Weak Signal Detection with Duffing Oscillator Based on Circular Boundary Counting Method and STFT State Identification Parameter

Zhi-qiang ZHU[1], Rui-li JIA[1], Xiao-peng YAN[1,*], Ke WANG[1] and Jian HOU[2]

[1]Science and Technology on Electromechanical Dynamic Control Laboratory, School of Mechatronical Engineering, Beijing Institute of Technology, Beijing 100081, China

[2]Beijing Jinghang Research Institute of Computing and Communication, Beijing 100074, China

*Corresponding author

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Abstract. To solve the problem of quantitative identification of the state of Duffing oscillator and determination of the critical threshold of reference amplitude, a combined weak signal detection method is proposed in this paper. The STFT statistical characteristics of Duffing oscillator in chaotic state, intermittent state and large-scale periodic state is analyzed and based on it the state identification parameter is selected to quantitatively present the state of Duffing oscillator. The step change of the number of the phase points on the boundary of circular regions in the phase plane is shown and the circular boundary counting method is proposed to determine the critical threshold. Based on above research, the algorithm of the combined weak signal detection is given, which uses Runge-Kutta algorithm to calculate the Duffing system. Simulation results show that this combined method is not only accurate at low SNR, but also robust when noise condition is variable.

Introduction

Recently, Duffing chaotic oscillator has become a hotspot in the field of weak signal detection, because of its strong advantage of being immune to noise and irrelavant signals while highly sensitive to the signal whose frequency is close to the internal reference frequency[1,2,3]. To make the best use of Duffing oscillator, the determination of the critical threshold of reference amplitude and the indentification among chaotic state, intermittent chaotic state and large-scale periodic state are of great significance[4,5]. However, many researchers address these tasks by their experience, experiments and observing, instead of quantitative analysis, which limits the engineering application of their methods.

To solve the above problem, a method with clear identification criteria and easy programming implementation is necessary. Literature [6] proposes a state identification parameter based on the statistical characteristics of short time Fourier transform (STFT). It is able to quantitatively distinguish Duffing oscillator among three states. However, the critical threshold determination of this method is subject to experience and experiments, which cannot be easily applied widely and whose performance may be susceptible to variable noise condition.

Therefore, a circular boundary counting method is proposed, which is not only less affected by noise, but also able to be easily calculated. In addition, an overall weak signal detection algorithm is also presented by combining these two methods with the Runge-Kutta method. Theoretical derivation and simulation results show the detection accuracy and robustness of this method.

Weak Signal Detection with Duffing Oscillator Based on STFT State Identification Parameter

Consider the extended Duffing oscillator, which aims to detect weak signal at any frequency. Its Duffing equation is given in Eq. 1.

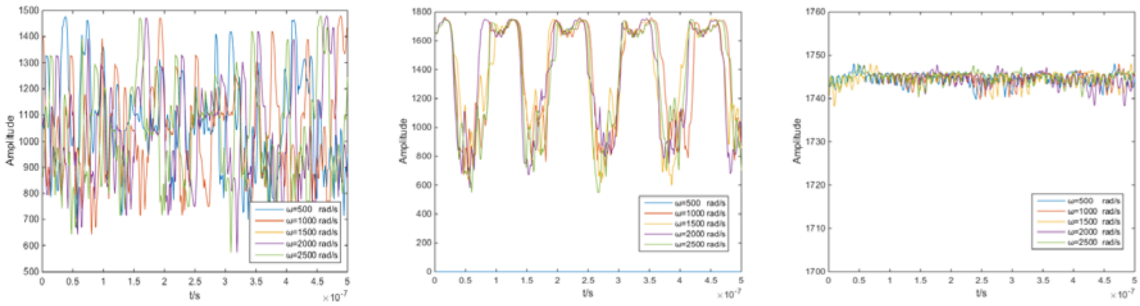
$$\ddot{x}(t)+k\omega\dot{x}(t)-\omega^2x(t)+\omega^2x^3(t)=\omega^2\gamma\cos(\omega t) \tag{1}$$

Where k is the damping ratio, $\gamma\cos(\omega t)$ is the internal reference signal, and γ and ω are its amplitude and angular frequency, respectively.

In Eq. 1, let $k=0.5$, and as the reference amplitude γ increases, Duffing oscillator system varies from chaotic state to chaotic critical state, i.e., chaos, but on the verge of changing to large-scale periodic state and then large-scale periodic state once γ increases to exceed the critical threshold γ_c .

Due to the initial value sensitivity and noise immunity of the above state transition, if the amplitude of reference signal γ is set very close to the critical threshold γ_c , we can determine whether or not there exists periodic signal whose frequency is close to the reference frequency in the external to-be-detect signal, and calculate the frequency of this periodic signal, based on the state transition of the Duffing oscillator system: if the system is in the chaotic state, there is only noise and irrelevant signals; if large-scale periodic state, there is periodic signal whose frequency equals to the reference; if intermittent chaotic state, there is periodic signal whose frequency can be calculate by $T = 2\pi/\omega\Delta\omega$ [1], where T is the period of the intermittent chaotic state.

