

Development of an Optimal Solution for a Location-Routing-Problem Based on 3rd Party Providers

Qin SUN and Bing BAI

Yuancheng Road 211, Hangzhou Economic Development Area, Hangzhou, Zhejiang, China

Liuhe Road 318, Hangzhou, West Lake District, Zhejiang, China

Keywords: Routing, Optimization, Transportation.

Abstract. The transportation is the movement of people, animals and goods from one place to another. Nowadays it includes several modes: air, rail, road, water and other modes. In mathematically, the transportation theory is to optimize transportation and allocation of results. In reality, the problem is more complicated: firstly, the distance between 2 places is not always the straight-line distance, that means the transport route will be divided into several parts; secondly the cost of each parts will not be always the same. For example, the high-way transportation cost is not equal to the city or state road cost, if a road transport plan includes both of high-way and normal road, then cost of each parts will be completely different. This means the total cost need to be calculate by adding the cost of all the parts, and the cost of each parts should be figured out first, it will be very tedious if the route is divided into a lot of parts. This paper will give a solution to solve this problem; it includes software using computer navigation technology, and shows how to get the optimize transportation mode by this software.

Brief Introduction of Technology Background

GPS Technology

The main concept of GPS is to use the transmitted data by the satellite to figure out the location of the GPS receiver, the GPS receiver can measure the distance from a satellite to itself, and with this distance, the position of the receiver can be determined. In 2-dimension space, we need at least 3 satellites to determine the position of receiver. Figure 1 shows this situation^[2].

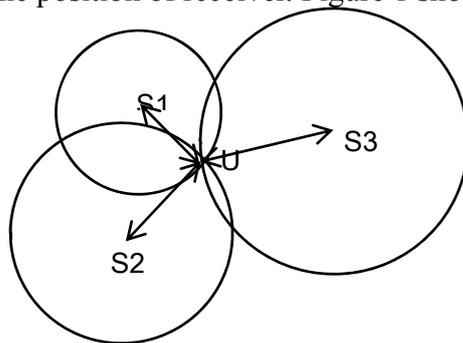


Figure 1. Working principle of GPS

With 3 equation^[4], the location of receiver can be figured out:

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} \quad (1)$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} \quad (2)$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} \quad (3)$$

Here, the x_u, y_u, z_u are the 3-dimension coordinate of the receiver's location, they are unknowns. ρ_1, ρ_2, ρ_3 are the distances between 3 satellites and the receiver. Other variables are known as the 3 satellites' coordinates.

The distance between a satellite and the receiver can be determined by the following formula:

$$\rho = c(t_u - t_s) \quad (4)$$

Here, ρ is the distance, c is the speed of light. t_u, t_s are variable of time. At time t_s , the satellite sends a signal, and at time t_u , the receiver gets it.

Since both the satellite and receiver may have clock bias error, we need to consider this situation, so the final equations to calculate the location of receiver are:

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + b_u \quad (5)$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + b_u \quad (6)$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + b_u \quad (7)$$

$$\rho_4 = \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + b_u \quad (8)$$

Here b_u is the user clock bias error expressed in distance. The reason why we have 4 equations here is that the b_u is unknown so we need another equation to figure out the value.

Now GPS has 70 satellites, and the accuracy is 7.8 meters at a 95% confidence level when in worst case^[3].

GPS can tell the location of 2 points, that's the 1st step of navigation; the 2nd step is figure out the shortest way between 2 points.

Route Algorithm

The most common algorithm used in a navigation system is Dijkstra's algorithm.

Dijkstra's algorithm is an algorithm for finding the shortest path between 2 nodes^[4] in a graph that all the edges' value are not negative. To use it, the graph's data struct must be:

$$\mathbf{G} = (\mathbf{V}, \mathbf{E}) \quad \text{where} \quad \begin{array}{l} \cdot \mathbf{V} \text{ is a set of vertices and} \\ \cdot \mathbf{E} \text{ is a set of edges.} \end{array}$$

The advantage of this algorithm is that, comparing to the enumeration method, it has a very small time complexity. The worst case performance of this algorithm is

$$\mathbf{O}(n^2) \quad \text{where} \cdot \mathbf{n} \text{ is the number of nodes}$$

But this algorithm is not recursive; make sure that the shortest path is a one-way solution before use it.

