

# A Trajectory Planning Method based on B-spline Algorithm for Automatic Parking Systems

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**Abstract.** This paper proposes a Trajectory planning method for automatic parallel parking system. The planning method consists of two part. First, the feasible area of parking, is used to find the starting position and azimuth of the parking. Second, planning the parking trajectory. the vehicle can be safely and accurately parked in the parking space, with the avoidance of sudden changes of speed and steering angle. The experimental results show that this algorithm finds a suitable starting point and parking trajectory in each parking space environment.

**Keywords:** automotive engineering, automatic parking, B-spline theory, Differential flat theory.

## 1. Introduction

The automatic parking system is an important part of autopilot vehicles. In order to improve the precision of the automatic parking system, [1] & [2] proposed a controller model based on fuzzy logic and [3] was proposed based on the network-fuzzy theory. Literature [4] proposed an easy way to plan the parking trail based on two tangent circular arcs. Literature [5] compute the parking path through fitting sinusoidal curve based on regression algorithm. Multi-constrained optimization method is used in [6] to plan the ideal parking path through fitting arctangent curve. Literatures [7-8] calculate continuous multi-segment parking paths based on vehicle geometric parameters. In order to conform the computed parking trail to vehicle dynamic characteristics [9-10] combined differential flatness theory with B-spline theory.

There are many difficulties among automatic parking path planning. Firstly, all parts of vehicles should avoid collisions with obstacles in the parking process. Secondly, the path should match the kinematics and dynamics of vehicles with each other. Finally, the algorithm must be as simple as possible and time-saving. In this paper, the B-spline path planning method based on differential flatness theory is chose to transform the estimation planning problem into the parameter optimization problem in order to obtain the safe and feasible trail in a short time. In addition, a new calculation method of parking starting area is proposed, which increases the flexibility of the automatic parking system.

## 2. Automatic Parking Path Planning

### 2.1 Constraints of Parking Path

#### 2.1.1 Contraints of Avoiding Obstacles

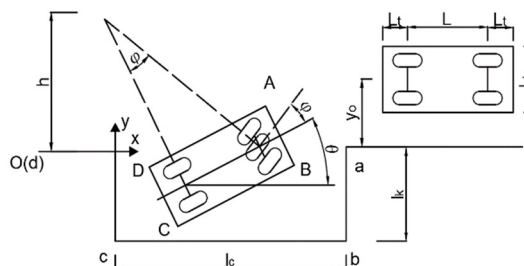


Fig. 1 Diagram of parallel parking

Fig.1 shows the process of parallel parking. The constraints for collision avoidance:

Q (1) The body of the vehicle should not exceed the current lane, which means ;(2) The edge of BC should not have any collision with point a, which means when and ;(3) The vehicle should avoid collision with three edges of the parking lot:

$$\begin{cases} x_D > 0; x_C > 0 \\ y_C > -l_k; y_B > -l_k \\ \text{when } y_B < 0: x_A < l_C; x_B < l_C \end{cases}$$

$$(4) \quad y_D < 0; y_A < 0; \text{ when } x = x_{\text{end}} .$$

### 2.1.2 Constraints of Vehicles

The input of the whole automatic parking system is the speed and steering angle of the vehicle. Consequently, constraints of vehicles have two parts:

$$\begin{cases} |v| < v_{\text{max}} \\ |\dot{v}| = |a| < a_{\text{max}} \end{cases} \quad (1)$$

$$\begin{cases} |\varphi| < \varphi_{\text{max}} \\ |\dot{\varphi}| = |w_f| < w_{\text{max}} \end{cases} \quad (2)$$

Another constraint makes the speed of the vehicle equal to 0 in the start and end point of the parking process. Last but not least the speed and steering angle of the vehicle should avoid transience in the whole parking process.

