A New Multi-sensor Particle CPHD Filtering Algorithm for Bearings-only Multi-target Tracking

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Abstract. Aiming at bearings-only multi-target tracking, a new multi-sensor particle CPHD filtering algorithm is proposed, which analyses the structure information of mixed linear/nonlinear state space models and combines particle filter and Kalman filter to predict and estimate the states of multiple targets to enhance the estimating performance of the PHD and cardinality distribution. The target state estimates are extracted by utilizing the kernel density estimation theory and mean-shift method. Simulation results are presented to demonstrate the improved performance of the proposed filtering algorithm.

Introduction

In bearings-only multi-target tracking, the number of targets is unknown and vary with time due to the uncertainty of target information. In addition, the problem of model nonlinearity caused by coordinate transformation of target motion modeling and measurement modeling, and the physical characteristics of passive sensors themselves, as well as the incompleteness of measurement information, all bring great difficulties to target tracking. How to track multi-target effectively based on bearings-only measurement information has always been a popular and difficult topic in both academic and engineering research[1].

Compared with other traditional multi-target tracking algorithms, probability hypothesis density (PHD) filtering algorithm based on random set theory can transform complex multi-target state space operations into single-target state space operations, effectively avoiding complex data association and combination problems in multi-target tracking[2-4]. The cardinalized probability hypothesis density (CPHD) filtering algorithm, which can make full use of the information of multi-target density and does not need to limit the number of targets to obey Poisson distribution, has attracted more attention. Many scholars have carried out relevant research[5-9].

In this essay, a new multi-sensor particle CPHD filtering algorithm is proposed which uses centralized fusion strategy. By mining the structural information of mixed linear/non-linear state model and combining particle filter (PF) [10] and Kalman filter (KF) to predict and estimate the state of each target, the PHD and cardinality distribution of multi-target can be better estimated. Simulation results show that the proposed filtering algorithm is effective in the challenging bearings-only multi-target tracking scenario.

Problem Formulation

Consider the following passive multi-sensor bearings-only multi-target tracking system:

$$x_{n,k+1} = Fx_{n,k} + Gw_{n,k}\,,\ z^{o}_{m,k} = \begin{cases} h\left(x_{n,k}\right) + v^{o}_{k}, & \varsigma_{m,k} = n \\ clutter, & \varsigma_{m,k} = 0 \end{cases}, \tag{1}$$

Where $x_{n,k}$ is the system state vector of target n at k-time, $x_{n,k} = \left[x_n(k), \dot{x}_n(k), y_n(k), \dot{y}_n(k)\right]^{\mathrm{T}}$, $w_{n,k}$ is a white Gaussian noise with zero mean and covariance matrix Q_n, $\left\{z^{o}_{m,k}, m = 1, \cdots, M_k\right\}$ is the set of target bearings generated by sensor o at time k which contains C_k clutters,

$h\left(x_{n,k}\right)=\arctan\left(\frac{y(k)-y_n^o}{x(k)-x_n^o}\right)$, $\left(x_n^o, y_n^o\right)$ is the location of sensor o, $\varsigma_{m,k}$ represents the target indicator associated with measurement m. The measurement noise v_k^o is a white Gaussian process with zero mean and covariance matrix R^o, and it is uncorrelated with process noise $w_{n,k}$.

Multi-target Tracking With Particle CPHD Filter

In many applications, the target state space contains both linear and non-linear parts. Aiming at this kind of mixed linear/non-linear state model, the linear state and non-linear state of the target are estimated separately by combining Kalman filter and particle filter, which improves the estimation accuracy and reduces the estimation variance [11]. In this paper, a multi-target tracking particle CPHD filtering algorithm is proposed.