The detail of Dijkstra's algorithm is^[5]:

Step 1: make a list for all the nodes that the route can visit, the source is the initial node and the destination is the end node. Set the distance of initial node to 0 and infinity to every other node.

Step 2: Mark all the nodes except initial node as unvisited nodes.

Step 3: Start from the initial node, consider all of its neighbors, calculate the distances. If the distance is smaller than the distance of this node, then change the distance.

Step 4: After considering all of the neighbors of the current node, mark it as visited node.

Step 5: If the destination node has been visited or all the nodes have been visited, then stop, the algorithm finished.

Step 6: Otherwise, select the unvisited node which has the smallest distance, set it as the current node and go back to Step 4.

But instead of doing Dijkstra's once from source to destination, most of the map providers (like Google) will start at both source and destination until they meet in the middle. That means the map provider will choose both the origin and the destination as the start node to use this algorithm. When the current node of both sides are the same (it means they meet in the middle of the way), the algorithm stops, that will make the time complexity become to:

$$O\left(\frac{n^2}{2}\right) \text{ where } n \text{ is the number of nodes}$$

Distance between 2 Points in Rectangular Coordinate System

In a rectangular coordinate system, 2 points have shortest distance (straight line distance), the solution below assumes the length of air transportation line as the shortest distance of 2 points, in order to improve the speed of calculation.

As the distance between 2 point in rectangular coordinate system is:

$$D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2} \quad (9)$$

D is the distance, $x_1, x_2, y_1, y_2, z_1, z_2$ are the coordinates of the 2 points.

Suspect the earth is a sphere, then the coordinate system transformation functions are:

$$\begin{cases} x = R \cos \theta \cos \delta \\ y = R \cos \theta \sin \delta \\ z = R \sin \theta \end{cases} \quad (10)$$

R is the radius of earth, θ is the longitude, δ is the latitude.

As the latitude and longitude have negative value (in computer system, normally the west longitude, and the south latitude are negative), so the functions can be transformed to:

$$\begin{cases} x = \begin{cases} R \cos(2\pi - \theta) \sin\left(\frac{\pi}{2} + \delta\right), \delta < 0 \\ R \cos(2\pi - \theta) \sin\left(\frac{\pi}{2} - \delta\right), \delta \geq 0 \end{cases} \\ y = \begin{cases} R \sin(2\pi - \theta) \sin\left(\frac{\pi}{2} + \delta\right), \delta < 0 \\ R \sin(2\pi - \theta) \sin\left(\frac{\pi}{2} - \delta\right), \delta \geq 0 \end{cases} \\ z = R \cos(2\pi - \theta) \end{cases} \quad (11)$$

To analyze a case, some software development frameworks are needed, the frameworks below are used in the case study:

Google Map API: Google Map API is an application that allows the developer to embed a Google Maps onto their own web pages. The Google Map API gives JavaScript interface and Flash interface, and Google also provides Java interface, which means it is designed to work on both mobile devices and traditional desktop browser applications.

Amadeus: This is the API for air transportation. It provides the solution of the shortest way between 2 places by air.

The API uses both HTTP and SOAP protocol and responses are formatted in JSON. Clients can simply send the request by an http url, and get the response. Because the responses are formed in JSON, it is very easy to decode. This keeps the normalization and expansibility of the API.

Aquaplot: This is the interface for ship transportation. It provides the solution of the shortest way between 2 places by ship. Like Amadeus, it also uses both HTTP and SOAP protocol, and the responses are formed in JSON. That makes the API expansible and easy to use. Although it provides the function like google maps, this program just uses the API part to solve the transportation problem.

Location-Routing-Problem's Solution

The idea of the solution is that using computer navigation service to get the lowest cost route of each transportation mode, and compare the result of every mode to get the optimal answer. So to achieve

the target, writing a program with algorithm encoded.

As the program should be cross-platform, a B/S framework (Browser/Server) based program will be the best choice.