## 2.2 Differential Flatness

### 2.2.1 Definition of Differential Flatness

The definition of differential flatness is as follows: For a nonlinear system:

$$\dot{x} = f(x, u) \quad x \in R^n, \quad u \in R^m \quad (3)$$

If you can find the output in the form

$$z = F(x, u, \dot{u}, \dots, u^{(l)}) \quad z \in R^m \quad (4)$$

the input quantity  $u$  and the state quantity  $x$  of the system can be expressed by the finite order derivative of the output quantities  $z$  and  $z$  as follows

$$\begin{cases} x = x(z, \dot{z}, \dots, z^{(j)}) \\ u = u(z, \dot{z}, \dots, z^{(j)}) \end{cases} \quad (5)$$

the system (3) is a differential flat system. In the system,  $z$  is called flatness outputs. The necessary conditions are given in essay[11], and a complex algorithm for solving flat outputs is given in essay[12]. in practical applications, a set of outputs is often selected according to physical meaning, and then it is verified whether the set of outputs conforms to formula (5).

Derived from the above, the motion trajectory of the system can be determined by  $z$ .

### 2.2.2 Flat Output of the Vehicle Kinematics Model

Automobiles are nonlinear systems that are subject to non-holonomic constraints. The vehicle movement must meet the following constraints:

$$\begin{cases} \dot{x}_f \sin(\theta + \varphi) - \dot{y}_f \cos(\theta + \varphi) = 0 \\ \dot{x} \sin \theta - \dot{y} \cos \theta = 0 \end{cases} \quad (6)$$

formula (6) indicates that the front and rear wheels of the vehicle have no lateral sliding,  $(x_f, y_f)$  indicates the coordinates of the front axis center.  $(x, y)$  indicates the coordinates of the rear axis center,  $\theta$  indicates the pose angle of the car body,  $\varphi$  indicates front wheel steering angle.

According to the actual driving operation,  $v$  and  $w_f$  are usually used as inputs to the vehicle model. There are the following kinematic model expressions:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ \tan \varphi / l & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w_f \end{bmatrix} \quad (7)$$

And  $l$  indicates the wheelbase. This paper directly selects the flat output as  $\mathbf{z} = [x, y]^T$ . Then we will verify if  $Z$  satisfies formula (3):

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} \quad (8)$$

$$\dot{\theta} = \frac{\dot{y} \cdot \ddot{x} - \dot{x} \cdot \ddot{y}}{v^2} = \frac{v}{l} \tan \varphi \quad (9)$$

$$\varphi = \arctan \left[ \frac{(\dot{y} \cdot \ddot{x} - \dot{x} \cdot \ddot{y}) \cdot l}{v^3} \right] \quad (10)$$

$$w_f = \dot{\varphi} \quad (11)$$

From formula (8) to (11), we can get  $\mathbf{z} = [x, y]^T$  is the flat output of the car kinematics model (formula (7)).

### 2.3 B-spline Curve

The B-spline curve is a flexible curve which retains the advantages of the Bezier curve, and overcomes shortcomings of the the Bezier curve. The B-spline expression is as follows:

$$Q_{i,n}(u) = \sum_{k=0}^n [P_{i+k} F_{k,n}(u)] \quad (12)$$

$$F_{k,n}(u) = \frac{1}{n!} \sum_{j=0}^{n-k} [(-1)^j C_{n+1}^j (u+n-k-j)^n] \quad (13)$$

In the formula,  $0 \leq u \leq 1$ ;  $k=0,1,2,\dots,n$ ;  $i=1,2,\dots,m-n$ ;  $m$  is the number of control points;  $n$  is the spline curve order;  $P_{i+k}$  is the coordinate of the  $(i+k)$ -th control point;  $F_{k,n}$  is the basic function of  $N$ -th B-spline curve;  $Q_i$  is the coordinate of any point on the curve of the  $i$ -th segment. For the calculation of path planning, the B-spline basis function is shown in formula (14):

$$F_{i,3}(u) = \frac{1}{6} \begin{bmatrix} u^3 \\ u^2 \\ u \\ 1 \end{bmatrix}^T \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} P_i \\ P_{i+1} \\ P_{i+2} \\ P_{i+3} \end{bmatrix}, i=1,2,\dots,m-3 \quad (14)$$

The expression of cubic B-spline and the 1-3th order derivative function as follows:

$$(x, y) = \frac{1}{6}(u^3, u^2, u, 1)GP \tag{15}$$

$$\left(\frac{dx}{du}, \frac{dy}{du}\right) = \frac{1}{6}(3u^2, 2u, 1, 0)GP \tag{16}$$

$$\left(\frac{d^2x}{du^2}, \frac{d^2y}{du^2}\right) = \frac{1}{6}(6u, 2, 0, 0)GP \tag{17}$$

$$\left(\frac{d^3x}{du^3}, \frac{d^3y}{du^3}\right) = \frac{1}{24}(6, 0, 0, 0)GP \tag{18}$$

In the formula,  $P_i \sim P_{i+3}$  are four control points which generate the i-th segment of B-spline curve.