The target filtering model can be described as linear and non-linear forms.

$$x_k^n = f_{k-1}\left(x_{k-1}^n\right)+A_{k-1}^n x_{k-1}^l + B_{k-1}^n w_{k-1}^n,\ x_k^l = A_{k-1}^l x_{k-1}^l + B_{k-1}^l w_{k-1}^l,\ z_k = h_k\left(x_k^n\right)+v_k, \tag{2}$$

Where x_k^n and x_k^l represent the nonlinear and linear state of the target at time k, respectively, $x_k=\left[x_k^n, x_k^l\right]^{\mathrm{T}}$, $w_k=\begin{bmatrix} w_k^n \\ w_k^l \end{bmatrix} \sim N\left(0,\begin{pmatrix} Q_k^n & S_k \\ S_k^{\mathrm{T}} & Q_k^l \end{pmatrix}\right)$, $v_k \sim N\left(0, R_k\right)$, w_k and v_k are independent of each other.

Suppose $x_0^l \sim N\left(x_{0|-1}^l, P_{0|-1}^l\right)$, and the target state distribution x_0^n is given. The models described in (2) can be presented as

$$x_k^l = A_{k-1}^l x_{k-1}^l + B_{k-1}^l w_{k-1}^l,\ \overline{z}_k = A_{k-1}^n x_{k-1}^l + B_{k-1}^n w_{k-1}^n, \tag{3}$$

Where $\overline{z}_k = x_k^n - f_{k-1}\left(x_{k-1}^n\right)$. The systems described in (3) are linear Gaussian process, KF can be used to estimate the optimal value.

For non-linear state x_k^n, the particle filter is used for estimation. The particles predicted from time $k-1$ to time k obey the Gaussian distribution, i.e,

$$p\left(x_k^n \mid x_{k-1}^n\right)=N\left(f_{k-1}\left(x_{k-1}^n\right)+A_{k-1}^n \hat{x}_{k-1|k-2}^l, R^n\right),\ R^n = A_{k-1}^n P_{k-1|k-2}^l \left(A_{k-1}^n\right)^{\mathrm{T}} + B_{k-1}^n Q_{k-1}^n \left(B_{k-1}^n\right)^{\mathrm{T}}, \tag{4}$$

Where $\hat{x}_{k-1|k-2}^l$ and $P_{k-1|k-2}^l$ represent the one-step predicted value and its covariance of the linear state, respectively.

The proposed particle CPHD filter contains the following two steps.

Prediction: Assuming that the posterior intensity D_{k-1} and the posterior cardinality p_{k-1} are known at $(k-1)$-time , and that D_{k-1} can be expressed as $D_{k-1}\left(x\right)=\sum_{i=1}^{L_{k-1}}\omega_{k-1}^{(i)}\delta\left(x-x_{k-1}^{(i)}\right)$.

Then the predicted PHD of multi-target random sets is $D_{k|k-1}\left(x\right)=\sum_{i=1}^{L_{k-1}}P_S\omega_{k-1}^{(i)} f_{k|k-1}\left(x \mid x_{k-1}^{n,(i)}, \hat{x}_{k-1|k-2}^{l,(i)}\right)+\gamma_k\left(x\right)$.

We use RB method [11] for each item in $D_{k|k-1}\left(x\right)$. Firstly, for the survival target, the non-linear state particles are predicted by (4), $x_k^{n,(i)} \sim N\left(f_{k-1}\left(x_{k-1}^{n,(i)}\right)+A_{k-1}^n \hat{x}_{k-1|k-2}^{l,(i)}, R^{n,(i)}\right)$.

Linear state particles can be obtained by KF equation as follows,

$$\hat{x}_{k|k-1}^{l,(i)} = A_{k-1}^l[\hat{x}_{k-1|k-2}^{l,(i)} + G_{k-1}^{(i)}(x_k^{n,(i)} - f_{k-1}(x_{k-1}^{n,(i)}) - A_{k-1}^n \hat{x}_{k-1|k-2}^{l,(i)})], \tag{5}$$

$$G_{k-1}^{(i)} = P_{k-1|k-2}^{l,(i)}\left(A_{k-1}^n\right)^{\mathrm{T}}\left[A_{k-1}^n P_{k-1|k-2}^{l,(i)}\left(A_{k-1}^n\right)^{\mathrm{T}} + B_{k-1}^n Q_{k-1}^n\left(B_{k-1}^n\right)^{\mathrm{T}}\right]^{-1}. \tag{6}$$

