PFAHP-based Index Weighting in Operation Quality Evaluation of GSM-R

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Abstract. GSM-R is a railway mobile communication network adapted to railway informatization and security. Determining the weight value of each index is the key step for comprehensive evaluation of GSM-R network operation quality. The traditional fuzzy analytic hierarchy process (FAHP) is not suitable for the situation which has huge number of experts and the experts are unfamiliar with each other. So, we use the peer-to-peer consensus reaching model to improve it and put forward the peer-to-peer fuzzy analytic hierarchy process (PFAHP). This method uses the peer-to-peer consensus reaching model to modify the expert matrix to increase the consistency of expert judgment. The modified expert evaluation matrix is transformed into a triangular fuzzy judgment matrix, and the weighted triangular fuzzy judgment matrix of each expert is used to obtain the average triangular fuzzy judgment matrix. Finally, the weight distribution value of the GSM-R network operation quality evaluation index is obtained. It can be seen from the results that the handover success rate, voice quality and reception level value have a larger proportion in evaluating the GSM-R network operation quality, which is consistent with the actual situation. The final index weight value calculated by FAHP has a referential significance in the GSM-R network operation quality evaluation.

Keywords: PFAHP; Index weight determination; GSM-R network; FAHP; Peer-to-Peer Consensus Reaching Model.

1. Introduction

Railway informationization and security is an inevitable trend of railway development. The GSM-R network is the railway mobile communication network that adapts to this tendency. The comprehensive evaluation of the operation quality of the GSM-R network makes the train operation safer and more efficient. The assessment should take into account the impact of various factors. Assigning appropriate weights according to the contribution of each factor is an important part to ensure the evaluation method of effectiveness and credibility.

As mentioned in [1], Lin Siyu proposed a scheme for assessment of quality of service (QoS) for train control data transmission based on grey Clustering (GC) theory, which assesses QoS of the GSM-R network through a simple way. The entropy-based weighting method proposed by Hou Weisheng et al. [2] can improve the rationality of weight distribution. This method is suitable for the existence of more objective data. Literature [3] proposed an AHP- based index weight calculation method, which divided decision-making objectives, considerations, and decision-making objects into the highest layer, the middle layer, and the bottom layer according to their mutual relations. Different elements in the same layer can be compared in pairs, but the method is greatly influenced by subjective factors; Negin Salimi et al. [4] proposed an index weight analysis method based on FAHP, which considers the ambiguity of people's judgment on complex things and can better overcome the influence of subjective factors on decision-making, but it does not consider the work of various experts. Different experience will lead to the deviation between the analysis results and the actual situation. Combining FAHP with group decision-making, comprehensively considering the opinions of each expert, the obtained weight index [5, 6] is not only effectively solves the ambiguity problem in the expert review process, but also reduces the subjective randomness of experts in the scoring process. However, this method is only applicable to the case with a small number of experts.

This paper improves the FAHP according to the characteristics of GSM-R network. 0n the basis of triangular fuzzy number, we use the peer-to-peer consensus model to determine the weight of each expert, and the expert weight to weight the expert triangle fuzzy judgment matrix and then take the mathematical expectation. This method not only takes into account the fuzziness of expert judgment, and overcomes the effects of the subjectivity and limitations of expert judgment, but also is more robust when there are a large number of experts.

2. Related Works

2.1 Fuzzy Analytic Hierarchy Process

Del phi Technique [7], also known as expert investigation, is a set of team communication processes proposed by Norman Dalkey et al. to solve a specific problem and its procedures system is to use anonymous comments. Experts cannot discuss with each other and horizontal linkages must not happen. Experts can only communicate with the investigators. By collecting opinions on issues raised by experts with several rounds of questionnaire survey experts and repeated consultation, induction and revision, and finally summed up the consensus of the experts to form the result of the forecast. This method is widely representative and reliable.

Analytic Hierarchy Process [8], an analytical method proposed by T. L. Saaty, is used to solve the problem of complex hierarchical weight decision making. AHP method firstly divides the decision-making objectives, factors (decision criteria) and objects into the highest level, the middle level and the lowest level according to their mutual relations, followed by pairwise comparison of the different elements in the same layer, which determines the influence of each element on the upper elements. According to the influence degree, all judgment matrices in each level are constructed, and weight indexes are calculated, and then hierarchical ordering and consistency test are conducted. Finally, the priority of different decisions relative to the overall goal can be obtained. However, it depends on people's experience to a great extent, which cannot exclude the one-sidedness of decision-makers and is greatly affected by subjective factors. And the comparison and judgment process of AHP is relatively rough, which cannot be applied to the decision-making problem with high precision.