Figure 2 shows the flow chart of the solution:

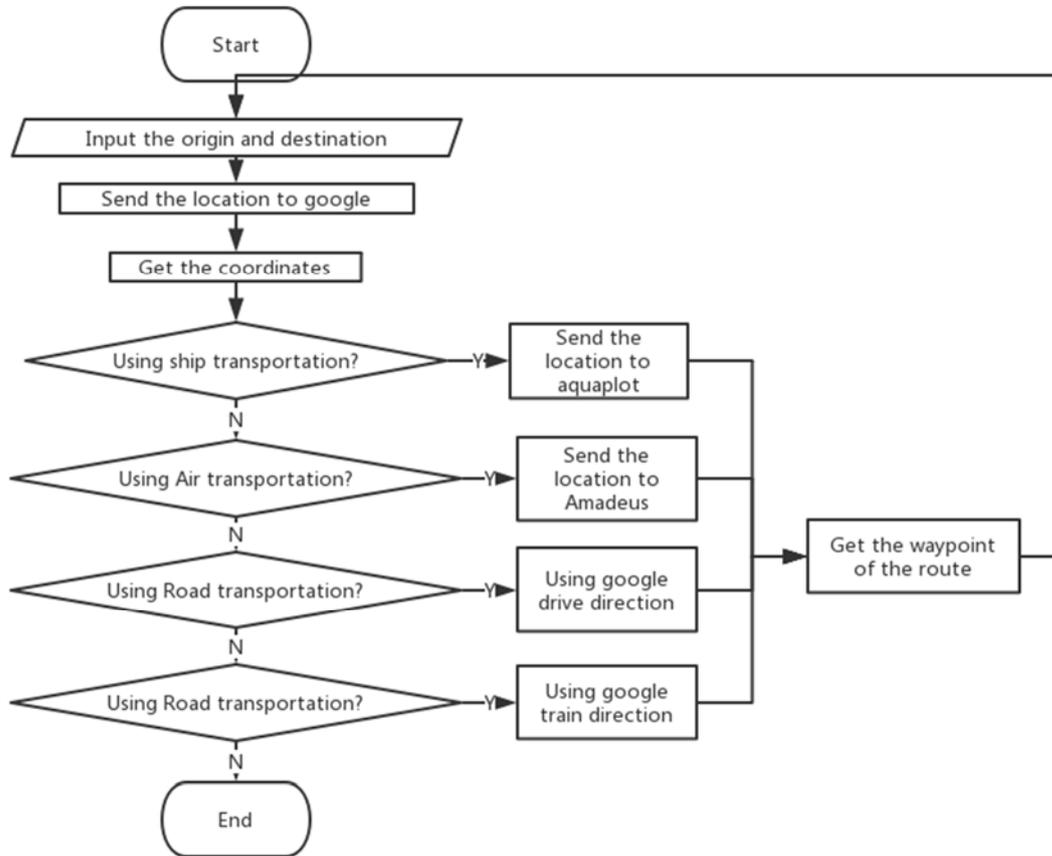


Figure 2. The flow chart of program

Case Study

Assume that 10 tons’ goods are to be delivered from Schweizer strasse Duisburg to Nanjing XiLu Shanghai China. So here is the data of the operation:

Table 1. The Cost of Different Transportation Way between Schweizer strasse Duisburg and Nanjing XiLu Shanghai

The transportation way	Total cost (k Euro)
The air transportation	357.18
The road transportation	None
The railway transportation	None
The ship transportation	103.51

And the following data is the total distance for each transportation way:

Table 2. The Distance of Different Transportation Way between Schweizer strasse Duisburg and Nanjing XiLu Shanghai

The transportation way	Total distance (km)
The air transportation	8929.405
The road transportation	None
The railway transportation	None
The ship transportation	10351.1875

The cost matches the distance multiply cost per km.

As the cost of ship transportation is 103k Euro, and the cost of air transportation is 357k Euro, and the other two types of transportation way is unreachable, so the best solution is to transport the goods with ship.

Here is another case: assume that 10 tons of goods to be delivered , from Nanjing XiLu Shanghai, to Xi zhi men Beijing. Then here is the data:

Table 3. The Cost of Different Transportation Way between Nanjing XiLu Shanghai and Xi zhi men Beijing

The transportation way	Total cost (k Euro)
The air transportation	44.03
The road transportation	36.63
The railway transportation	27.17
The ship transportation	6.25

And the following data is the total distance for each transportation way:

Table 4. The Distance of Different Transportation Way between Nanjing XiLu Shanghai and Xi zhi men Beijing

The transportation way	Total distance (km)
The air transportation	1100.78
The road transportation	1216.96
The railway transportation	1341.26
The ship transportation	625.125

The cost matches the distance multiply cost per km.