The covariance of particle $\hat{x}_{k|k-1}^{l,(i)}$ is $P_{k|k-1}^{l,(i)} = A_{k-1}^{l}\left(P_{k-1|k-2}^{l,(i)} - G_{k-1}^{(i)}A_{k-1}^{n}P_{k-1|k-2}^{l,(i)}\right)\left(A_{k-1}^{l}\right)^{\mathrm{T}} + B_{k-1}^{l}Q_{k-1}^{l}\left(B_{k-1}^{l}\right)^{\mathrm{T}}$.

Secondly, for the birth targets, suppose the intensity $\gamma_k(x) = \gamma_k^n(x_k^n)\gamma_k^l(x_k^l)$. Select $\gamma_k^n(x_k^n)$ as the important density function and extract particles $x_k^{n,(i)}$, $i = L_{k-1}+1,\cdots,L_{k-1}+J_k$. The initial value of KF is set to $\left\{\hat{x}_{k|k-1}^{l,(i)}, P_{k|k-1}^{l,(i)}\right\}_{i=L_{k-1}+1}^{L_{k-1}+J_k} = \left\{\overline{x}_{0|-1}^{l}, \overline{P}_{0|-1}^{l}\right\}$.

Calculate the weights of particles,

$$\omega_{k|k-1}^{(i)} = \begin{cases} p_{S,k}\omega_{k-1}^{(i)}, & (i = 1,\cdots,L_{k-1}) \\ \dfrac{1}{J_k}, & (i = L_{k-1}+1,\cdots,L_{k-1}+J_k) \end{cases} . \tag{7}$$

The cardinality distribution is calculated as $p_{k|k-1}(n) = \sum_{j=0}^{n} p_{\Gamma,k}(n-j)\Pi_{k|k-1}[D_{k-1}, p_{k-1}](j)$.

Update: Assuming that the prediction intensity $D_{k|k-1}$ and the prediction cardinality $p_{k|k-1}$ at time $k-1$ are known , and that $D_{k|k-1}$ can be expressed by particles $\left\{\omega_{k|k-1}^{(i)}, x_k^{(i)}\right\}_{i=1}^{L_{k-1}+J_k}$. Using KDE theory [12] to update PHD of multi-target random set, $D_k(x) = \sum_{i=1}^{L_{k-1}+J_k} \omega_k^{(i)} K_{\sigma_d}^{x}\left(x - x_k^{(i)}\right)$, where $x_k^{(i)} = \left[x_k^{n,(i)}, \hat{x}_{k|k-1}^{l,(i)}\right]^{\mathrm{T}}$, $K_{\sigma_d}^{x}$ is a Parzen-Rosenblatt kernel function [13]. The weights $\omega_k^{(i)}$ are calculated as follows,

$$\omega_k^{(i)} = \frac{(1-P_d)\left\langle \Upsilon_k^1\left[D_{k|k-1}, Z_k\right], p_{k|k-1}\right\rangle}{\left\langle \Upsilon_k^0\left[D_{k|k-1}, Z_k\right], p_{k|k-1}\right\rangle}\omega_{k|k-1}^{(i)} + \sum_{z\in Z_k}\Psi_{k,z}\left(x_k^{(i)}\right)\frac{\left\langle \Upsilon_k^1\left[D_{k|k-1}, Z_k \setminus \{z\}\right], p_{k|k-1}\right\rangle}{\left\langle \Upsilon_k^0\left[D_{k|k-1}, Z_k\right], p_{k|k-1}\right\rangle}\omega_{k|k-1}^{(i)} . \tag{8}$$

