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Application of an Extended Constant Elasticity of Substitution Production Function Model

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Abstract. Adopting constant elasticity of substitution production function model to calculate the contribution rate of each influential factor on economic growth when analyzing factors of economic growth is usually a major subject frequently researched. This paper makes an extension under general meaning of constant elasticity of substitution production function model structure, which is named "extended production function". Compared with traditional method, this one is featured by fast convergence and high precision. When it comes to application of production function model to calculate factors' contribution rate, traditional method generates big errors, while this paper gives a relatively precise method to calculate factors' contribution rate. Finally we adopted this method to calculate the contribution rate of factors influencing economic growth in China, which resulted in a good effect and conformed to reality.

1. Introduction

The general form of production function is $Y = f(X_1, X_2, \cdots, X_n)$, $X_1, X_2, \cdots, X_n$ are input factors, Y is output quantity. The form of a constant elasticity of substitution production function with n input factors is

$$Y = A(\delta_1 X_1^{-\rho} + \delta_2 X_2^{-\rho} + \cdots + \delta_n X_n^{-\rho})^{-\frac{\mu}{\rho}},$$

where A is efficiency coefficient, and $A > 0$; $\delta_i (i = 1, 2 \cdots, n)$ indicates the intensive degree of each factor technically; μ indicates homogeneous orders or scale return rate of the function, and $\mu > 0$; ρ is the substitutional parameter and $\rho \geq -1$. $A, \delta_i (i = 1, 2, \cdots, n), \rho, \mu$ are parameters to be estimated of the model.

For production functions with the input of multiple factors, their elasticity of substitution among different factors differs. So this paper makes an extension under general meaning of constant elasticity of substitution production function model structure, which is named "extended production function". Its form(Cheng and Han 2014;Hang et al.1997;Kemfert 1998;Kmenta 1997; Li and Pang 2010;Sun and Ma 2004) is

$$Y = A_0 e^{\gamma t} [\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}]^{-\frac{\mu}{\frac{\rho_1 + \rho_2 + \cdots \rho_m}{m}}},$$

where X_i is input factor, Y is output quantity, $A(t) = A_0 e^{\gamma t}$ is technical level; δ_i indicates the intensive degree of each factor technically; ρ_i is the substitutional parameter; New added λ_i is gain factor of input factor, and $(A_0, \gamma, \delta_i, \lambda_i, \rho_i, \mu_)$ $(i = 1, 2, \cdots, m)$ are parameters to be estimated of the model. This paper adopts nonlinear least squares method to estimate the model parameters. When it

comes to application of constant elasticity of substitution production function to calculate contribution rate of economic growth factors, traditional method generates big errors, while this paper has made some progress and has theoretically given a precise general formula for using production function to calculate the contribution rate of economic growth factors and a relatively precise specific calculation method(Cheng 2004;Cheng 2013;Cheng and Han 2009;Cheng and Xiang 2014; Yan and Wang 2009;Zhou and Hu 2010)in practical application. With extended constant elasticity of substitution production function model and according to calculation method of factor contribution rate as mentioned above, this paper has calculated the contribution rate of the factors influencing economic growth in China.

2. Parameter Estimation of Extended Constant Elasticity of Substitution Production Function Model and Calculation Method of Contribution Rate of Economic Growth Factors

Extended constant elasticity of substitution production function method is denoted as:

$$Y_t = f(X_t, \eta) + \varepsilon_t,$$

where $X_t = (X_{1t}, X_{2t}, \cdots, X_{mt})$, $\eta = (A_0, \gamma, \delta_1, \delta_2, \cdots, \delta_m, \lambda_1, \lambda_2, \cdots, \lambda_m, \rho_1, \rho_2, \cdots, \rho_m, \mu)$,and ε_t are fitting errors of the model.

$G(\eta) = \sum_{t=1}^{n} \varepsilon_t^2 = \sum_{t=1}^{n} [Y_t - f(X_t, \eta)]^2$, acquire the parameter η by getting minimum value of the function.

