# GMSK Coherent Demodulation Technology based on Laurent Decomposition

Fei Teng[1, a], Yurong Liao[2, b] and Yuntao Li[3, c]

[1]School of master student, Space Engineering University, Beijing 100000, China;

[2]School of associate professor, Space Engineering University, Beijing 100000, China;

[3]School of lecture, Space Engineering University, Beijing 100000, China.

[a]tfcyg_10@163.com, [b]1622175028@qq.com, [c] skyskyzenith@sina.com

**Abstract.** Using the Laurent method, the GMSK signal can be expressed as the sum of a series of pulse amplitude modulation (PAM) signals. The GMSK signal suboptimal coherent detection technology based on Laurent decomposition is improved. Under different phase correlation lengths, the BER performance and implementation complexity of GMSK signal optimization, suboptimization and improved coherent detection algorithms are simulated. The results show that compared with the suboptimal detection algorithm, the proposed algorithm improves the demodulation error performance without increasing the complexity of the demodulation module.

**Keywords:** GMSK modulation, Laurent decomposition, coherent demodulation.

## 1. Introduction

Gaussian minimum frequency shift keying (GMSK) is a continuous phase modulation (CPM) method with a modulation index of 0.5. It has the advantages of constant signal envelope and fast spectral roll-off. Therefore, it is used in wireless local area networks and spatial information transmission. Has a wide range of applications. The demodulation of GMSK has two modes: non-coherent demodulation and coherent demodulation. Non-coherent demodulation does not require phase synchronization. The main methods include differential demodulation and limiting frequency demodulation. literature [1] using the Viterbi decoding method, the performance of the non-coherent demodulation is greatly improved, but the demodulation performance still has room for improvement due to the influence of the front end "differential" or "limited amplitude frequency discrimination". Coherent demodulation requires carrier phase synchronization, and its demodulation performance is better than non-coherent demodulation, but the implementation structure is more complicated. literature [2] the Maximum Likelihood Sequence Detection (MLSD) demodulator using Viterbi processor is presented, which greatly reduces the implementation complexity of the coherent demodulation machine, and can easily give real-time symbol-by-symbol decision.

In 1986, Pierre A. Laurent demonstrated that any binary CPM signal can be expressed in the form of a finite number of pulse amplitude modulation (PAM) signals[1]. By using this decomposition method, the complexity of the detection module of the CPM receiving end can be effectively reduced, and the specific process can be divided into optimized coherent detection and suboptimal coherent detection[3]. The optimal detection is to bring all the pulse signals into the MLSD detection signal. The demodulation performance of the method is basically consistent with the performance of coherent demodulation directly for the GMSK signal. Since the energy of each pulse signal constituting the complete CPM signal is not uniformly distributed, most of the energy is concentrated in the first few pulses, so the influence of the subsequent pulse signal can be ignored, that is, the suboptimal detection, and the suboptimal method is relatively solved. The tuning performance has decreased, but the use of Viterbi detection greatly reduces the number of states generated and improves the demodulation efficiency. In this paper, the Viterbi detection method is used to improve the suboptimal PAM demodulation and improve the error performance.

## 2. Laurent Decomposition

The GMSK signal can be regarded as a special case of a CPM signal with a modulation index of 0.5, and its expression is

$$s_c(t,\boldsymbol{\alpha}) = \sqrt{\frac{2E}{T}}\cos(2\pi f_c t + \phi(t,\boldsymbol{\alpha})), \tag{1}$$

among them $T$ for symbol intervals, $E$ For symbolic energy, $f_c$ for the carrier frequency, $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, ...)$ a sequence of binary information symbols to be sent, $\alpha_i = \pm 1$.

Since continuous phase modulation is a constant envelope modulation method, for all the amplitude of the signal is constant. It should be noted that the CPM signal is defined over the entire time axis due to the continuous, time-varying phase function. $\phi(t,\boldsymbol{\alpha})$ not only affected by one symbol. The transmitted binary symbol sequence is included in the additional phase function as follows

$$\phi(t,\boldsymbol{\alpha}) = 2\pi h \sum_{k=-\infty}^{\infty} \alpha_k q(t-kT) \tag{2}$$

among them

$$q(t) = \int_{-\infty}^{t} g(\tau)d\tau \tag{3}$$

$h$ for the modulation index, the value in the GMSK modulation is 0.5. Phase pulse $q(t)$ modulation index $h$ and input symbols $\alpha_k$ it determines how the phase function changes over time. $q(t)$ frequency derivative $g(t)$, $g(t)$ usually in $0 \le t \le LT$ has a smooth pulse shape for the time, and takes a value of 0 outside this interval. $L$ is called phase correlation length.