In order to quantitatively identify the state of extended Duffing osillator, a STFT state identification parameter is proposed, which is based on the phenomenon that the time-amplitude diagrams of STFT of Duffing oscillator in the three states show quite different characteristics, while these characteristics remain the same when Duffing oscillator select different reference frequency, as is shown in Figure 1.



(a) The chaotic state (b) The intermittent state (c) large-scale periodic state

Figure 1. The time-amplitude diagrams of three different states

Therefore, the mathematical expectation of the maximum amplitude curve, E_m , can be taken as the identification parameter. Its expression is as shown in Eq. 2, where *i* and *j* present the i-th time point and the j-th frequency point, respectively, N is the length of sequence, and g_{Ham} is the expression of Hamming window function.

$$E_{\mathrm{m}} = \max_j\left(\frac{1}{N}\sum_{i=1}^{N}\left|\sum_{k=0}^{N-1} x(k) g_{Ham}(k-i) e^{-j2\pi j/N}\right|\right) \tag{2}$$

As is shown in Figure 2, based on the relationship between state identification parameter E_m and the difference frequency between to-be-detect signal and reference signal, when $\Omega_1:\{E_{\mathrm{m}} \mid E_{\mathrm{m}} \le 1300\}$ Duffing oscillator system is in chaotic state; when $\Omega_2:\{E_{\mathrm{m}} \mid 1300 < E_{\mathrm{m}} < 1730\}$, intermittent chaotic state; and when $\Omega_3:\{E_{\mathrm{m}} \mid E_{\mathrm{m}} \ge 1730\}$, large-scale periodic state.

Based on the above analysis, the weak signal detetion can be described as follow: First, set the reference frequency to the critical threshold γ_c, making the system in critical chaotic state; Then input the to-be-detect signal at a low SNR , and perform STFT to Duffing oscillator system; Finally distinguish the state of the system and calculate the frequency of the signal if Duffing oscillator is in the intermittent chaotic state or large-scale periodic state.

Circular Boundary Counting Method and Combined Weak Signal Detection

The phase trajectories of the Duffing oscillator system in chaotic state and large-scale periodic state are very different, which can be used to determine the critical threshold. a series of circular regions with radii of R=0.4~1.0 are drawn inside the outer trajectory of the phase plane of the Duffing oscillator system and the number of the phase points falling on the boundary of the circular domain is counted. For convenience, the number is indicated by Num. As shown in Figure 3, for these circular boundaries with different radii, when the noise condition is fixed, Nums show the same trend with the increase of the reference amplitude, which includes a step change around $\gamma = 0.826$, which indicates that $\gamma = 0.826$ is the critical threshold.

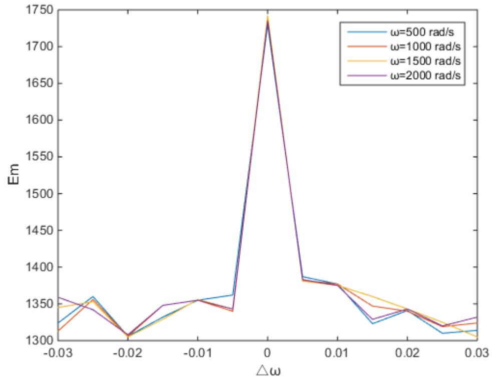


Figure 2. The relationship between the frequency difference and E_m

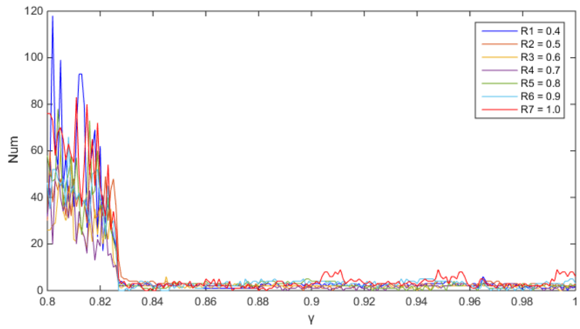


Figure 3. The phase points distribution on the circular boundary with different radii

Set the radius of the circle as 0.5, as is shown in Table 1, even if the value of σ^2 varies, as long as the reference amplitude γ increases to a certain point, Num decreases rapidly and remains in a very small value. Set determination threshold as $N_t = 6$ and therefore the critical threshold of reference amplitude is determined as 0.826, 0.830, 0.850 and 0.860 when the value of σ^2 is no more than 10^{-3}, 10^{-2}, 10^{-1} and 0.25, respectively. This determination results are the same as those of the traditional maximum Lyapunov exponential method. However, circular boundary counting method is less affected by noise, and its calculation process is relatively simple, which makes it possible to quickly adjust the critical threshold according to the change of noise power.