Set extended constant elasticity of substitution production function model as:

$$Y = A_0 e^{\gamma t} [\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}]^{-\frac{\mu}{\frac{\rho_1 + \rho_2 + \cdots \rho_m}{m}}},$$

Let $A = A_0 e^{\gamma t}$, $\rho = \frac{\rho_1 + \rho_2 + \cdots + \rho_m}{m}$,

then

$$Y = A[\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}]^{-\frac{\mu}{\rho}}.$$

Differential as

$$\begin{aligned} dY = {} & [\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}]^{-\frac{\mu}{\rho}} dA \\ & + \frac{\mu \delta_1 \lambda_1^{-\rho_1} \rho_1 Y}{[\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}] \rho X_1^{\rho_1 + 1}} dX_1 \\ & + \cdots + \frac{\mu \delta_m \lambda_m^{-\rho_m} \rho_m Y}{[\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}] \rho X_m^{\rho + 1}} dX_m. \end{aligned}$$

Elastic coefficient is

$$\alpha_i = \frac{\partial Y}{\partial X_i} \cdot \frac{X_i}{Y} = \frac{\mu \delta_i \lambda_i^{-\rho_i} \rho_i}{[\delta_1 (\lambda_1 X_1)^{-\rho_1} + \delta_2 (\lambda_2 X_2)^{-\rho_2} + \cdots + \delta_m (\lambda_m X_m)^{-\rho_m}] \rho X_i^{\rho_i}},$$

so

$$dY = \frac{Y}{A}dA + \alpha_1 \frac{Y}{X_1}dX_1 + \alpha_2 \frac{Y}{X_2}dX_2 + \cdots + \alpha_m \frac{Y}{X_m}dX_m.$$

Set economic vector quantity $(A, X_1, X_2, \cdots, X_m, Y)$ in year t to change to $M_{t+1}(A^{(t+1)}, X_1^{(t+1)}, X_2^{(t+1)}, \cdots, X_m^{(t+1)}, Y^{(t+1)})$ in year $t+1$ according to a continuous curve $L^{(t)}$, then

$$\Delta Y_A = \int_{L^{(t)}} \frac{\partial Y}{\partial A}dA = \int_{L^{(t)}} \frac{Y}{A}dA,\ \Delta Y_{X_i} = \int_{L^{(t)}} \frac{\partial Y}{\partial X_i}dX_i = \int_{L^{(t)}} \alpha_i \frac{Y}{X_i}dX_i\ (i = 1, 2, \cdots, m).$$

Obviously ΔY_A indicates the absolute influence value of technical progress on economic growth; ΔY_{X_i} indicates the absolute influence value of factor i on economic growth, so the growth; ΔY_{X_i} indicates the absolute influence value of factor i on economic growth, so the contribution rate of input factor i on economic growth is $\frac{\Delta Y_{X_i}}{\Delta Y}$

Here is a specific circumstance. Set extended constant elasticity of substitution production function model as

$$Y = f(A, K, L, E) = A[\delta_1(\lambda_1 K)^{-\rho_1} + \delta_2(\lambda_2 L)^{-\rho_2} + \delta_3(\lambda_3 E)^{-\rho_3}]^{-\frac{\mu}{\rho}},$$

where $A(t) = A_0 e^{\gamma t}$ technical progress level, K is capital input, L is labor force input and K is energy input. Set $L^{(i)}$ is the curve connecting point M_i and $M_{i+1}(i = 1, 2, \cdots, n-1)$, the curvilinear equation is

$$\{A^t = A^i(\frac{A^{i+1}}{A^i})^t, K^t = K^i(\frac{K^{i+1}}{K^i})^t, L^t = L^i(\frac{L^{i+1}}{L^i})^t, E^t = E^i(\frac{E^{i+1}}{E^i})^t, Y^t = Y^i(\frac{Y^{i+1}}{Y^i})^t, 0 \le t \le 1\}$$