In order to improve AHP, some scholars combined the fuzzy mathematics thought with it and put forward FAHP [7,9]. FAHP quantifies some factors that have unclear boundaries and are difficult to be quantified according to the membership degree theory of fuzzy mathematics. That is, using fuzzy mathematics to make a general analysis of the things or objects restricted by many factors. In the general AHP, the construction of the pairwise comparison judgment matrix usually does not consider the ambiguity of human judgment, and only considers two possible extreme situations that people judge: selecting an indicator with membership degree 1 and negating other scale value with membership degree 0. Since the opinions given by the experts are often not the determined membership degrees 0 and 1, but some fuzzy quantities, the use of fuzzy mathematical theory such as triangulation to analyze the expert judgment is helpful to rationalize the problem.

2.2 Peer-to-peer Consensus Reaching Model

We combine the peer-to-peer consensus reaching model with FAHP to propose the PFAHP, which can comprehensively take the different opinions into account when there are big differences in diverse experts. Firstly, based on the gap of each expert, the expert judgment matrix is rectified by the peer-to-peer consensus reaching model to ensure that the consistency of expert judgment can meet the requirement. According to the consistency of expert judgment, the weight of each expert in the evaluation is calculated. Secondly, the expert evaluation matrix is transformed into a triangular fuzzy judgment matrix, and the calculated expert weights are used to weight the mathematical expectation of the triangular fuzzy matrix, and then the initial index weight is obtained and used to gain the final index weight by defuzzification. PFAHP also has better robustness when the number of experts increases.

3. PFAHP

Considering the characteristics of GSM-R network operation quality evaluation, the diversity and even opposition of expert opinions due to the working years of experts and their understanding of the field. The PFAHP can further improve the reliability and effectiveness of gsm-r network operation quality evaluation index weight. The specific steps and process are as follows:

Step 1 Determine the evaluation matrix

Every expert compares all the indicators of each level in pairs and uses AHP to determine the weight evaluation matrix of each factor. For AHP, there are a variety of scale methods in evaluating the relative importance of each index, among which the reciprocal scale includes 1~5 scale, x^2 scale and exponential scale, etc. The scale of complementarity is 0.1~0.9 and 0~2, etc. At present, the most widely used scale is 1~9. In this paper, considering that the scale is too fine for experts to distinguish, the scale is divided into very vital importance, moderate importance, equal importance, unimportance and strong unimportance in order to better apply to practice. The scale of relative importance is shown in table 1.

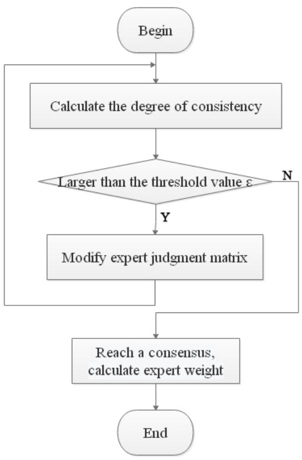
Table 1. AHP pairwise comparison scale used for questionnaires

Quantitative Representation	Description of relative importance used in pairwise comparison
5	Very vital importance
3	Moderate importance
1	Equal importance
1/3	Unimportance
1/5	Strong unimportance

Note: Take 4, 2, 1/2, 1/4 as the median value.

Step 2 Determine the expert weight

The peer-to-peer consensus model is used to determine the expert weight. This method improves the accuracy of the results by increasing the consistency of expert judgment. The process is shown in Fig. 1:



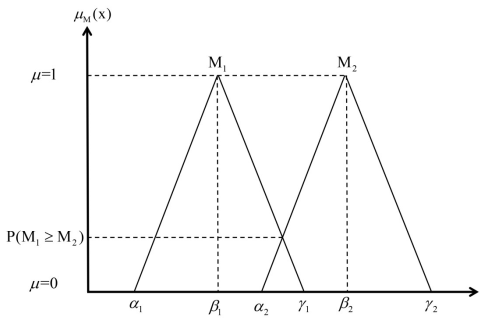


Fig. 1 The peer-to-peer consensus model   Fig. 2 Representation of the degree of possibility

Calculate the consistency of the judgment of any two experts *a* and *b*:

$$ICI_{ab} = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij(a)} q_{ji(b)} \tag{1}$$

Where ICI_{ab} indicates the degree of consistency of judgment between expert a and expert b, and the greater the value, the smaller the consistency. And n represents the number of judgment indexes; q is the judgment scale.

