

# Dynamicity of Economic Speed Based on Continuous Variation of Ship Functional Coefficient Factors

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**Abstract.** Economic speed of a ship is a fixed value in existing studies, but it is changing. The ship's functional coefficient formula is imported into the economic speed expression to study the actual variability of the ship's economic speed, and get 5 main influencing factors. Taking a typical ship type as an example, first the dynamics of the economic speed and its changing trend under the single-factor variation are analyzed by two-dimensional curve. The fluctuation range of economic speed caused by the variation range of fuel consumption rate is relatively obvious. Economic speed has a convex decreasing status on displacement, fuel consumption and oil price, and a concave increasing status on daily fixed cost and average profit. The impact of displacement on economic speed should be paid attention on ocean shipping routes and ships with low cargo load, and the economic speed fluctuates relatively large when the everyday fixed cost and profit are low. After that, the dynamics and trend of the economic speed under the two-factor variation are analyzed by three-dimensional surface, and the impacts of distance, shipping market, age, cargo capacity, business model, maintenance level, fiscal and tax policies on economic speed can be obtained.

## 1. Introduction

Under certain operating environment and economic conditions, different ship speeds will lead to different economic effects on specific freight transportation tasks, which are mainly affected by the increase and decrease of operating costs and market revenue opportunities. Therefore, there is an optimal or better speed under certain operational objectives and constraints, and the optimal speed has different concepts with different operational objectives. The relatively basic and common optimal speeds are the economic speed with the lowest cost per nautical mile and the profitable speed with the greatest profit per operating day, in addition, there are also revised optimal speeds considering no-load voyage, variable cargo volume, and backlog of funds in transit etc. But the current research and application of the optimal speed is about a static optimal one, that is, the optimal speed of the entire voyage is relatively stable and unchanged. However, this optimal speed is based on the premise that the relevant factors are fixed. In fact, some factors change with the navigation of the ship, such as displacement and some cost items. Therefore, the optimal speed will inevitably change, so how will it change? What is the difference between the results with static optimal speed? Further research is needed in theory. The dynamics of economic speed would be analyzed based on concepts, calculation methods and influencing factors in this paper, so would the dynamic trend, trajectory and the effect of factors change on the economic speed.

## 2. Economic Speed Model and Its Influencing Factors

Set the speed  $v$ , the ship's fixed daily cost  $C_f$ , including crew salary, insurance, maintenance, management fee, etc., fuel price  $b$  (yuan/ton), ship displacement  $z$ , the unit fuel consumption rate of the engine containing the lubricant consumption is  $a$  (g/kWh), and the effective power of the engine is  $N$  (kW).  $\lambda$  is the Navy constant, and  $r$  is average daily profit. The ship's function coefficient  $K$  is

expressed in Eq. (1), the cost of navigation per mile is expressed in Eq. (2), and the economic speed  $v^*$  is expressed in Eq. (3).

$$k = 24 \times 10^{-6} abz^{2/3} / \lambda. \quad (1)$$

$$s = (C_f + kv^3) / (24v) = (C_f + 24 \times 10^{-6} abz^{2/3} v^3 / \lambda) / (24v). \quad (2)$$

$$v^* = [(C_f + r) / (2k)]^{1/3} = [(C_f + r) \lambda / (48 \times 10^{-6} abz^{2/3})]^{1/3}. \quad (3)$$

The economic speed is constant in a shipping voyage in original theory. However, in the process of ship running, fuel consumption and displacement change are continuous, and the unit fuel consumption rate of engine may also change continuously. It is necessary to further explore and compare how the economic speed changes with the influencing factors in theory, which is also conducive to more precise adjustment of ship operation plan and grasp the quantitative influence of factors in practice.

$$(\partial v^* / \partial C_f) = (\partial v^* / \partial r) = (1/3) \times \lambda^{1/3} \times [48 \times 10^{-6} abz^{2/3} (C_f + r)^2]^{-1/3}. \quad (4)$$