Suppose that growth rates of K, L, E, Y in stage i are respectively k_i, l_i, e_i, y_i

Then the influence value of factor A on variation of aggregate indicator Y in stage i is

$$\Delta Y_{iA} = \int_{L^{(i)}} \frac{Y}{A}dA = \int_0^1 \frac{Y^i(\frac{Y^{i+1}}{Y^i})^t}{A^i(\frac{A^{i+1}}{A^i})^t} \cdot A^i(\frac{A^{i+1}}{A^i})^t \cdot \ln(\frac{A^{i+1}}{A^i})dt$$

$$= \int_0^1 Y^i(1+y_i)^t \gamma dt = \frac{\gamma Y^i y_i}{\ln(1+y_i)} = \gamma \frac{\Delta Y^i}{\ln(1+y_i)},$$

And the influence value of factor K on variation of aggregate indicator Y in stage i is

$$\Delta Y_{iK} = \int_{L^{(i)}} \alpha_1 \frac{Y}{K} dK$$
$$= \int_0^1 \alpha_1 \frac{Y^i (\frac{Y^{i+1}}{Y^i})^t}{K^i (\frac{K^{i+1}}{K^i})^t} \cdot K^i (\frac{K^{i+1}}{K^i})^t \cdot \ln(\frac{K^{i+1}}{K^i}) dt$$
$$= \int_0^1 \alpha_1 Y^i (1+y_i)^t \cdot \ln(1+k_i) dt$$
$$= \alpha_1 Y^i \ln(1+k_i) \frac{y_i}{\ln(1+y_i)}$$
$$= \alpha_1 \Delta Y^i \frac{\ln(1+k_i)}{\ln(1+y_i)}$$

The influence value of factor L on variation of aggregate indicator Y in stage i is

$$\Delta Y_{iL} = \int_{L^{(i)}} \alpha_2 \frac{Y}{L} dL$$
$$= \int_0^1 \alpha_2 \frac{Y^i (\frac{Y^{i+1}}{Y^i})^t}{L^i (\frac{L^{i+1}}{L^i})^t} \cdot L^i (\frac{L^{i+1}}{L^i})^t \cdot \ln(\frac{L^{i+1}}{L^i}) dt$$
$$= \int_0^1 \alpha_2 Y^i (1+y_i)^t \cdot \ln(1+l_i) dt$$
$$= \alpha_2 Y^i \ln(1+l_i) \frac{y_i}{\ln(1+y_i)}$$
$$= \alpha_2 \Delta Y^i \frac{\ln(1+l_i)}{\ln(1+y_i)}$$

The influence value of factor E on variation of aggregate indicator Y in stage i is

$$\Delta Y_{iE} = \int_{L^{(i)}} \alpha_3 \frac{Y}{E} dE$$
$$= \int_0^1 \alpha_3 \frac{Y^i (\frac{Y^{i+1}}{Y^i})^t}{E^i (\frac{E^{i+1}}{E^i})^t} \cdot E^i (\frac{E^{i+1}}{E^i})^t \cdot \ln(\frac{E^{i+1}}{E^i}) dt$$
$$= \int_0^1 \alpha_3 Y^i (1+y_i)^t \cdot \ln(1+e_i) dt$$
$$= \alpha_3 Y^i \ln(1+e_i) \frac{y_i}{\ln(1+y_i)}$$
$$= \alpha_3 \Delta Y^i \frac{\ln(1+e_i)}{\ln(1+y_i)}$$