Then, according to the formula (1), the judgment consistency degree of all experts in the expert team is calculated. The expert consistency control parameter ε is introduced. For any two experts a and b, if $ICI_{ab} > \varepsilon$, the judgment matrix A_a and A_b need to be corrected:

$$q_{ij(a)}^{l+1} = (q_{ij(a)}^{l})^{\tau_a^l}(q_{ij(b)}^{l})^{1-\tau_a^l}, \quad q_{ij(b)}^{l+1} = (q_{ij(b)}^{l})^{\tau_b^l}(q_{ij(a)}^{l})^{1-\tau_b^l} \tag{2}$$

Where, l is the number of iterations; τ_a^l and τ_b^l are parameters calculated as follows:

$$\tau_a^l = 1 - \frac{\sum_{i=1,i\neq a,b}^{m} ICI_{ai}}{2(\sum_{i=1,i\neq a,b}^{m} ICI_{ai} + \sum_{i=1,i\neq a,b}^{m} ICI_{bi})}, \quad \tau_b^l = 1 - \frac{\sum_{i=1,i\neq a,b}^{m} ICI_{bi}}{2(\sum_{i=1,i\neq a,b}^{m} ICI_{ai} + \sum_{i=1,i\neq a,b}^{m} ICI_{bi})} \tag{3}$$

When any $ICI_{ab} \leq \varepsilon$, that is, the consistency of expert matrix meets the requirements, experts are considered to have reached a consensus. Saaty thinks that 1.1 is the upper bound of ε [7], and ε generally takes as 1.01[12]. The weight of each expert shall be calculated according to the following formula:

$$\omega_i = \frac{1/ICI_{ij}}{\sum_{j=1}^{n}(1/ICI_{ij})} \tag{4}$$

Step 3 Determine fuzzy matrix

Assuming that the fuzzy number on R is M, and μ_M, the membership function of M, makes R→ [0, 1] are determined by following the rules expressed by (7):

$$\mu_M(x) = \begin{cases} \frac{1}{\beta - x}x - \frac{\alpha}{\beta - \alpha} & x \in [\alpha, \beta] \\ \frac{1}{\beta - \gamma}x - \frac{\gamma}{\beta - \gamma} & x \in [\beta, \gamma] \\ 0 & x \in (-\infty, \alpha) \cup [\gamma, +\infty) \end{cases} \tag{5}$$

Where α and γ represent the upper and lower bounds respectively, the membership of β is the median of 1, and when x=β, x completely belongs to M. It does not belong to the fuzzy set M except for α and β.

According to the above definition of fuzzy number, the fuzzy judgment matrix is constructed on the basis of expert comparison matrices. The corresponding relationship between expert comparison value and triangular fuzzy number is shown in Table 2.

Table 2. Linguistic terms and corresponding fuzzy values for weight evaluation

Linguistic values	Rating level	Fuzzy values	reciprocal
Same index	1	(1,1,1)	(1,1,1)
Equal importance	1	(1/2,1,3/2)	(2/3,1,2)
Moderate importance	3	(1,3/2,2)	(1/2,2/3,1)
Very vital importance	5	(3/2,2,5/2)	(2/5,1/2,2/3)

The average triangular fuzzy number is obtained by weighting the triangular fuzzy number with the expert weight, which is used as the weight value of each index.

The number of experts is n. After being evaluated by experts, the average triangular fuzzy number is:

$$\overline{M} = (\sum_{i=1}^{n}\alpha_i\omega_i, \sum_{i=1}^{n}\beta_i\omega_i, \sum_{i=1}^{n}\gamma_i\omega_i) \tag{6}$$

Step 4 Calculate the initial weight of each index and defuzzify

The evaluation system of GSM-R network operation quality is divided into two layers. According to formula (9), the initial weights of corresponding indicators in each layer are calculated.

$$D_i^k = \sum_{j=1}^{n} \overline{M}_{ij}^k / (\sum_{i=1}^{n}\sum_{j=1}^{n} \overline{M}_{ij}^k),\ i = 1,2,...,n \tag{7}$$

Where, D_i^k is the initial weight of index I of layer K.