In the case of given x_k^n, measurement z_k and target x_k^l are independent of each other, so $\Psi_{k,z}\left(x_k^{(i)}\right)$ in (8) can be simplified as follows,

$$\Psi_{k,z}\left(x_k^{(i)}\right) = \frac{P_d\langle 1, \kappa_k\rangle}{\kappa_k(z)} g_k\left(z \mid x_k^{n,(i)}, x_k^{l,(i)}\right) = \frac{P_d\langle 1, \kappa_k\rangle}{\kappa_k(z)} g_k\left(z \mid x_k^{n,(i)}\right) . \tag{9}$$

Update the cardinality distribution, $p_k(n) = \Upsilon_k^0\left[D_{k|k-1}, Z_k\right](n)\,p_{k|k-1}(n) \Big/ \left\langle \Upsilon_k^0\left[D_{k|k-1}, Z_k\right], p_{k|k-1}\right\rangle$.

The number of targets is estimated as $n_k = \sum_{j=1}^{\infty} j p_k(j)$.

Using Mean-Shift algorithm, all the peak positions of the density function can be given accurately [14]. For particle $x_k^{(i)}$, its Mean-Shift vector is

$$m\left(x_k^{(i)}\right) = \sum_{j=1}^{L_{k-1}+J_k} \omega_k^{(j)} K_{\sigma_d}^{x}\left(x_k^{(i)} - x_k^{(j)}\right)x_k^{(j)} \Big/ \sum_{j=1}^{L_{k-1}+J_k} \omega_k^{(j)} K_{\sigma_d}^{x}\left(x_k^{(i)} - x_k^{(j)}\right) - x_k^{(i)} . \tag{10}$$

Equation (10) indicates the mean-shift vector $m\left(x_k^{(i)}\right)$ should be transferred to the spot of the maximum consistent change, which is also the direction of density gradient. The algorithm is to take $x_k^{(i)}$ as a starting point, then move to the densest place, i.e., $x_k^{(i)} \to x_k^{(i)} + m\left(x_k^{(i)}\right)$. After iterating repeatedly, we can obtain the optimal locations of the intensity $D_k(x)$. Targets states can be estimated from $D_k(x)$ by taking n_k local maxima with the highest weights.

Simulation Analysis

In this part, the simulation results and analysis of the proposed algorithm are given. For multi-target tracking performance evaluation, the statistics of cardinality estimates and OSPA measure are used.

Consider the following bearings-only multi-target tracking scenario. The system model is described by Eq.(1), in which the number of targets varies with time. The specific parameters of the system are as follows:a multi-target bearings-only tracking problem

$T_s = 1\text{s}$, $F = diag\left(\left[\tilde{F}, \tilde{F}\right]\right)$, $\tilde{F} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$, $G = \begin{bmatrix} T_s^2/2 & T_s & 0 & 0 \\ 0 & 0 & T_s^2/2 & T_s \end{bmatrix}^{\mathrm{T}}$, $Q = diag\left(\left[0.01, 0.01\right]\right)$.

The positions of the sensors are set to $(-8, -10)$ km、$(8, -10)$ km and $(0, 13.86)$ km. The standard deviation of measurement noise $\sigma_\beta = 0.005$.

We assume that the spontaneous birth RFS is Poisson with intensity, $\gamma_k(\mathbf{x}) = 0.2 \times \sum_{i=1}^{3} N\left(\xi; m_\gamma^{(i)}, P_\gamma\right)$, where $m_\gamma^{(1)} = \left[-3.5, 0, -2, 0\right]^{\mathrm{T}}$, $m_\gamma^{(2)} = \left[3, 0, 0, 0\right]^{\mathrm{T}}$, $m_\gamma^{(3)} = \left[-3.5, 0, 2, 0\right]^{\mathrm{T}}$, $m_\gamma^{(4)} = \left[-5, 0, 5, 0\right]^{\mathrm{T}}$, and $P_\gamma = diag\left(\left[4, 2, 4, 2\right]\right)$. Target survival probability is $P_s = 0.99$ and target detection probability is $P_d = 0.98$. Clutter can be modeled as a Poisson RFS with intensity $\lambda_k = 3$ over the measurement space. Particle number is $N = 500$. The OSPA parameters are set as $p = 2$ and $c = 50$.