Hence, the contribution rate of technical progress to economic growth from stage 1 to stage n is

$$\frac{\Delta Y_A}{\Delta Y}=\frac{\sum_{i=1}^{n-1}\Delta Y_{iA}}{\sum_{i=1}^{n-1}\Delta Y_{iA}+\sum_{i=1}^{n-1}\Delta Y_{iK}+\sum_{i=1}^{n-1}\Delta Y_{iL}+\sum_{i=1}^{n-1}\Delta Y_{iE}}$$
$$=\frac{\sum_{i=1}^{n-1}\gamma\frac{\Delta Y^i}{\ln(1+y_i)}}{\sum_{i=1}^{n-1}\gamma\frac{\Delta Y^i}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_1\Delta Y^i\frac{\ln(1+k_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_2\Delta Y^i\frac{\ln(1+l_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_3\Delta Y^i\frac{\ln(1+e_i)}{\ln(1+y_i)}}.$$

And the contribution rate of capital to economic growth from stage 1 to stage n is

$$\frac{\Delta Y_K}{\Delta Y}=\frac{\sum_{i=1}^{n-1}\Delta Y_{iK}}{\sum_{i=1}^{n-1}\Delta Y_{iA}+\sum_{i=1}^{n-1}\Delta Y_{iK}+\sum_{i=1}^{n-1}\Delta Y_{iL}+\sum_{i=1}^{n-1}\Delta Y_{iE}}$$
$$=\frac{\sum_{i=1}^{n-1}\alpha_1\Delta Y^i\frac{\ln(1+k_i)}{\ln(1+y_i)}}{\sum_{i=1}^{n-1}\gamma\frac{\Delta Y^i}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_1\Delta Y^i\frac{\ln(1+k_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_2\Delta Y^i\frac{\ln(1+l_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_3\Delta Y^i\frac{\ln(1+e_i)}{\ln(1+y_i)}}$$

And the contribution rate of labor force to economic growth from stage 1 to stage n is

$$\frac{\Delta Y_L}{\Delta Y}=\frac{\sum_{i=1}^{n-1}\Delta Y_{iL}}{\sum_{i=1}^{n-1}\Delta Y_{iA}+\sum_{i=1}^{n-1}\Delta Y_{iK}+\sum_{i=1}^{n-1}\Delta Y_{iL}+\sum_{i=1}^{n-1}\Delta Y_{iE}}$$
$$=\frac{\sum_{i=1}^{n-1}\alpha_2\Delta Y^i\frac{\ln(1+l_i)}{\ln(1+y_i)}}{\sum_{i=1}^{n-1}\gamma\frac{\Delta Y^i}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_1\Delta Y^i\frac{\ln(1+k_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_2\Delta Y^i\frac{\ln(1+l_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_3\Delta Y^i\frac{\ln(1+e_i)}{\ln(1+y_i)}}$$

And the contribution rate of energy to economic growth from stage 1 to stage n is

$$\frac{\Delta Y_E}{\Delta Y}=\frac{\sum_{i=1}^{n-1}\Delta Y_{iE}}{\sum_{i=1}^{n-1}\Delta Y_{iA}+\sum_{i=1}^{n-1}\Delta Y_{iK}+\sum_{i=1}^{n-1}\Delta Y_{iL}+\sum_{i=1}^{n-1}\Delta Y_{iE}}$$
$$=\frac{\sum_{i=1}^{n-1}\alpha_3\Delta Y^i\frac{\ln(1+e_i)}{\ln(1+y_i)}}{\sum_{i=1}^{n-1}\gamma\frac{\Delta Y^i}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_1\Delta Y^i\frac{\ln(1+k_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_2\Delta Y^i\frac{\ln(1+l_i)}{\ln(1+y_i)}+\sum_{i=1}^{n-1}\alpha_3\Delta Y^i\frac{\ln(1+e_i)}{\ln(1+y_i)}}$$

3. Empirical Analysis on Contribution Rate of Chinese Economic Growth Factors

In order to make in-depth research of economic growth status of China's secondary industry ,discussion of its growth patterns and calculation of contribution rate of input factors to economic growth, we conduct an analysis by selecting added value Y (thousand million RMB) of secondary industry as output quantity, and fixed-asset investment K (thousand million RMB), number of employed personnel L (thousand million people) and total energy consumed E (ten thousand tons of standard coal) as influential factors of economy. The analysis was conducted from 1996 to 2017. Table 1 shows related data coming from *Chinese Statistical Yearbook*.