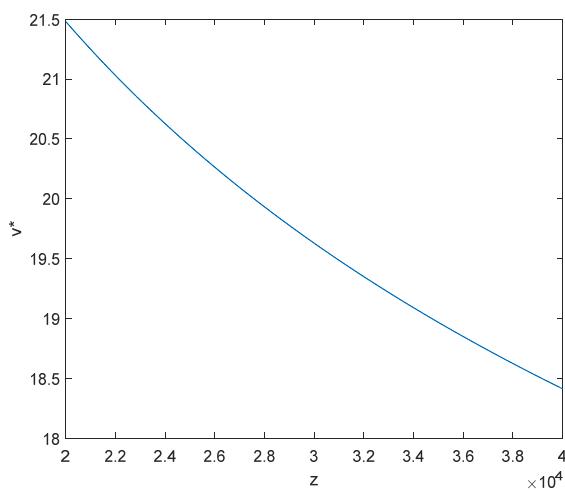
$$(\partial v^* / \partial a) = (-1/3) \times [\lambda(C_f + r)]^{1/3} \times (48 \times 10^{-6} a^4 b z^{2/3})^{-1/3}. \quad (5)$$

$$(\partial v^* / \partial b) = (-1/3) \times [\lambda(C_f + r)]^{1/3} \times (48 \times 10^{-6} a b^4 z^{2/3})^{-1/3}. \quad (6)$$

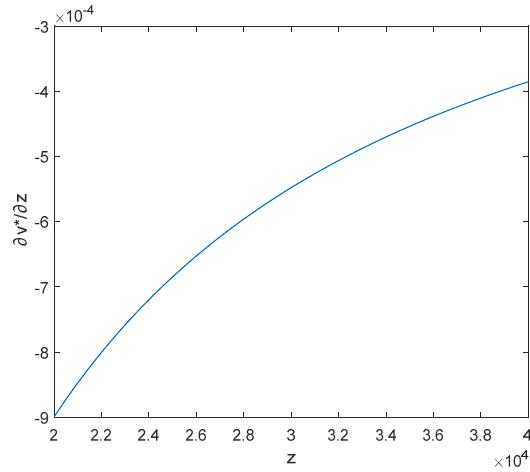
$$(\partial v^* / \partial z) = (-2/9) \times [\lambda(C_f + r)]^{1/3} \times (48 \times 10^{-6} a b)^{-1/3} z^{-11/9}. \quad (7)$$

### 3. Dynamics and Trends of Economic Speed on Continuous Variation of Single Factor

#### 3.1. Dynamics of Economic Speed with Continuous Variation of Displacement



**Figure 1.** Dynamic  $v^*$  under continuous varying  $z$

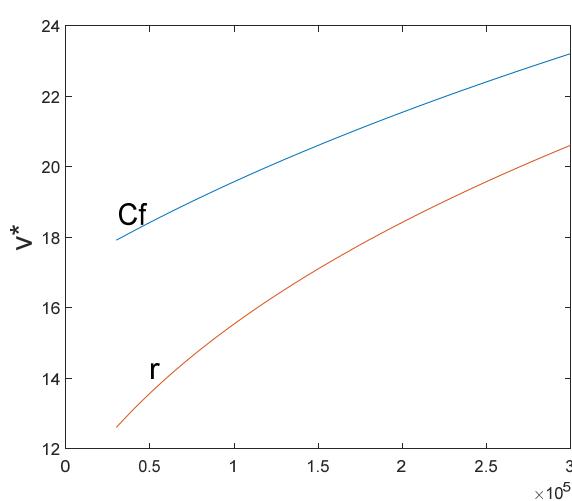


**Figure 2.** Rate of change of  $v^*$  to  $z$

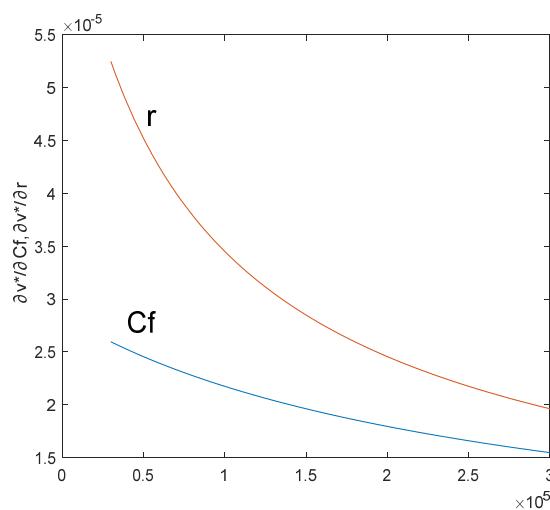
Considering a 40,000-ton carrier.  $a$  is 190g/kwh;  $b$  is 1500 yuan;  $\lambda$  is 400;  $C_f$  is 50000 yuan;  $r$  is 200000 yuan. The change of  $v^*$  with  $z$  is shown in Fig. 1 and its rate of change is shown in Fig. 2. It can be seen that  $v^*$  is a convex reduction function for  $z$ , and the change rate of  $v^*$  to  $z$  is a concave increase function. During the voyage, with the decrease of fuel, fresh water and materials, the