Table 1. The variation of the number of points on the circle with the internal reference amplitude under different noise conditions

Num σ^2 / γ	0	10^{-5}	10^{-4}	10^{-3}	10^{-2}	10^{-1}	0.25
0.824	37	41	48	42	49	444	916
0.826	36	33	34	31	39	429	994
0.828	5	5	5	5	27	34	105
0.830	5	5	5	3	18	27	50
0.832	4	4	4	4	4	29	51
0.848	3	3	3	1	3	35	48
0.850	1	1	1	2	2	35	43
0.852	3	3	3	3	2	4	47
0.854	2	2	2	2	2	2	45
0.860	2	2	2	2	2	3	50
0.862	2	2	2	2	2	4	2

Based on the research above, the combined weak signal detection method can be shown as in Figure 4:

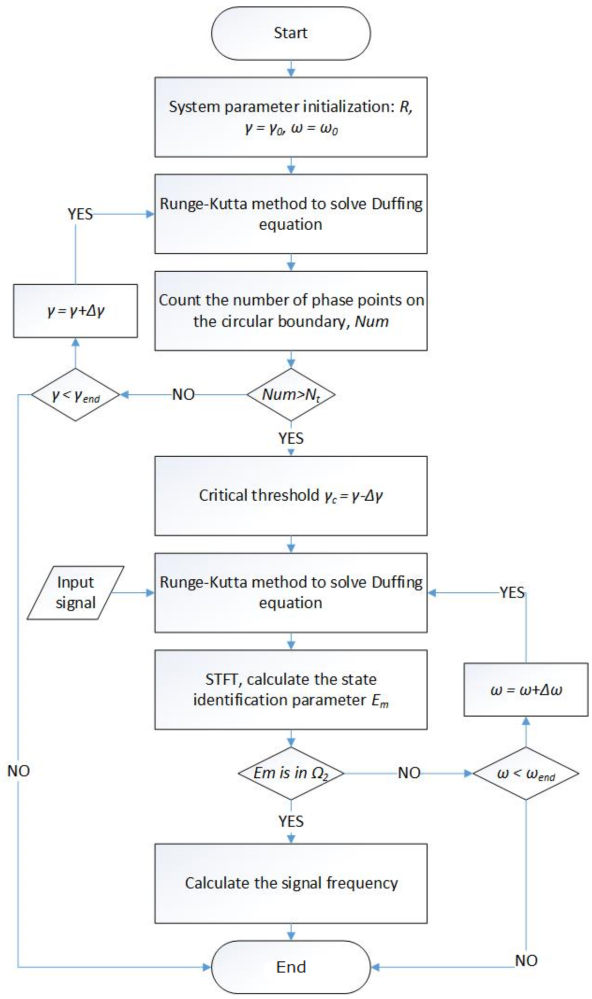


Figure 4. The combined weak signal detection algorithm

a) Estimate the frequency range of the input signal and noise power, setting the initial frequency ω_0, initial critical threshold γ_0, and step sizes $\Delta\omega$ and $\Delta\gamma$;

b) Calculate the critical threshold of Duffing oscillator, adjusting the amplitude γ equal to γ_c; With each increase, the Runge-Kutta algorithm is used to calculate the output of the system;

c) Input the to-be-detect signal, performing STFT to the output of Duffing oscillator, calculating the state identification parameter E_m and subsequently the reference frequency ω;

d) Calculate the signal frequency if Duffing oscillator is in intermittent chaotic state.

Simulation Results

Express the to-be-detect signal as $y(t) = A\sin(2\pi f_0 t) + n(t)$, where the signal amplitude $A = 0.04$, frequency $f_0 = 400$ Hz , $n(t)$ is Gaussian white noise with mean value of 0 and σ^2 varying from 0.0008 to 0.8 (SNR from 0 dB to -30 dB). To detect this signal, we use both the STFT state identification parameter method with a critical threshold of 0.826 and the combined weak signal detection method whose critical threshold is determined by circular boundary counting. The detection result is as shown in Table 2.

Table 2. Weak signal detection results at different noise conditions

σ^2	SNR(dB)	Critical threshold	Average errors(%)	
			STFT	Combined
0.0008	0	0.826	0.015	0.017
0.008	-10	0.826	0.019	0.018
0.08	-20	0.832	1.447	0.085
0.8	-30	0.850	3.538	0.157

From the detection results, it is obvious that the critical threshold is subject to the noise power condition. Since the reference amplitude of the STFT state identification parameter method based is fixed, which may be gotten from experiments at -10 dB SNR, though its detection accuracy is high at 0 dB and -10 dB, it degrades when SNR varies to -20 dB and -30 dB. And if the reference amplitude is set to 0.850 manually, though the average errors of the STFT state identification parameter method will also be reduced to 0.157% at -30 dB SNR, the average errors at -10 dB and -20 dB will increase.

In contrast, the combined method can not only make use of the easily quantitative calculation of the STFT state identification parameter method, but also detect weak signal with high resolution at different noise conditions, without manual adjusting of the reference amplitude. So the detection accuracy and noise robustness is attest by the simulation results.

Summary

In this paper, we propose a combined weak signal detection method with Duffing chaotic oscillator, based on the analysis of the STFT state identification parameter and circular boundary counting method. Critical threshold determination and state identification of it can be quantitative calculated, and the algorithm is given. Simulation results show that the combined method has high accuracy at low SNR and robustness at variable noise condition.

Acknowledgement

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