Table 1. Information of Chinese economic growth

Year	Y	L	K	E	Year	Y	L	K	E
1996	71176.6	68950	22913.5	135192	2007	249529.9	75321	137239	280508
1997	78973	69820	24941.1	135909	2008	316228.8	75564	172828.4	291448
1998	84402.3	70637	28406.2	136184	2009	343464.7	75828	224598.8	306647
1999	89677.1	71394	29854.7	140569	2010	401512.8	76105	251683.8	324939
2000	99214.6	72085	32917.7	145531	2011	473104.0	76420	311485.1	348002
2001	109655.2	72797	37213.5	150406	2012	519470.1	76704	374694.7	361732
2002	120332.7	73280	43499.9	159431	2013	568845.0	76977	447074.0	375252
2003	135822.8	73736	55566.6	183792	2014	636462.7	77253	512760.70	426000
2004	159878.3	74264	70477.4	213456	2015	676780.0	77451	562000.0	430000
2005	183217.5	74647	887736	235997	2016	744127.0	77603	606466.0	436000
2006	211923.5	74978	109998.2	258676	2017	827122.0	77640	641238.0	449000

Set extended CES production function model as

$$Y = A[\delta_1(\lambda_1 K)^{-\rho_1} + \delta_2(\lambda_2 L)^{-\rho_2} + \delta_3(\lambda_3 E)^{-\rho_3}]^{-\frac{\mu}{\frac{\rho_1+\rho_2+\rho_3}{3}}}$$

$$= A_0 e^{\gamma t}[\delta_1(\lambda_1 K)^{-\rho_1} + \delta_2(\lambda_2 L)^{-\rho_2} + \delta_3(\lambda_3 E)^{-\rho_3}]^{-\frac{\mu}{\rho}}.$$

$$\eta = (A_0, \gamma, \delta_1, \delta_2, \delta_3, \lambda_1, \lambda_2, \lambda_3, \rho_1, \rho_2, \rho_3, \mu)$$
$$= (0.1999, 0.0639, 0.5004, 1.7870, 0.4730, 1.9989, 0.5259, 0.8084, 0.6027, 0.7408, 0.5549, 1.2967).$$

Namely

$$Y = 0.1999e^{0.0639t}[0.5004(1.9989K)^{-0.6027} + 1.7870(0.5259L)^{-0.7408}$$
$$+0.4730(0.8084E)^{-0.5549}]^{-\frac{1.2967}{0.6328}}.$$

Coefficient of determination of the model is $R^2 = 1 - \frac{\sum(Y_t - \hat{Y}_t)^2}{\sum(Y_t - \bar{Y})^2} = 0.9965$

We can see that the model's fitting precision is high, coefficient of determination approaches1. The contribution rates of all factors to economic growth from 1996 to 2017 are respectively:

Contribution rate of technical progress A to economic growth is $\frac{\Delta Y_A}{\Delta Y} = 55.24\%$;contribution rate of factor K to economic growth is $\frac{\Delta Y_K}{\Delta Y} = 24.86\%$;contribution rate of factor L to economic growth is $\frac{\Delta Y_L}{\Delta Y} = 2.30\%$;contribution rate of factor E to economic growth is $\frac{\Delta Y_E}{\Delta Y} = 17.60\%$.

From the estimation results, we can see that economic growth in China mainly depends on technical progress， and secondly depend on capital input and energy input, and the contribution rate of labor force is relatively low. The results are relatively consistent with the reality in China.

4. Conclusion

This paper gives an extended constant elasticity of substitution production function model ,so that the model has better adaptability. Theoretically this paper gives a precise formula for the production function model to calculate the contribution rate of economic growth factors, which provides a good thought for actual calculation. Finally this paper takes advantage of the given model and method to calculate the contribution rates of economic growth factors in China.

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6. References

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