Algebraic operations can be accomplished among fuzzy numbers. For instance, considering two numbers M_1 and M_2, the following operation (10) may be defined:

$$M_1 \oplus M_2 = (\alpha_1 + \alpha_2, \beta_1 + \beta_2, \gamma_1 + \gamma_2) \tag{8}$$

However, the initial weights calculated are still triangular fuzzy numbers. By deblurring them, we can obtain the specific initial weights.

According to literature [3], if M_1 and M_2 are triangular fuzzy numbers, the possibility degree of $M_1 > M_2$ is defined by triangular fuzzy function as:

$$P(\overline{M}_1 \geq \overline{M}_2) = \begin{cases} 1 & \beta_1 \geq \beta_2 \\ \dfrac{\alpha_2 - \gamma_1}{(\beta_1 - \gamma_1) - (\beta_2 - \alpha_2)} & \beta_1 \leq \beta_2, \gamma_1 \geq \alpha_2 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Furthermore, the possibility degree that a fuzzy number is greater than k fuzzy numbers corresponds to:

$$P(\overline{M} \geq \overline{M}_1, \overline{M}_2, \cdots\cdots \overline{M}_k) = \min P(\overline{M} \geq \overline{M}_i),\ \ i = 1,2,\cdots,\ \ k \tag{10}$$

According to formula (11-12), the initial weights are defuzzified.

Step 5 Standardize

Let us observe that these obtained weights have to be normalized with respect to their total so that their sum equals one. The final normalized index weights, called d_j, will be obtained by:

$$d_j^{'} = \frac{d_j}{\sum_{j=1}^{n} d_j} \tag{11}$$

Where j is the number of subordinate indicators that corresponding to their superior indicator.

Step 6 Determine the final weight

The weights of each layer are calculated according to steps 1-5, and then the final standardized weights are obtained by multiplying the weights of each layer with the weights of the upper indexes of the path.

4. Application

4.1 Hierarchical Structure

According to the characteristics of the GSM-R railway mobile communication network, the secondary indicator layer is set. Figure 3 shows the hierarchical structure of the GSM-R network operation quality evaluation system. Based on the expert's scoring matrix, the improved FAHP method is used to calculate the weight of each index.

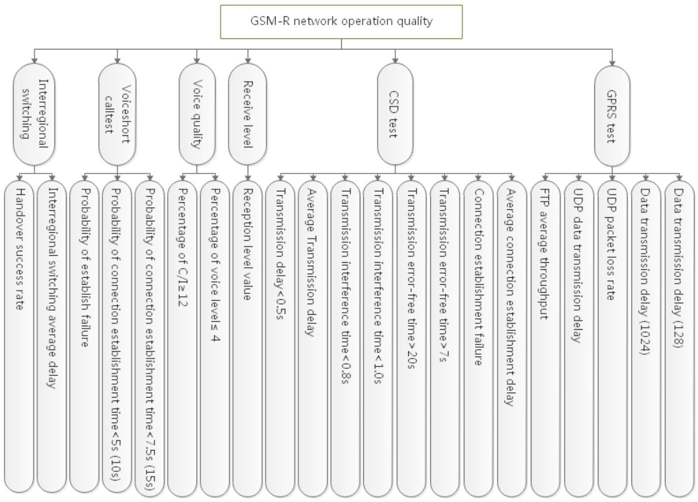


Fig. 3 Hierarchical definition of GSM-R network operation quality evaluation system

4.2 Determine Expert Weights

According to the GSM-R network operation quality evaluation architecture of Fig. 3, fifteen experts with authority in this field are invited to compare all the indicators of each level and give the evaluation results. Since each expert has different working years in the field of railway communication or signal, and the degree of understanding of each type of data is different, there are data with large differences in values. According to formula (1), the degree of consistency of any two experts is calculated, and the judgment scale with the degree of consistency greater than 1.01 is corrected. Use the first-level indicators of network operation quality, and take expert a and expert b as examples to calculate the final judgment matrix.