In this case, it can be see, the cost of ship transportation is 6k Euro, the cost of air transportation is 44k Euro, the cost of road transportation 36k Euro, the cost of railway cost is 27k Euro, so here a decision to choose the ship transportation can be made.

From the cases above the following data can be shown:

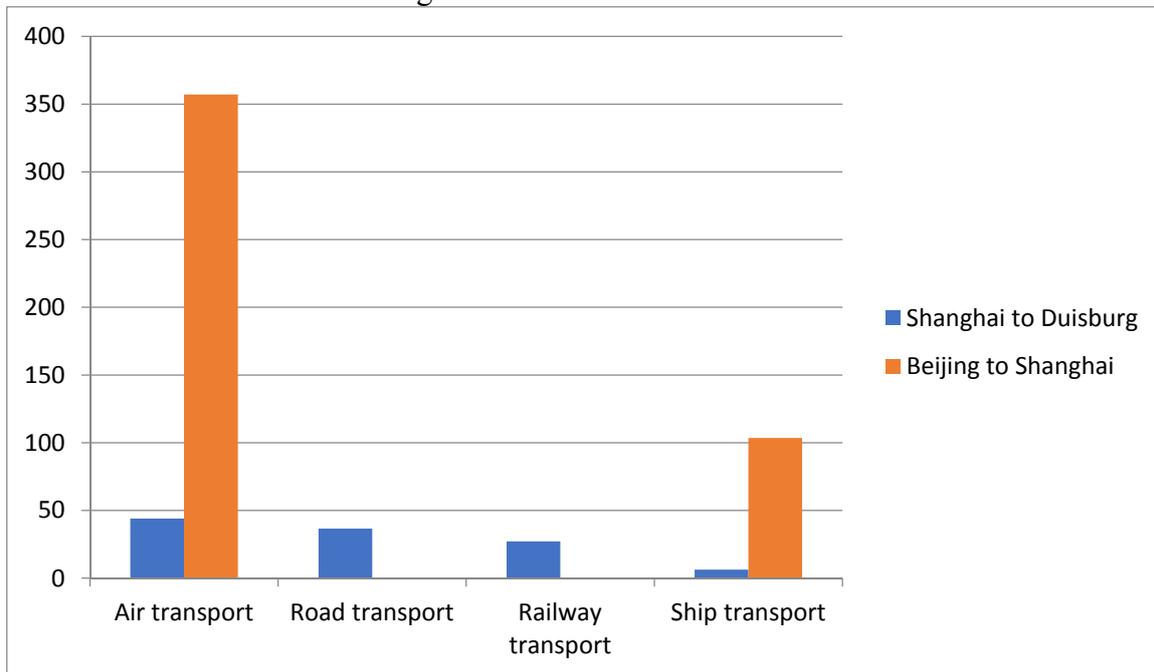


Figure 3: Compare of Different Transportation Ways' Cost Between “Shanghai to Duisburg” and “Beijing to Shanghai”

From the result the following conclusion can be drawn: Since the cost per km of ship transport is the lowest, when carrying same volume goods, this way will always be the cheapest, although the distance is always the longest. Ship transportation is good for large volume good transportation, but the speed and distance will become problem, the air transportation is always very expensive, but the speed and distance will be very fast and short.

The results of 2 cases show that the solution can figure out the costs of different ways of transportation correctly, because the cost equals to distance multiply the cost per km per unit. They

all choose the ship transportation as the best way of transportation, this is because the solution focuses on the lowest cost of the transportation, it is the only variable in this problem, and the data assumes that the volumes of goods in different transportation ways are the same. Because the cost per km per unit of the ship transportation is the lowest one, although the distance of it maybe longer than other transportation ways, the total cost will still be the lowest. If there is some accurate data, the result will be more credible.

Conclusion

In traditional way, when a solution for a transportation problem is required, three things are required: Figure out the best route of different transportation way, get the cost of different transportation way's detail, and figure out the final answer. In this way, solving a transportation problem will be quite tedious, but with this application, what the user needs to do is only get the detail of the costs; other things can be finished by the program automatically. This application uses the available data resources on the internet, in this way, the solution can be figured out very quickly. That improves the efficiency.

References

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