The GMSK phase pulse is obtained by delaying, truncating and normalizing the Gaussian frequency function. $g(t)$ defined as

$$g(t) = \frac{1}{K} g_0(t - \frac{LT}{2}) \tag{4}$$

among them

$$g_0(t) = erf(\beta_0(t)) - erf(\beta_m(t)) \tag{5}$$

$$\beta_p(t) = C_0\left[\frac{t}{T} + \frac{1}{2}\right] \tag{6}$$

$$\beta_m(t) = C_0\left[\frac{t}{T} - \frac{1}{2}\right] \tag{7}$$

$$C_0 = BT\pi\sqrt{\frac{2}{\ln(2)}} \tag{8}$$

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy \tag{9}$$

$B$ single sideband for Gaussian pulses $3dB$ bandwidth, $BT$ for the time bandwidth product, $BT$ the smaller the value, the denser the spectrum will be, but the more the inter-symbol interference (isi) is, the more the bit error probability performance is degraded. So, for a given practical application problem, $BT$ choice of value must be a compromise between spectral efficiency and bit error rate performance. In a general wireless transmission environment, $BT = 0.3$, $L = 3$. $K$ is the normalized constant, the value is

$$K = 8[q_0\left(\frac{LT}{2}\right) - q_0\left(-\frac{LT}{2}\right)] \tag{10}$$

Using Laurent decomposition, you can represent the CPM signal as $M = 2^{L-1}$ sum of the PAM signals, when the modulation index $h = 0.5$ in the form of:

$$s(t) = \sum_{i=-\infty}^{\infty} \sum_{m=0}^{M-1} a_{m,i} h_m (t - iT) \tag{11}$$

among them

$$h_m(t) = c(t) \prod_{l=1}^{L-1} c(t + lT + \gamma_{m,l} LT) \tag{12}$$

$$c(t) = \begin{cases} \sin(\pi \cdot q(t)) & 0 \le t \le LT \\ \sin(\pi / 2 - \pi \cdot q(t - LT)) & LT < t \le 2LT \\ 0 & 其他 \end{cases} \tag{13}$$

$\gamma_{m,l}$ value is 0 or 1, and is satisfied

$$m = \sum_{l=1}^{L-1} 2^{l-1} \cdot \gamma_{m,l} \tag{14}$$

PAM form factor $a_{m,i}$ satisfy

$$a_{m,i} = \exp\left\{ j \frac{\pi}{2} [\sum_{l=-\infty}^{i} \alpha_l - \sum_{l=1}^{L-1} \alpha_{i-l} \cdot \gamma_{m,l}] \right\} \tag{15}$$

The waveform of each PAM component can be obtained by calculation. Fig. 1 Shown as GMSK signal $BT = 0.3$, $L = 3$ waveform of each PAM component at the time. As can be seen from the figure, $h_0(t)$ the largest amplitude, its energy accounts for more than 95% of the total energy of the signal.

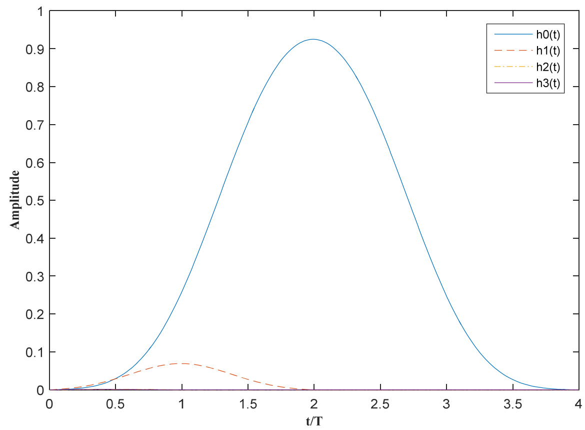


Fig. 1 The PAM component of the GMSK signal

## 3. Coherent Detection of Gmsk Signals

The coherent detection of CPM signals based on Laurent decomposition can be divided into two schemes: optimized coherent detection and suboptimal coherent detection. Optimized coherent detection has better BER performance than suboptimal coherent detection, but suboptimal coherent detection has a simpler structure and less computation. Receiver system block diagram using coherent detection Fig. 2 Shown.