The initial judgment matrices of expert a and expert b are:

$$A_a^1=\begin{bmatrix}1.000 & 0.333 & 0.333 & 0.333 & 1.000 & 0.333\\ 3.000 & 1.000 & 3.000 & 3.000 & 5.000 & 1.000\\ 3.000 & 0.333 & 1.000 & 0.333 & 3.000 & 0.333\\ 3.000 & 0.333 & 3.000 & 1.000 & 5.000 & 0.333\\ 1.000 & 0.200 & 0.333 & 0.200 & 1.000 & 0.333\\ 3.000 & 1.000 & 3.000 & 3.000 & 3.000 & 1.000\end{bmatrix}\quad A_b^1=\begin{bmatrix}1.000 & 0.333 & 5.000 & 5.000 & 1.000 & 0.333\\ 3.000 & 1.000 & 5.000 & 5.000 & 1.000 & 1.000\\ 0.200 & 0.200 & 1.000 & 5.000 & 0.333 & 0.200\\ 0.200 & 0.200 & 0.200 & 1.000 & 0.333 & 0.200\\ 1.000 & 1.000 & 3.000 & 3.000 & 1.000 & 0.333\\ 3.000 & 1.000 & 5.000 & 5.000 & 3.000 & 1.000\end{bmatrix}$$

Because the expert consistency index ICI_{ab}=2.7679> ε =1.01, the judgment matrices of the two experts are corrected according to equations (6), (7), (8) and (9), and the iterative operation is performed until the condition $ICI_{ab}\leq \varepsilon$ is true. The final calculated correction factors are:

$$q_a^1=0.8111,\quad q_b^1=0.6889$$

The corrected judgment matrices are:

$$A_a^2=\begin{bmatrix}1.000 & 0.486 & 1.741 & 1.438 & 1.773 & 0.456\\ 2.058 & 1.000 & 2.931 & 2.026 & 2.334 & 0.891\\ 0.574 & 0.341 & 1.000 & 1.695 & 1.010 & 0.379\\ 0.695 & 0.494 & 0.590 & 1.000 & 1.300 & 0.435\\ 0.564 & 0.428 & 0.990 & 0.769 & 1.000 & 0.408\\ 2.192 & 1.123 & 2.640 & 2.297 & 2.450 & 1.000\end{bmatrix}\quad A_b^2=\begin{bmatrix}1.000 & 0.481 & 1.787 & 1.483 & 1.748 & 0.453\\ 2.077 & 1.000 & 2.970 & 2.072 & 2.285 & 0.893\\ 0.560 & 0.337 & 1.000 & 1.741 & 0.983 & 0.373\\ 0.674 & 0.483 & 0.574 & 1.000 & 1.257 & 0.427\\ 0.572 & 0.438 & 1.018 & 0.796 & 1.000 & 0.406\\ 2.209 & 1.119 & 2.683 & 2.342 & 2.462 & 1.000\end{bmatrix}$$

The revised expert consistency indicator ICI_{ab}=1.0002, which satisfies $ICI\leq\varepsilon$. Finally, the expert weight is calculated by means of equation (10).

When the degree of consistency of any two experts meets the requirements, the weight values of fifteen experts are calculated as shown in Table 3.

Table 3. Expert weights of the first-level indicators

Expert	1	2	3	4	5	6	7	8
Weight	**0.06619**	**0.06669**	**0.06670**	**0.06669**	**0.06669**	**0.06670**	**0.06671**	**0.06670**
Expert	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
Weight	**0.06670**	**0.06671**	**0.06671**	**0.06670**	**0.06670**	**0.06670**	**0.06671**	

4.3 Construct Judgment Matrix

The correspondence between the comparison value and the triangular fuzzy number given by the expert is shown in Table 2. The triangular fuzzy number matrix of each level of indicators can be obtained on the basis of it. Take the first-level indicators of network operation quality as an example, as shown in Table 4.