displacement will decrease, and the economic speed will change accordingly. It also reflects the range of changes in the economic speed when the ship type changes. It indicates that: (1) In voyages with lower speed and shorter transit distance, because fuel and material consumption is not much, the displacement change has little effect on the economic speed; (2) the worse the utilization of load capacity, the greater the influence of displacement change and the greater the range of economic speed change; (3) in the ocean routes with high speed, long distance and few refueling ports, attention should be paid to dynamic adjustment of speed in order to maintain the optimal navigation state and results; (4) the fleet with smaller tonnage should pay more attention to the adjustment of economic speed.

### **3.2. Dynamics of Economic Speed with Continuous Variation of Average Daily Fixed Cost or Profit**



**Figure 3.** Dynamic  $v^*$  under continuous varying  $C_f$  or  $r$



**Figure 4.** Rate of change of  $v^*$  to  $C_f$  or  $r$

The change of  $v^*$  with  $C_f$  or  $r$  is shown in Fig. 3, and the change rate of  $v^*$  with  $C_f$  or  $r$  is shown in Fig. 4. The effect of both factors decreases with the increase of its value, and the increase of economic speed tends to slow down. It indicates: (1) if the depreciation of the new ship is higher, or the maintenance cost of the old ship is higher, the range of economic speed change is very small when other conditions are unchanged, at either end of a ship's life cycle, properly increasing the wages and benefits of the crew, increasing the cost of ship insurance, maintenance and moistening materials will not have a significant impact on the optimization of ship navigation; (2) the economic speed of vessels of medium age is low; (3) the economic speed does not change much when the market is good or when transporting between large hub ports with other conditions unchanged; (4) the influence of freight fluctuation on economic speed is relatively obvious when the shipping market is close to break-even.

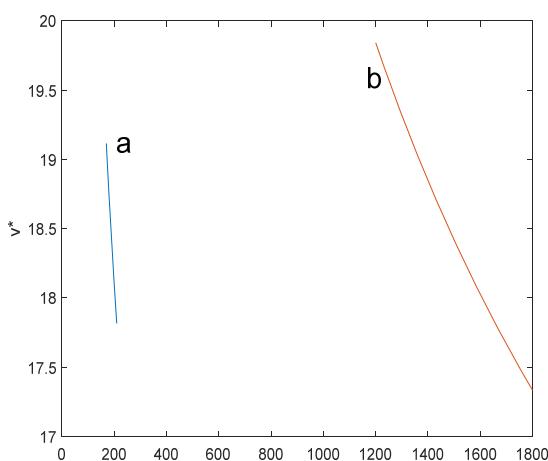
### **3.3. Dynamics of Economic Speed with Continuous Variation of Fuel Consumption Rate or Fuel Price**

The change of  $v^*$  with  $a$  or  $b$  is shown in Fig. 5, and the change rate of  $v^*$  with  $a$  or  $b$  is shown in Fig. 6. Within a certain range of relevant parameters of a particular ship type, the economic speed is a convex reduction function for both factors, and the rate of change is a concave increase function. It can be seen that: (1) the effect of ship fuel consumption rate on pure numerical value is obvious, and if the oil price fluctuates drastically, the economic speed will show some dynamics; (2) the economic speed of a ship will gradually decrease with its aging, because the fuel consumption rate will increase; (3) the dynamic characteristics of economic speed should be paid attention to when a ship needs to refuel in different areas for a long voyage.