The coherent detection of the CPM signal usually uses the maximum likelihood sequence estimation, and combined with the Viterbi algorithm, the soft decision is performed on the signal. The maximum likelihood sequence estimation is based on the Bayesian formula, and the signal is sequenced by taking the maximum value of the conditional probability function.

The basic principle of coherent detection using the Laurent algorithm is the same as the maximum likelihood sequence detection, except that the known signal space in the detection is decomposed into a weighted sum of multiple PAM signals according to the Laurent algorithm. The coherent detection of the receiver is performed by using the signal weighted sum, thereby achieving optimal coherent detection and suboptimal coherent detection.

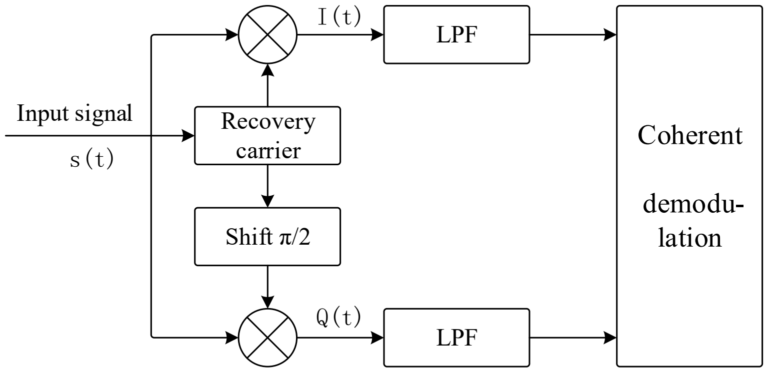

Fig. 2 Coherent demodulation schematic

### 3.1 Optimized Coherent Detection.

Assuming that the system channel is an additive white Gaussian noise channel, the received GMSK signal can be expressed as follows:

$$r(t) = s(t) + n(t) \tag{16}$$

$n(t)$ zero-mean Gaussian white noise, since all possible transmitted signals have the same energy and probability of occurrence, according to the requirements of coherent detection, If formula (17) has a sequence of maximum values, such $\alpha$ is considered to be a sequence of symbol information transmitted by the transmitting end.

$$\delta_i = \int \mathrm{Re}\{r(t) \cdot s^*(t)\} dt \tag{17}$$

Using the principle of Laurent decomposition, Bring the formula (11) into the formula (17).

$$\delta_i = \sqrt{\frac{E_s}{T}} \sum_{n=0}^{Ns-1} \lambda_i(n) \tag{18}$$

$$\lambda_i(n) = \mathrm{Re}\{\sum_{m=0}^{M-1} r_{m,n} a_{m,n}^*\} \tag{19}$$

$$\begin{aligned} r_{m,n} &= [r(t) * g^*(-t)]_{t=nT} \\ &= \int r(t) g_m^*(t - nT) dt \end{aligned} \tag{20}$$

As can be seen from equation (19), you only need to know $r_{m,n}$ with $a_{m,n}$ the value can be counted and judged to find a sequence of information that meets the requirements. Calculation $\lambda_i(n)$ need to know at $nT$ time $a_{m,n}$ all possible values, $a_{m,n}$ symbol information $a_n$ and state vector $\{\theta_n, a_{n-1+L}, ..., a_{n-1}\}$ decide, therefore need $M = 2^{L-1}$ matching filters for processing. The optimized coherent detection model is shown in the Fig. 3.

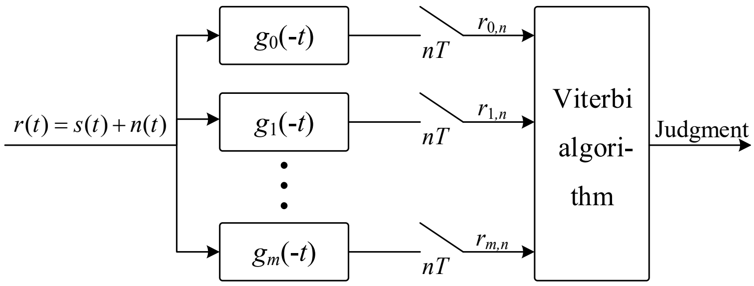

Fig. 3 Coherent detection model

In the above optimization model, the number of states of the Viterbi algorithm is determined by the number of signal phase states. When the modulation index is 0.5, the number of states is $4 \cdot 2^{L-1} = 2^{L+1}$.

In the optimization model, the signal is processed by the complete Laurent decomposition. Since the Laurent decomposition is a very accurate decomposition of the original signal, the coherent detection method is based on the BER performance and the traditional maximum likelihood sequence detection. It is consistent, but it achieves a large simplification in both structure and computation.