### 2.4 Initial Distribution of B-spline Control Points

There are about ten control points, or twenty parameters, in a parking process. If we divide the control points evenly into two circular arc trajectories and solve its shortcomings by small adjustments, a good parking trajectory can be get in a short time, as shown in Fig. 2.

In Fig.2, the three sections of the road in different colors represent three stages of parking. L1 indicates that the vehicle accelerates to a constant speed and the front wheel angle is 0. The steering wheel starts to rotate at the junction of L1 and L2. The steering wheel angle of junction of L2 and L3 is the maximum positive value. Through L3 the front wheel angle of the vehicle is unchanged and speed decelerates to 0.

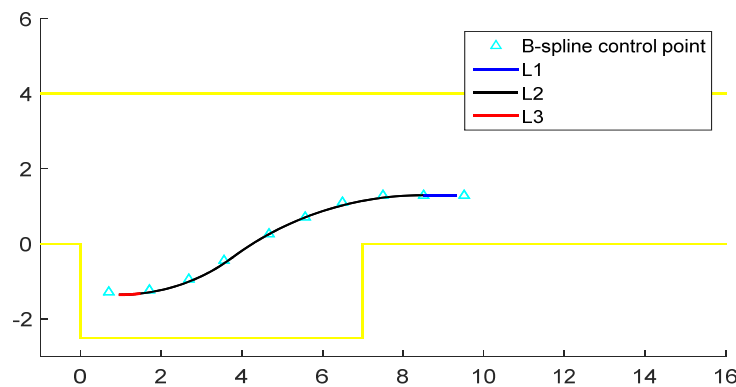


Fig. 2 Schematic diagram of initial distribution of B-spline control points

### 2.5 The Study on Parking Starting Area

The automatic parking environment is complex and variable. Therefore, this paper innovatively proposes a calculation method of the parking starting point area, which is three-dimensional, including horizontal and vertical coordinates and azimuth range. We try to plan two tangential arcs from a starting point and azimuth to connect the end of the parking by double-arc planning method, as shown in Figure 3. In fig.3,  $R_1, \theta_1$  indicates the radius and central angle of the arc near the end of the parking. Set  $R_1$  equal to the minimum turning radius of the vehicle.  $R_2, \theta_2$  indicates the radius and center angle of the arc near the starting point of the parking. We use some constraints to determine whether a point can be used as a starting point for parking:  $d_1$  represents the closest distance between trajectory and the lane line, and  $d_2$  represents the closest distance between trajectory and the parking space angle,  $d_3$  indicates the closest distance from the left front corner of the vehicle to the lane line,  $d_4$  indicates the closest distance from the right front corner of the vehicle to the lane line. In order to make the vehicle safe to avoid collision, the following requirements are required:

$$\begin{cases} d_1 > \frac{L_K}{2}; d_2 > \frac{L_K}{2}; d_3 > 0; d_4 > 0 \\ \theta_1 < \frac{p_i}{2}; \theta_1 + \theta_2 < p_i \end{cases} \tag{19}$$

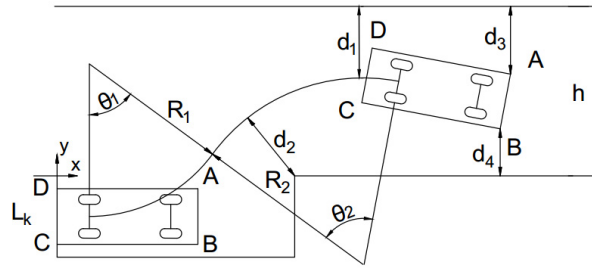


Fig. 3 Schematic diagram of the calculation method of the starting point area

### 3. Simulation Examples and Analyses

#### 3.1 Simulation Examples

There are two types of parameters required to calculate the automatic parking starting area: vehicle parameters and environmental parameters. Vehicle parameters can be stored before use. Then this algorithm can calculate the starting area of the parking when the automatic parking system senses the length  $l_c$ , width  $l_k$  of the parking space and the width of the road  $l_w$ . The vehicle parameters are shown in Table 1, and the environmental parameters are shown in Table 2.