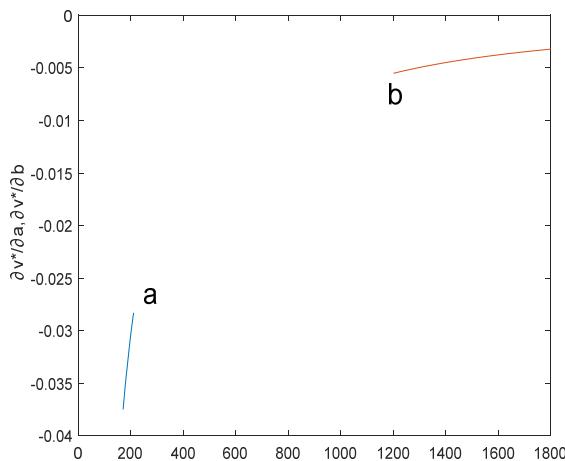
## 4. Dynamics and Trends of Economic Speed on Continuous Variation of Two Factors

### 4.1. The Effect of Displacement and Average Daily Cost or Profit

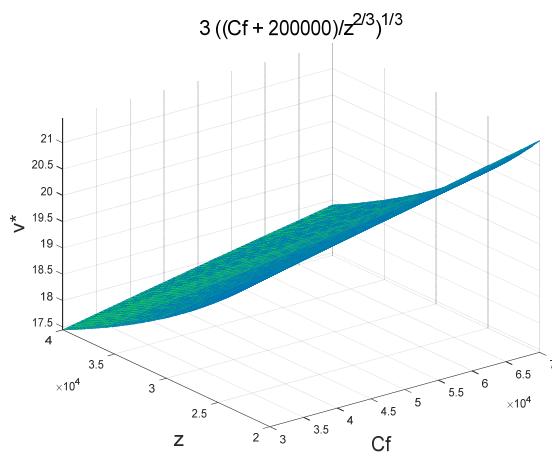
The changes of  $v^*$  with  $C_f$  and  $z$ ,  $r$  and  $z$  are shown in Fig. 7 and 8 respectively. Fig. 7 shows how the economic speed varies with the ship's newness and tonnage or the amount of cargo, indicating that a large ship of moderate age or a large cargo load should adopt a lower speed, and a smaller old or new ship or a smaller cargo load should adopt a higher speed. Fig. 8 shows how the economic speed varies with the shipping market and the size of the tonnage or the amount of cargo. If the ship age is moderate and the fixed cost is not high when the market is good, ships of large tonnage should also adopt a higher sailing speed. When the market is not good, newer or older small ships or empty vessels should also use a lower speed. In addition, significant changes in fixed costs may occur due to upgrades in key technologies of ships, large fluctuations in voyage risks, or unstable supply of market capacity.



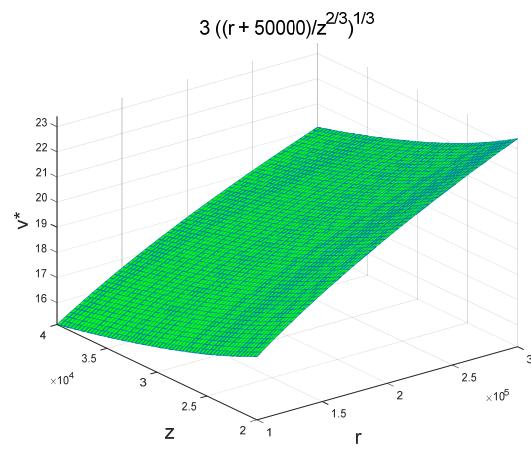
**Figure 5.** Dynamic  $v^*$  under continuous varying a or b



**Figure 6.** Rate of change of  $v^*$  to a or b



**Figure 7.** Dynamic  $v^*$  under continuous varying  $C_f$  and  $z$



**Figure 8.** Dynamic  $v^*$  under continuous varying  $r$  and  $z$

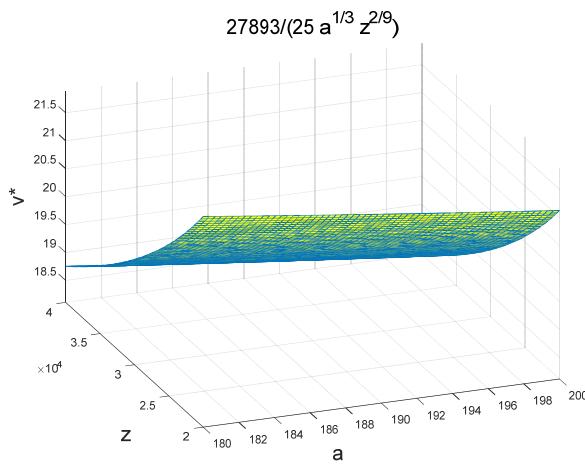
### 4.2. The Effect of Displacement and Fuel Consumption Rate or Price

