets, correlation matrices, invariants, etc. Through these analysis techniques, many important properties of Petri net can be understood. The first analysis

method used in this section is the reachability tree. A complete business process must be reachable, and there is no conflict source [6]. The reachability tree of the out-put and in-put of warehouse business process is shown in "Fig. 6" and "Fig. 7".

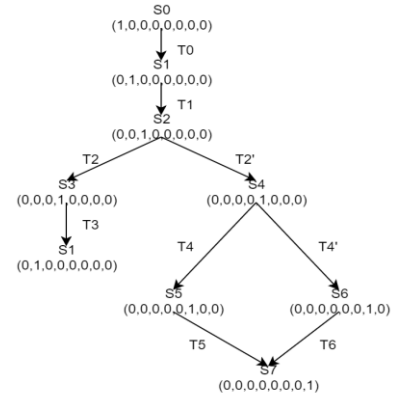


Fig. 6. Reachability tree of warehousing business process.

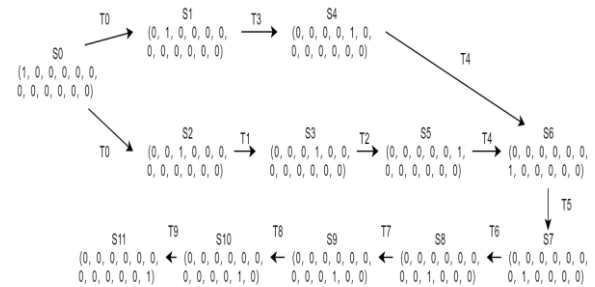


Fig. 7. Reachability tree of outbound business process.

The OR\_split structure in the inbound business process is actually formed by merging two transitions. Therefore, T2 and T4 are split into two transitions in the reachable tree of fig. 6 to describe. As can be seen from the reachable tree, since the transitions in the model represent the transfer of information or forms, the reachable tree is composed of numbers 0 or 1, and the number of token contained in each node is always no more than 1. The Petri net model of warehousing business is bounded and safe. The whole inbound and outbound business process is reachable, and at the end of the state, only the final place has token.

In the outbound business process model, the number of times transitions are triggered is limited, so the model is fair. However, the existence of "account adjustment" task in warehousing business results in a cycle of S1, S2 and S3 places, which may be executed indefinitely. This is unfair to other tasks. After specific analysis of the process, a limit should be set in the process, for example, limiting the number of workshop adjustments.

C. Quantitative Analysis of Petri Net Model

This section takes Petri net model of outbound business as an example, and quantitative analysis is based on Markov

chain. Before MC is established, the following assumptions are made:

- The business data obtained through on-site investigation represent the average work efficiency of warehouse keepers.
- The implementation rate includes the average waiting time.
- There are multiple order shipment plans every day, and the goods of one order will not enter the next outbound business process until they are outbound.

Based on the Petri net model as shown in "Fig. 5", MC is constructed, i.e. transition  $T_i$  is converted into implementation rate  $\lambda_i$ . At the same time, the initial identification of the model, namely  $M_0 = (1,0,0,0,0,0,0,0,0,0,0,0,0)$  is marked as (0), indicating that there is a token in place  $S_0$ . Thus, different reachable sets are obtained through different transitions, and further state sets  $M_0, M_1, M_2 \dots M_{11}$  are obtained. According to these state sets, isomorphic Markov chain is established, and the directed arcs in the chains represent the transition from one state to another, as shown in "Fig. 8".

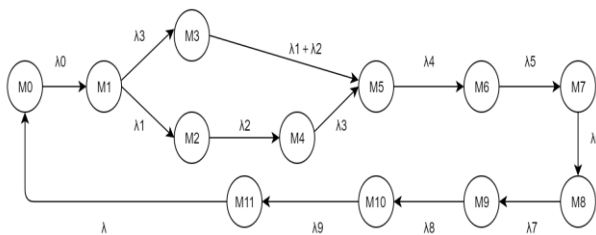


Fig. 8. Markov chain of outbound business.

Establish a linear equation, as shown in formula (1):

$$\begin{cases} SQ = 0, \\ \sum_i P_i = 1, \quad 0 \leq i \leq 11. \end{cases} \quad (1)$$

$S$  in the formula is vector  $(P(M_0), P(M_1), \dots, P(M_{11}))$ , which is the probability that the reachable marker reaches a stable state;  $Q$  is the rate matrix of Markov process transfer. According to the collected data, the implementable frequency of each task in unit time (30min) is calculated to obtain the implementation rate  $\lambda_i$ , as shown in "Table II".