Table 4. The triangulation fuzzy judgment matrix of the first-level indicators

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	(1,1,1) (1,1,1) ⋮ (1,1,1)	(1.5,2,2.5) (0.5000,1,1.5000) ⋮ (0.5000,1,1.5000)	(1.500,2,2.5000) (0.5000,1,1.5000) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (1.5000,2,2.5000) ⋮ (0.5000,1,1.500)	(0.5000,1,1.5000) (1,1.5000,2) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (0.5000,0.6667,1) ⋮ (1.5000,2,2.5000)
C_2	(0.5000,0.6667,1) (0.5000,1,1.5000) ⋮ (0.5000,1,1.5000)	(1,1,1) (1,1,1) ⋮ (1,1,1)	(0.5000,0.6667,1) (0.5000,1,1.5000) ⋮ (0.5000,0.6667,1)	(0.5000,0.6667,1) (1.5000,2,2.5000) ⋮ (1.5000,2,2.5000)	(0.4000,0.5000,0.6667) (1.5000,2,2.5000) ⋮ (0.5000,0.6667,1)	(0.5000,1,1.5000) (0.5000,1,1.5000) ⋮ (1.5000,2,2.5000)
C_3	(0.5000,0.6667,1) (0.5000,1,1.5000) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (0.5000,1,1.5000) ⋮ (1.5000,2,2.5000)	(1,1,1) (1,1,1) ⋮ (1,1,1)	(1.5000,2,2.5000) (1.5000,2,2.5000) ⋮ (1.5000,2,2.5000)	(0.5000,0.6667,1) (1.5000,2,2.5000) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (0.5000,1,1.5000) ⋮ (1.5000,2,2.5000)
C_4	(0.5000,0.6667,1) (0.5000,0.6667,1) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (0.5000,0.6667,1) ⋮ (0.4000,0.5000,0.6667)	(0.5000,0.6667,1) (0.5000,0.6667,1) ⋮ (0.4000,0.5000,0.6667)	(1,1,1) (1,1,1) ⋮ (1,1,1)	(0.4000,0.5000,0.6667) (1.5000,2,2.5000) ⋮ (0.4000,0.5000,0.6667)	(1.5000,2,2.5000) (0.5000,1,1.5000) ⋮ (1.5000,2,2.5000)
C_5	(0.5000,1,1.5000) (0.4000,0.5000,0.6667) ⋮ (0.5000,1,1.5000)	(1.5000,2,2.5000) (0.5000,0.6667,1) ⋮ (1.5000,2,2.5000)	(1.5000,2,2.5000) (0.5000,0.6667,1) ⋮ (0.5000,1,1.5000)	(1,1.5000,2) (0.5000,0.6667,1) ⋮ (1.5000,2,2.5000)	(1,1,1) (1,1,1) ⋮ (1,1,1)	(1.5000,2,2.5000) (0.5000,0.6667,1) ⋮ (1.5000,2,2.5000)
C_6	(0.5000,0.6667,1) (1.5000,2,2.5000) ⋮ (0.5000,0.6667,1)	(0.5000,1,1.5000) (0.5000,1,1.5000) ⋮ (0.5000,0.6667,1)	(0.5000,0.6667,1) (0.5000,1,1.5000) ⋮ (0.5000,0.6667,1)	(0.5000,0.6667,1) (0.5000,1,1.5000) ⋮ (0.5000,0.6667,1)	(0.5000,0.6667,1) (1.5000,2,2.5000) ⋮ (0.5000,0.6667,1)	(1,1,1) (1,1,1) ⋮ (1,1,1)

4.4 Weight Calculation

The fuzzy consistent judgment matrix is built on the basis of formula (12):

$$\boldsymbol{D}=\begin{bmatrix} (0.9995,0.9995,0.9995) & (1.1996,1.6771,2.1657) & (0.6727,1.0212,1.4098) & (0.7193,1.0768,1.4876) & (0.5264,0.8218,1.1660) & (1.1596,1.6326,2.1102) \\ (0.5598,0.8107,1.1771) & (0.9995,0.9995,0.9995) & (0.5398,0.7775,1.1106) & (0.6398,0.8996,1.2438) & (0.4331,0.5997,0.8440) & (0.8533,1.2996,1.7549) \\ (0.7666,1.1998,1.6662) & (1.0990,1.5766,2.0652) & (0.9995,0.9995,0.9995) & (0.7927,1.1991,1.6432) & (0.5398,0.8440,1.2105) & (1.1596,1.6327,2.1102) \\ (0.8266,1.2331,1.6774) & (1.0262,1.4548,1.9100) & (0.7265,1.1108,1.5440) & (0.9995,0.9995,0.9995) & (0.5932,0.8443,1.1665) & (1.0929,1.5660,2.0435) \\ (0.8327,1.3103,1.7989) & (1.0326,1.5323,2.0320) & (0.8994,1.3769,1.8655) & (0.9659,1.4211,1.8987) & (0.9995,0.9995,0.9995) & (1.1996,1.6994,2.1991) \\ (0.5064,0.7217,1.0437) & (0.5464,0.8772,1.2658) & (0.5064,0.7217,1.0437) & (0.5064,0.7440,1.0770) & (0.4664,0.6550,0.9548) & (0.9995,0.9995,0.9995) \end{bmatrix}$$

