

# Logical and Mathematical Concept of the Time Value of Money and Discounting

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**Abstract**—In the conditions of non-stationary economy during the investment analysis there is a need to take into account the factors of changes in the value of money in time, uncertainty and risk. This requires a comprehensive and systematic analysis based on the logical and mathematical concept of the time value of money and discounting in the formation of the evaluation apparatus of investment projects. The authors propose the principle of logical-mathematical concept, which consists in the fact that you can only discount the increased amount, not capital gains. It is argued that it is mathematically incorrect to compensate for the impact of inflation, risk and alternative cost of capital on the cash flows of a real investment project by discounting cash inflows. The principal point in assessing the effectiveness of investment projects is the organization of a clear algorithm of cash flows. The authors have compiled such an algorithm.

**Keywords**—investment project, the time value of money, uncertainty, risk, methods of assessing the effectiveness of investment projects

## I. INTRODUCTION

Economic growth requires the implementation of effective investment projects. The Russian economy is characterized as non-stationary [1]. In this regard, to assess the economic efficiency of investment projects, it is necessary to take into account the factors that determine this unsteadiness, for example, uncertainty, risk, changes in the value of money over time and inflation for different currencies.

The problem of dynamic methods of evaluation of investment projects is to underestimate their effectiveness through the use of controversial mathematical mechanism of discounting, which is fundamentally different from the classical discounting used in financial mathematics. This problem is still unsolved and hushed up.

In accordance with the “Methodological recommendations for assessing the effectiveness of investment projects” (second edition) “discounted cash flows is called bringing their multi-time (relating to different steps of the calculation period) values to their value at a certain point in time, which is called the moment of reduction” [2]. “The cash flow of an investment project

is a dependence on the time of cash receipts and payments during the implementation of the project generating it, determined for the entire settlement period” [2].

Thus, based on these two definitions, it follows that different time values of cash flows are subject to discounting, that is, the dependence of cash receipts and payments on time. Simply put, you need to discount cash receipts and payments, the value of which varies over time.

On the one hand, the authors of the recommendations write that  $\Phi(m)$  is the cash flow. On the other hand, in the formula of net present value  $\Phi(m)$  is net cash inflow. So which of the two statements is true and which is false? This question is important from the standpoint of logical rigor and the laws of logic.

According to the law of logic of the “excluded third” statement is true or it is false. And according to the law “contradictions” statement can not be both true and false.

And what should be discounted: cash flow or cash inflow? This question is fundamental, since the answer to it depends on the mathematical accuracy of the results of evaluating the effectiveness of investment projects.

Despite this discrepancy, it turns out that in the calculations it is used the NPV formula, and the formula is discounted net cash inflow [3]. The legality of its discounting is highly questionable.

## II. LOGICAL-MATHEMATICAL CONCEPT OF THE TIME VALUE OF MONEY AND DISCOUNTING

In financial mathematics, discounting is considered as a reverse compounding process. Compounding or augmentation is the process of calculating future value based on present or present value or value. Discounting is the process of calculating present or present value based on a known future value [4].

Logical-mathematical concept of the time value of money and discounting lies in the fact that the initial capital invested in financial investments with time grows and you receive the accumulated amount, including the initial capital and its growth, and which can be discounted in

order to answer the question: “How much initial capital in financial investments to create the required (planned) the accrued amount and, accordingly, a capital gain?” In the case of real investments, the issue is somewhat different: “How much alternative initial capital will be required in risk – free financial investments to form the necessary (planned) increased amount and, accordingly, the capital growth of the real investment project?”

Thus, the purpose of discounting in the evaluation of real investment projects, in our opinion, is not to bring the time value of cash flows at a certain point in time, and finding the value of alternative capital, which will bring the same capital gains, but at a risk-free rate of return.

The main idea and (or) constructive principle of the logical and mathematical concept is the premise that it is possible to discount only the increased amount, but not capital gains.

When calculating the efficiency of a financial project, for example, a bank deposit, the use of a complex interest scheme is quite logical and reasonable, because interest is charged for the use of the bank's funds, compensating for deferred consumption and the expectation of a refund, and the use of capitalization is a mechanism that stimulates the interests of customers. Discounting in this case is intended to find out the value of the initial capital, which when investing will increase to the desired value of the accrued amount.

It is mathematically incorrect to compensate for the impact of inflation, risk and alternative cost of capital on the cash flows of a real investment project by discounting for the following reasons.

First, in the case of an assessment of a real investment project, the net cash flow is discounted, which is the difference between total inflows and outflows. In financial mathematics, the increased amount is discounted, that is, the amount of the initial capital that is positioned as an outflow, and the capital increment that is the net cash inflow. Accordingly, to discount the amount of outflows and inflows logically and mathematically true. To discount the difference between inflows and outflows from a mathematical and logical point of view is not correct, since the increased amount is discounted, not its increment.

Secondly, to take into account inflation and risk, the capitalization mechanism with a complex interest scheme

using the power function is impractical, because inflation and risks can both increase, but not exponentially, and decrease over time.

Third, when finding the value of lost profits on alternative investment in a risk-free replicable project, which, for example, are investments in government bonds or a Deposit in a reliable Bank, a financial and mathematical operation, reverse discounting, which is called compounding, is used.

#### *A. Step-by-Step Scheme of Formation of Cash Flows*

The principal point in assessing the effectiveness of investment projects is the organization of a clear algorithm of cash flows from the time of investment to obtain the desired effect. Consider the organization of this algorithm in figure 1.

Cash inflow from financing activities is formed from own and borrowed funds. This inflow is a source of capital for investment and operating activities.