TABLE II. SCHEDULE OF IMPLEMENTATION RATES

Transition process	Average working time /min	Implementation rate	Rate value
T0	2.0	$\lambda_0$	15.0
T1	4.0	$\lambda_1$	7.5
T2	2.0	$\lambda_2$	15.0
T3	12.0	$\lambda_3$	2.5
T4	13.0	$\lambda_4$	2.3
T5	0.5	$\lambda_5$	60.0
T6	15.0	$\lambda_6$	2.0
T7	1.0	$\lambda_7$	30.0
T8	8.0	$\lambda_8$	3.8
T9	2.0	$\lambda_9$	15.0
T	2.5	$\lambda$	12.0

The rate matrix drawn in this paper is an  $11 \times 11$  matrix, and the calculation rules of the elements in the matrix are as follows: if there is a directed arc between the markers  $M_i$  and  $M_j$ , the matrix element  $q_{ij}(i \neq j)$  is the rate value on the arc, and if there is no directed arc between the markers, the matrix element is 0; The matrix element  $q_{ij}$  on the diagonal is the inverse of the sum of the rate values of all transitions that can be triggered from  $M_i$  [7]. The result rate matrix is as follows. (see "Fig. 9")

-15	15	0	0	0	0	0	0	0	0	0	0
0	-10	7.5	2.5	0	0	0	0	0	0	0	0
0	0	-15	0	15	0	0	0	0	0	0	0
0	0	0	-23	0	22.5	0	0	0	0	0	0
0	0	0	0	-2.5	2.5	0	0	0	0	0	0
0	0	0	0	0	-2.3	2.3	0	0	0	0	0
0	0	0	0	0	0	-60	60	0	0	0	0
0	0	0	0	0	0	0	-2	2	0	0	0
0	0	0	0	0	0	0	0	-30	30	0	0
0	0	0	0	0	0	0	0	0	-3.8	3.8	0
0	0	0	0	0	0	0	0	0	0	-15	15
12	0	0	0	0	0	0	0	0	0	0	-12

Fig. 9. Result rate matrix.

According to the rate matrix, by solving the linear equations, the steady-state probability of each reachable marker can be obtained as follows.

- $P(M_0)=P(M_{10})=0.0346,$
- $P(M_1)=0.0518,$
- $P(M_2)=0.0259,$
- $P(M_3)=0.0059,$
- $P(M_4)=0.1527,$
- $P(M_5)=0.2253,$
- $P(M_6)=0.0086,$
- $P(M_7)=0.2592,$
- $P(M_8)=0.0173,$
- $P(M_9)=0.1365,$
- $P(M_{11})=0.0432.$

Based on the steady-state probability, several performance indexes of the system can be calculated. In this section, the efficiency of the business process is analyzed by using two indexes —the busy probability of the warehouse and the transition utilization rate.

- 1) The place busy probability:
  - $P[M(S_0)=1]=P[M(S_{10})=1]=P(M_0)=0.0346$
  - $P[M(S_1)=1]=P[M(S_2)=1]=P(M_1)=0.0518$
  - $P[M(S_3)=1]=P(M_2)=0.0259$
  - $P[M(S_4)=1]=P(M_3)=0.0059$
  - $P[M(S_5)=1]=P(M_4)=0.1527$
  - $P[M(S_6)=1]=P(M_5)+P(M_6)=0.2339$

$$P[M(S7)=1]=P(M7)=0.2592$$

$$P[M(S8)=1]=P(M8)=0.0173$$

$$P[M(S9)=1]=P(M9)=0.1365$$

$$P[M(S11)=1]=P(M11)=0.0432$$

2) *Transition utilization rate*: The transition utilization rate indicates how much time each job takes up for the entire outbound business process, and is the sum of the steady-state probabilities of all reachable identifiers that enable the transition to be executed. The utilization rate of each transition is as follows.

$$U(T0)=P(M0)=0.0346$$

$$U(T1)=P(M1)=0.0518$$

$$U(T2)=P(M2)=0.0259$$

$$U(T3)=P(M1)=0.0518$$

$$U(T4)=P(M5)=0.2253$$

$$U(T5)=P(M6)=0.0086$$

$$U(T6)=P(M7)=0.2592$$

$$U(T7)=P(M8)=0.0173$$

$$U(T8)=P(M9)=0.1365$$

$$U(T9)=P(M10)=0.0346$$

$$U(T10)=P(M11)=0.0432$$

From the above two performance indicators, it can be seen that there are some key links in the outbound business process, such as T4 (packing), T6 (loading) and T8 (signature checking), which account for a large proportion of the total business time. The reason is that there are many personnel and physical documents involved in the product delivery business. Other departments provide the necessary documents required for packing. There is no unified logistics delivery time and logistics vehicles are called at any time according to the delivery needs, thus there is a lot of waiting and moving waste. Improving the key links of the product delivery process and shortening the delivery time has an immediate effect on improving the storage efficiency.

## V. CONCLUSION

Through the above modeling and analysis process, combined with industrial engineering methods, the following optimization suggestions are put forward for outbound business process.

- Cancel the manual transfer of shipping marks and contract information by the salesman, and transfer the electronic information of the documents to the finished product warehouse, which will be printed by the warehouse keeper, and use the electronic office mode to replace the paper transfer.
- Simplify the signature process for each delivery by the salesman. The warehouse keeper will save the delivery information and sign and check it once every day after the completion of all delivery plans.

- Statistical packing box size information, cancel the work of measuring box size (when goods exceed the box, actual measurement is required).
- On the basis of reserved loading time, a more concentrated departure time is determined.

Warehousing business process involves complicated personnel and complicated work. The model is established by using the relevant theoretical knowledge of Petri net, and then the performance of the model is analyzed by qualitative and quantitative methods respectively. The key links that need optimization in the process can be effectively identified. The model can provide direction for business improvement of traditional warehouse and decision basis for management to formulate optimization scheme. In this paper, only the outbound business process has been optimized. In future research, the scope of business research can be expanded, and the most influential work links can be found by combining the analytic hierarchy process. The model can be further optimized.

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