Fig. 1 shows the true trajectories of the targets. Fig. 2 shows the true number of targets and the mean of the estimated cardinality distribution. In Fig. 3, the comparison of the standard deviation (STD) of cardinality distribution for both algorithms is given. It can be seen that both algorithms can estimate the number of targets accurately, but the estimated STD of target number of the proposed algorithm is smaller than that of RBP-PHD filtering algorithm, which shows that the target number estimation of the proposed algorithm is more reliable.

In addition, Fig. 4 gives the MC average of OSPA distances for both filters. These results show that the proposed algorithm performs better than the RBP-PHD filtering algorithm due to that the proposed CPHD filtering algorithm provides more accurate estimates of target states and target number than the RBP-PHD filtering algorithm.

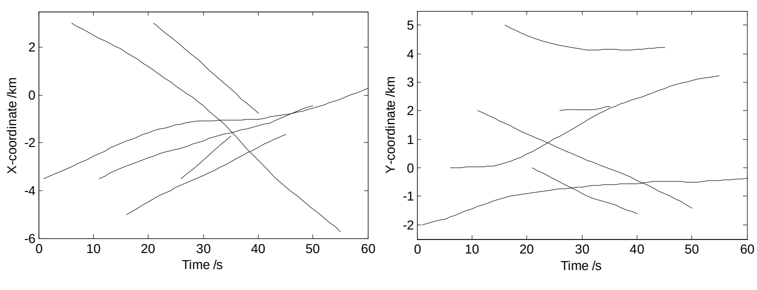

Figure 1. The true target trajectories in x- and y-coordinates

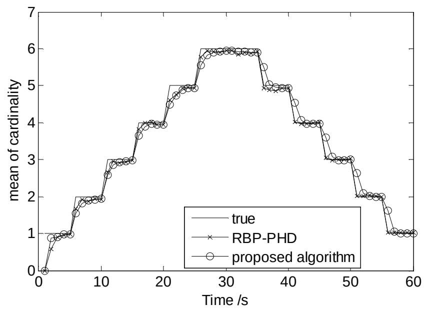

Figure 2. The mean of target number

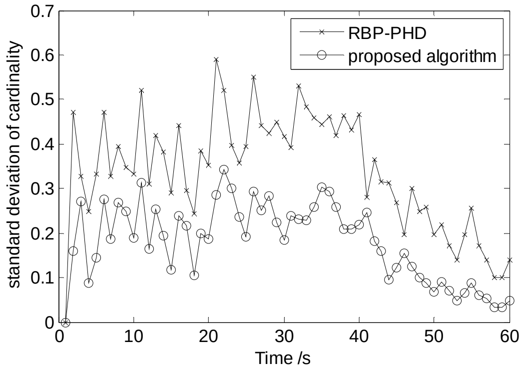

Figure 3. The STD of target number

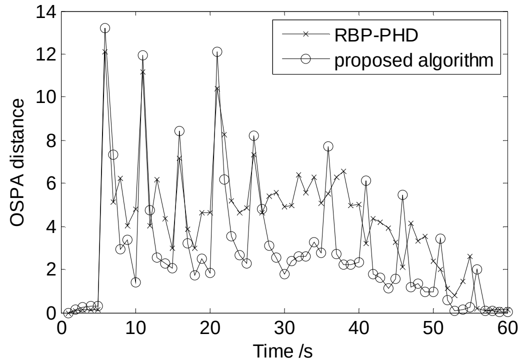

Figure 4. The average OSPA distance

Conclusion

In this paper, a new multi-sensor particle CPHD filtering algorithm is proposed as a solution to the bearings-only multi-target tracking problem for the class of mixed linear/nonlinear state space models. Simulations results demonstrate that the proposed CPHD filtering algorithm performs accurately and shows a significant reduction in the variance of estimation of target number.

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