According to formula (4), the initial weights of the first-level index layer of network operation quality are calculated:

$$\begin{aligned} \boldsymbol{D}_1 = & [(0.9995,0.9995,0.9995)+(1.1996,1.6771,2.1657)+...+(1.1596,1.6326,2.1102)]/ \\ & [(0.9995,0.9995,0.9995)+(1.1996,1.6771,2.1657)+...+(0.4664,0.6550,0.9548)+ \\ & (0.9995,0.9995,0.9995)]=[0.1002,0.1792,0.3178] \end{aligned}$$

Other initial weights are calculated in the similar way:

$$\boldsymbol{D}_2=[0.0764,0.1335,0.2426],\ \boldsymbol{D}_3=[0.1017,0.1847,0.3299],\ \boldsymbol{D}_4=[0.0999,0.1787,0.3179],$$
$$\boldsymbol{D}_5=[0.1126,0.2068,0.3673],\ \boldsymbol{D}_6=[0.0670,0.1170,0.2173]$$

The possibility degree of $D_1 > D_j$ (j=1, …, 6) can be calculated by using formula (2):

$$P(D_1 \geq D_2) = 1, \quad P(D_1 \geq D_2) = 1, \quad P(D_1 \geq D_3) = 0.9751$$

$$P(D_1 \geq D_4) = 0.1, \quad P(D_1 \geq D_5) = 0.8817, \quad P(D_1 \geq D_6) = 1$$

According to formula (3), the index is defuzzified:

$$d_1 = \min P(\mathrm{D}_1 \geq \mathrm{D}_2, \mathrm{D}_3, \mathrm{D}_4, \mathrm{D}_5, \mathrm{D}_6) = 0.8817$$

From the set of similar measurements, d_2, …, d_6 are calculated:

$$d_2 = 0.6399, \quad d_3 = 0.9080, \quad d_4 = 0.8798, \quad d_5 = 1, \quad d_6 = 0.5384$$

According to formula (13), the weight value is standardized, and the final weight of the first-level index layer of network operation quality can be obtained, which are 0.1819, 0.1320, 0.1873, 0.1815, 0.2063 and 0.1111, respectively.

Similarly, the weights of each index of the Criteria layer can be calculated, which are shown in table 5.

Table 5. Index weights of each layer

GSM-R network operation quality	Interregional switching	Handover success rate	0.0993
		Interregional switching average delay	0.0761
	Voice short call test	Probability of establish failure	0.0552
		Probability of connection establishment time <5s (10s)	0.0432
		Probability of connection establishment time<7.5s (15s)	0.0369
	Voice quality	Percentage of C/I⩾12	0.0859
		Percentage of voice level ≤ 4	0.1001
	Receive level	Reception level value	0.1819
	CSD test	Transmission delay<0.5s	0.0396
		Average Transmission delay	0.0423
		Transmission interference time<0.8s	0.046
		Transmission interference time<1.0s	0.0396
		Transmission error-free time>20s	0.0389
		Transmission error-free time>7s	0.0129
		Connection establishment failure	0.012
		Average connection establishment delay	0.0152
	GPRS test	FTP average throughput	0.015
		UDP data transmission delay	0.0157
		UDP packet loss rate	0.0158
		Data transmission delay (1024)	0.0151
		Data transmission delay (128)	0.0134

5. Conclusion

In this paper, PFAHP is proposed to analyze the index weights. Firstly, the peer-to-peer consensus reaching model is used to assign weights to the experts in the group. This method does not exclude any data but retains all the expert evaluation matrices, which can be well used when the number of experts is large and the experts are unfamiliar with each other. Secondly, the expert judgment matrix is transformed into a triangular fuzzy judgment matrix, and the initial weight is calculated and then the final weight is obtained by defuzzification, which overcomes the fuzziness and subjectivity of expert judgment and makes the result more reasonable. At the end of the paper applied the method to the calculation of the GSM-R network evaluation index weight to illustrate the rationality and feasibility of the proposed scheme. The results show that the ratio of handover success rate is 0.0993, the percentage of voice level less than or equal to 4 is 0.1001 and reception level value is 0.1819. The calculation results show that they have a large proportion in evaluating the operation quality of the GSM-R network, which is consistent with the actual situation. Therefore, the application of PFAHP in the comprehensive evaluation of GSM-R network has a referential significance.

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