### 3.2 Suboptimal Coherent Detection.

Compared to optimized coherent detection, suboptimal detection has a simpler structure and computational complexity. By Fig. 1 can be seen that the energy distribution of the PAM component constituting the GMSK signal is not uniform, wherein $h_0(t)$ the energy provided is much larger than the energy of other PAM waveforms. Therefore, in the optimization model, only the first matched filter can be considered, and the influence of the subsequent matched filter is ignored, which is the basic model of the suboptimal coherent detection.

Suboptimal coherent detection requires calculation of the number of phase states only by $\theta_n$ is decided that for a GMSK modulation with a modulation index equal to 0.5, the number of states is four. Therefore, the algorithm is greatly reduced in both the number of matched filters and the number of states of the Viterbi algorithm, thereby reducing the complexity of the demodulation module at the receiving end and the system cost, while ensuring a certain error performance.

## 4. Improved Suboptimal Coherent Detection Method

From the analysis in the previous section, the error performance of the optimized coherent detection is better, but the calculation of the demodulation end is more complicated and the implementation cost is higher; while the suboptimal detection is at the expense of a certain bit error rate performance, which greatly reduces the coherence. The computational complexity of the detection, however, when the transmission channel is severely affected by external interference, the signal-to-noise ratio of the received signal is relatively low, and the suboptimal detection method cannot meet the error rate requirement. Therefore, the suboptimal coherent detection needs to be further improved to improve the error performance.

The Laurent decomposition of the GMSK signal is visible, and the second PAM component waveform $h_1(t)$ the energy accounts for 2% to 5% of the total energy. $h_2(t)$ to $h_{2^{L-1}}(t)$ the sum of the energy accounts for about 1% of the total energy. The results of GMSK simulation analysis for different time bandwidth products and different phase correlation lengths are shown in Table. As shown, obviously the $h_1(t)$ energy is much larger than $h_2(t)$ to $h_{2^{L-1}}(t)$ the sum of the energy, therefore, the $h_1(t)$ energy occupied cannot be directly ignored, and is added to the judgment detection calculation. the $h_1(t)$ impact is necessary.

Table 1. PAM component energy ratio

| $BT=0.3$ | $h_0(t)$ | $h_1(t)$ | $h_2(t)+...+h_{2^{L-1}}(t)$ |
|---|---|---|---|
| $L=3$ | 95.70% | 4.21% | 0.09% |
| $L=4$ | 95.40% | 4.46% | 0.14% |
| $L=5$ | 95.39% | 4.47% | 0.15% |
| $BT=0.5$ | $h_0(t)$ | $h_1(t)$ | $h_2(t)+...+h_{2^{L-1}}(t)$ |
| $L=3$ | 97.88% | 2.11% | 0.01% |
| $L=4$ | 97.86% | 2.12% | 0.01% |
| $L=5$ | 97.86% | 2.12% | 0.01% |

Ignored in the suboptimal test $h_1(t)$ direct use $h_0(t)$ demodulation results in a large degree of error performance degradation.

Therefore, based on the suboptimal detection, this paper introduces the PAM component. $h_1(t)$ impact of the detection discriminant is corrected to improve the error performance. The signal form at this time is

$$\lambda_i(n)=\mathrm{Re}\{r_{0,n}a_{0,n}^{*}+r_{1,n}a_{1,n}^{*}\} \tag{21}$$

Compared with formula (19), less $2^{L-1}-1$ the process of single-item integration only adds the first two items, reducing the number of states detected.

## 5. Simulation Experiment and Performance Analysis

This paper separately simulates the optimization algorithm, suboptimal algorithm and improved algorithm. $BT=0.3$ or $BT=0.5$, $L=3$ compare the bit error rate performance under different SNR conditions, as shown in Fig. 4 and Fig. 5.It can be seen that the error rate performance of the improved algorithm is improved compared with the suboptimal detection algorithm, and the error performance of the optimization algorithm is almost the same.