Cash outflow from investment activities is an investment in fixed assets and an increase in working capital.

Cash outflows from operating activities are material costs, wages and insurance premiums. Then, in the course of the project, other taxes will be added to them.

Insurance premiums must be paid by the organization before revenue is received. Therefore, as well as wages to employees, these costs should be provided in advance.

All other taxes will be paid upon receipt of the revenue or profit, because the source of their payment is the financial result.

Thus, the outflow of funds from operating activities will initially include material costs, wages and insurance premiums.

The cash flows from investment and operating activities represents the total outflow of cash or capital that is necessary for the implementation of the project.

As part of the cash outflow from operating activities will be formed cost, which in addition to material costs, wages and insurance premiums will include depreciation, transport and land taxes. Further, the amount is added to the profit and the cost of production is obtained without taking into account indirect taxes.

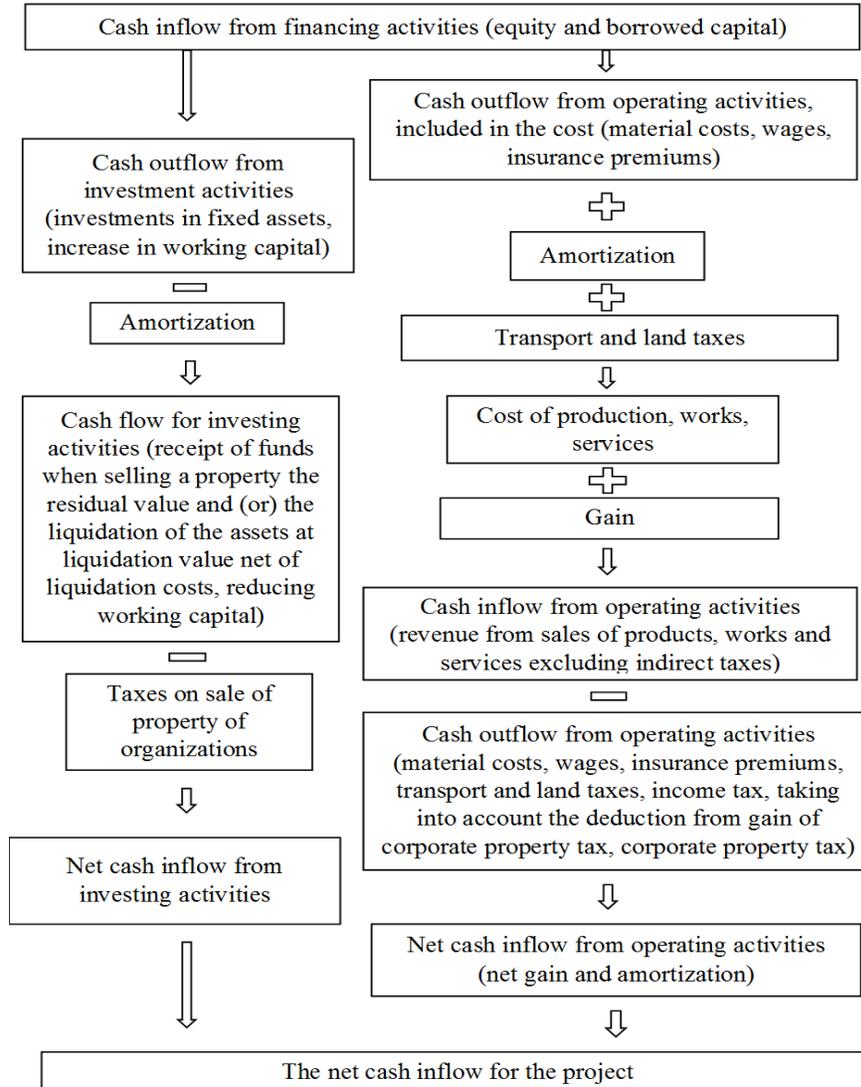


Figure 1. Step-by-step scheme of formation of cash flows

**B. Example of Investment Project Evaluation**

Let's consider an example of investment project evaluation. The initial investment in the project, forming investment and operating activities amounted to 731216 thousand rubles over the project life cycle – 9 years. Profitability of production is 20 %. Annual projected net cash inflows are estimated at 99818,744 thousand rubles, and for 9 year – 210531,315 thousand rubles. Net total cash inflow is equal to 1009561,267 thousand rubles. Net discounted income at an alternative cost of capital 7% takes a negative value (-18967,4 thousand rubles.). The internal rate of return is 6%, which is equal to and even in some cases lower than the deposit rate. Therefore, the project is economically impractical, and it is at profitability of production of 20%. If the calculation is carried out according to the classical financial and mathematical scheme, discounting the increased cost of capital, equal to 1740777,263 thousand rubles. At the same discount rate of

7%, the net capital gain, taking into account the opportunity cost will be 215651,492 thousand rubles. The profitability of the project on the mechanism of simple interest will be equal to 15%. According to the mechanism of compound interest (using the equivalent rate function), the profitability of the project will be 10%. Simple mathematical calculations show the effectiveness of this project and prove the fact that it is logically and mathematically correct to discount the increased amount, not the net cash inflow.

**III. CONCLUSION**

Thus, it is necessary to comprehensively and systematically review approaches to the formation of mathematical and analytical tools for assessing the effectiveness of investment projects, taking into account the logical and mathematical concept of the time value of money and discounting. We recommend the classical

scheme of discounting the increased amount of capital of the project, rather than the net cash inflow, which will significantly increase the accuracy of the forecast calculations for the investment project

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