The changes of  $v^*$  with  $a$  and  $z$ ,  $b$  and  $z$  are shown in Fig. 9 and 10 respectively. Figure 9 shows that the fuel consumption rate is low when the engine has high thermal efficiency, mechanical efficiency

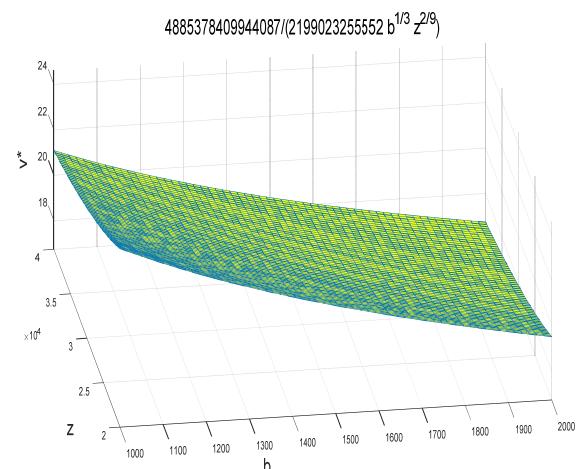
or low calorific value. It is suitable for high speed. Fig. 10 shows that a larger cargo load or a larger ship should adopt a lower speed when the oil price is high, whereas a higher speed is advisable.

#### 4.3. The Effect of Other Cases

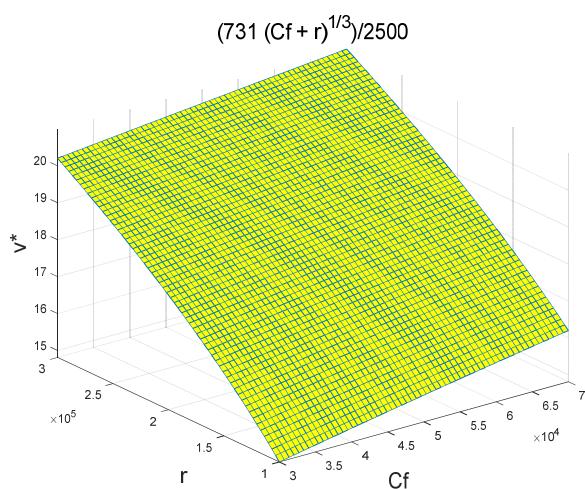
The changes of  $v^*$  with  $C_f$  and  $r$ ,  $a$  and  $b$  are shown in Fig. 11 and 12 respectively. When the freight rate or cargo volume changes greatly, such as the significant impact of economic and trade, the imbalance between supply and demand of transport capacity, the transfer of hub port trunk line to satellite port branch line, the dispatch of empty ships, the small imbalance coefficient of cargo flow direction, or the large difference of freight types between different voyages of multi-purpose ships leads to a distinct change in freight rates, the economic speed will change obviously. The fixed cost of a ship usually does not change much, but if the loan interest, depreciation and taxes are adjusted greatly, the maintenance of ships is insufficient, the aging wear and tear is faster, or the insurance and indemnity costs of different types of ships vary greatly, the economic speed will also change significantly. Ships with advanced technology and facilities cost more and should choose faster speed. In addition, when choosing or changing the way of chartering, the difference between daily fixed cost change and daily profit change should be considered, which determines the change degree of economic speed. Meanwhile, with the growth of ship age, the fuel consumption rate will increase year by year.