Table 1. The vehicle parameters

Parameter	value	Parameter	value
$L_f / m$	0.8	$V_{max} / (m \cdot s^{-1})$	2
$L / m$	2.4	$a_{max} / (m \cdot s^{-2})$	4
$L_r / m$	0.95	$\varphi_{max} / rad$	0.5
$L_k / m$	1.64	$\omega_{max} / (rad \cdot s^{-1})$	0.5

Table 2. The environmental parameters

Parameter	scene 1	scene 2	scene 3
$l_c / m$	8	7	7
$l_k / m$	2.5	2.5	2.5
$l_w / m$	5	4	3.5

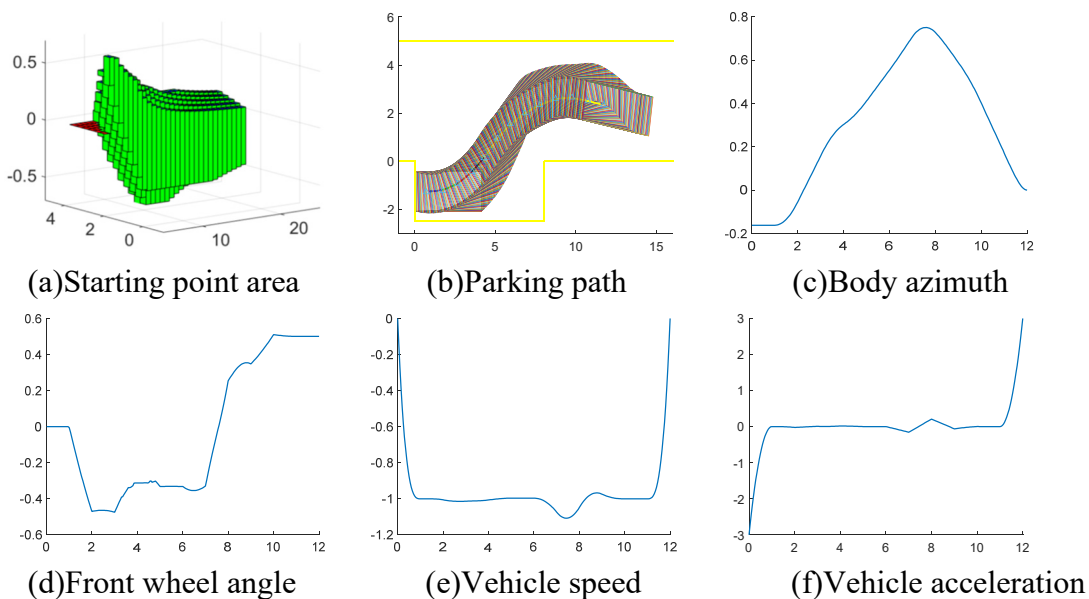


Fig. 4 Parking planning results of scene 1

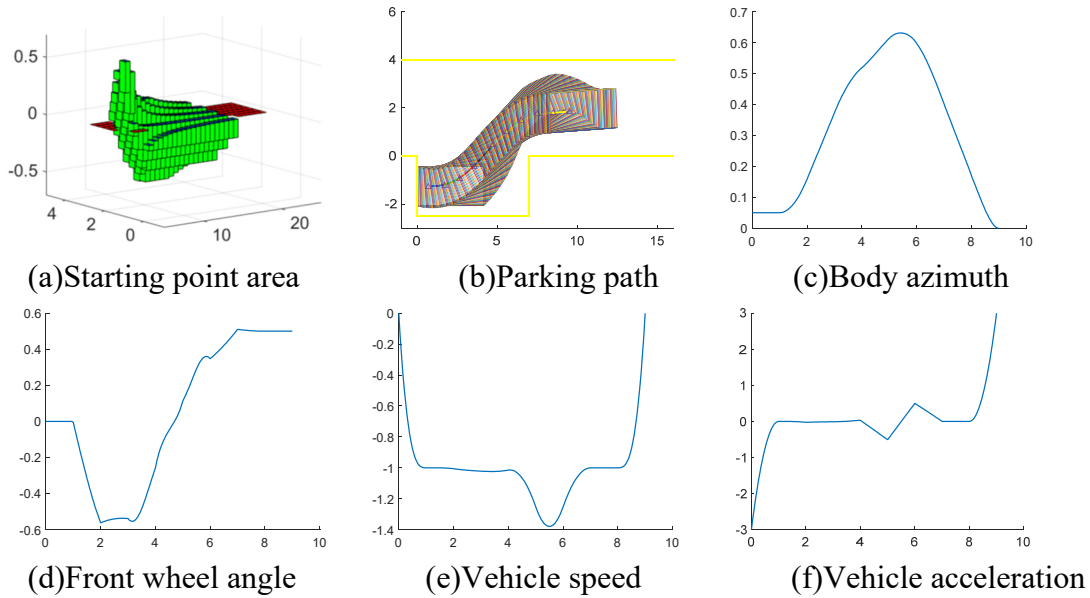


Fig. 5 Parking planning results of scene 2

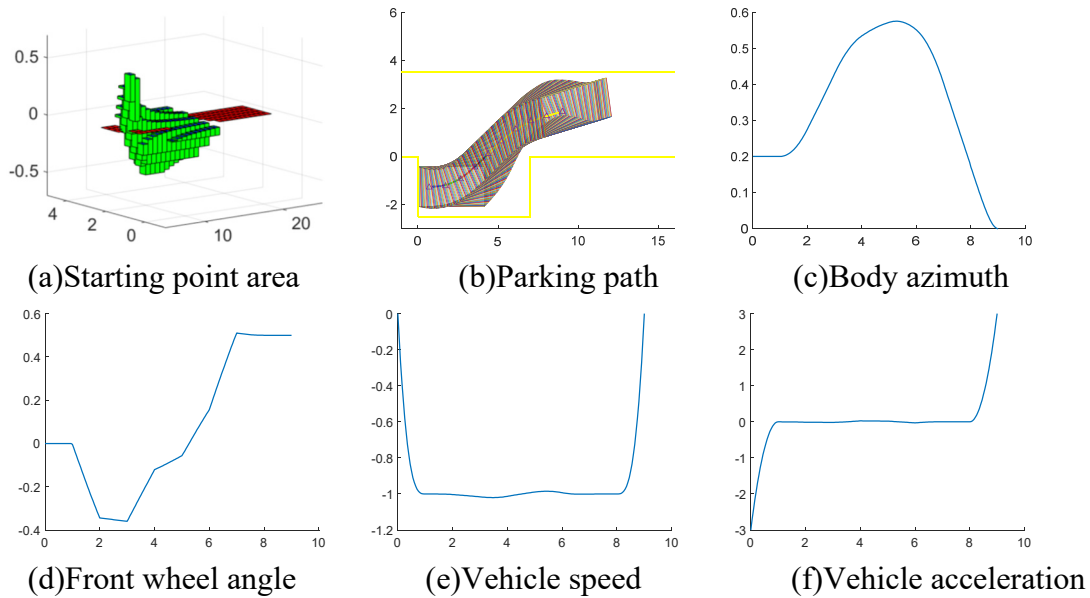


Fig. 6 Parking planning results of scene 3

### 3.2 Simulation Analysis

Fig. 4-6 shows the planning of parking trajectories in scenarios 1, 2, and 3. It can be seen from subfigures: 4(a)-6(a) that the size and azimuth range of the parking starting area are reduced in a narrow environment. From subfigures 4(b)-6(b), it can be seen that parking starts at different starting points of different vehicles in different environments, and finally ends parking in the same posture. It can be seen from other figures that the azimuth, front wheel angle, vehicle speed and acceleration of the vehicle are continuous during the parking process.

## 4. Summary

A complete approach of a Trajectory planning method was proposed. First, we need to determine which points with azimuth in this parking environment can be used as a starting point for parking. Secondly, we choose the starting point of parking in this feasible area. Finally, we use B-spline and differential flat theory to plan a safe trajectory that is consistent with automotive dynamics and kinematics. It turns out that this algorithm allows the vehicle to accurately reach the parking position from different starting points.

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