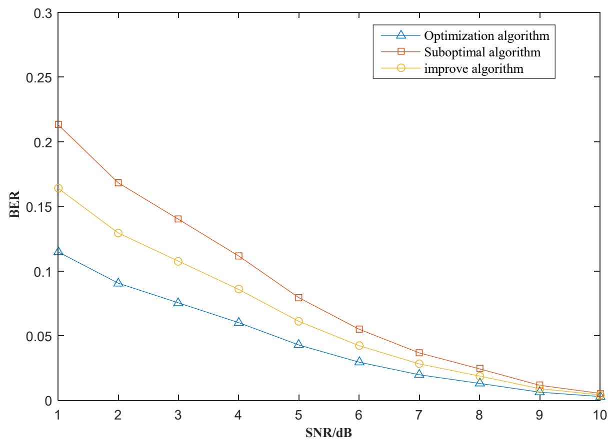


Fig. 4 $BT=0.3$ Time error rate and signal to noise ratio curve

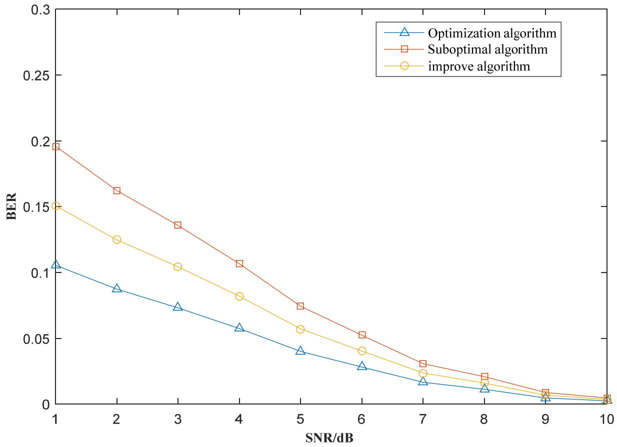


Fig. 5 $BT = 0.5$ Time error rate and signal to noise ratio curve

Table 2 Compared with the different phase correlation lengths, the number of matched filters and the number of Viterbi states required for coherent detection at the GMSK signal receiving end are optimized by optimal coherent detection, suboptimal coherent detection and improved suboptimal coherent detection. It can be seen that the application of the improved algorithm brings a greater simplification to the demodulation module of the receiving end than the optimization detection, thereby reducing the complexity of the system.

Table 2. Comparison of algorithm complexity

| Phase correlation length | algorithm | Matched filter Number | Number of states |
|---|---|---|---|
| $L = 2$ | Optimization algorithm | 2 | 8 |
| | Suboptimal algorithm | 1 | 8 |
| | improve algorithm | 2 | 4 |
| $L = 3$ | Optimization algorithm | 4 | 16 |
| | Suboptimal algorithm | 1 | 16 |
| | improve algorithm | 2 | 4 |
| $L = 4$ | Optimization algorithm | 8 | 32 |
| | Suboptimal algorithm | 1 | 32 |
| | improve algorithm | 2 | 4 |

## References


[1]. Fonseka J P. Noncoherent detection with Viterbi decoding for GMSK signals [J]. IEEE Proceedings on Communications,1996,143(6) : 373 - 379.

[2]. Aulin T, Sundberg C E. Continuous phase modulation-Part 1: Partial response signaling[J]. IEEE Trans. On Comm., 1981,29(3): 210-225.

[3]. Yang Jianfeng. Multi-ary continuous phase modulation and demodulation technology based on Laurent algorithm [D]. University of Electronic Science and Technology of China, 2007.

[4]. Dwi Putri Kusumadewi,Suwadi Suwadi,Titiek Suryani. Implementasi dan Evaluasi Kinerja Modulasi dan Demodulasi GMSK dengan menggunakan WARP[J]. Jurnal Teknik ITS,2015,4(1).

[5]. Cui Zhongpu, Dou Gaoqi, Gao Jun, He Mingjia.A Low Complexity Demodulation Method Based on Laurent Decomposition Multi-h CPM[J].Journal of Chongqing Technology and Business University(Natural Science Edition),2018,35(01):59-63.

[6]. Xie Chunlei, Zhang Tao. GMSK Timing Synchronous Loop Based on Laurent Decomposition [J]. Radio Engineering,2017,47(12):20-23+60.

[7]. LAURENT P A. Exact and Approximate Construc-tion of Digital Phase Modulations by Superposition of Amplitude Modulated Pulses (AMP) [J]. IEEE Trans. Commun. ,1986, 34( 2) : 150-160.

[8]. MORUTA K,HIRADE K. GMSK Modulation for Digital Mobile Radio Telephony[J]. IEEE Trans. Commun,1981,29 (6): 1044-1050.

[9]. Simon MK, Simon. High-bandwidth efficiency digital modulation and its application in deep space communication [M]. Tsinghua University Press, 2006.