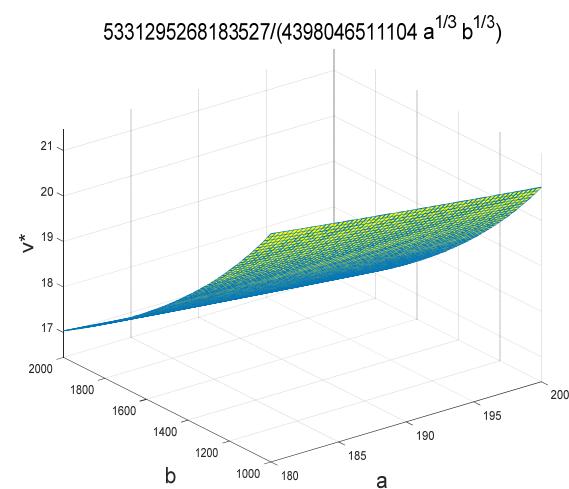
**Figure 9.** Dynamic  $v^*$  under continuous varying  $a$  and  $z$



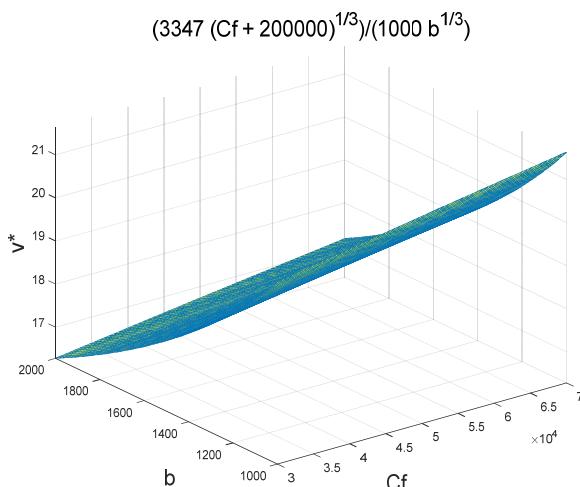
**Figure 10.** Dynamic  $v^*$  under continuous varying  $b$  and  $z$



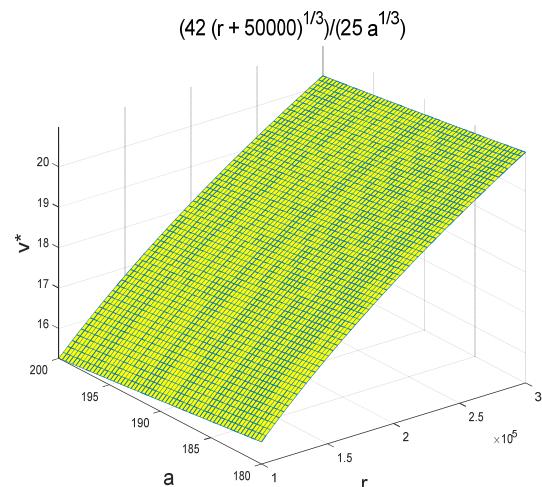
**Figure 11.** Dynamic  $v^*$  under continuous varying  $C_f$  and  $r$



**Figure 12.** Dynamic  $v^*$  under continuous varying  $a$  and  $b$



**Figure 13.** Dynamic  $v^*$  under continuous varying  $b$  and  $C_f$



**Figure 14.** Dynamic  $v^*$  under continuous varying  $a$  and  $r$

As mentioned earlier,  $C_f$  and  $r$ ,  $a$  and  $b$  are of the same properties respectively in structure, only different ranges of values make the surface shape of  $v^*$  different, but the basic trend is consistent, so here only list two cases as shown in Fig. 13 and 14.

## 5. Summary

The economic speed of a ship is not fixed in a voyage, but dynamic with the change of its structural factors, and shows a certain trend.

- (1) Economic speed has the same rate of change for daily fixed cost and average daily profit of a ship, and the structure of the rate of change to the oil consumption rate and the oil price is similar.
- (2) Economic speed is convex decreasing to displacement, small tonnage and high broken stowage vessels should pay more attention to the adjustment of economic speed.
- (3) Economic speed is concave increasing to daily fixed cost and average daily profit of the ship, and it is low on medium-age ships, and economic speed changes little on new, old ships and good market.
- (4) Economic speed is convex decreasing to oil price and oil consumption rate of a ship, ship's age growth and counter-current driving will reduce the economic speed.
- (5) Through 3D surface graph, the variation trend of economic speed under the combination of ship age, shipping market, ship tonnage, broken stowage, capacity supply and demand, policy adjustment, maintenance level, ship operation mode, etc. can be further obtained.

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