Fragile and Quasi-Brittle Fracture of Gypsum Boards with Circular Hole with Unevenly Distributed Compression

Suknev S.V.

Mining Institute of the North named after N.V. Cherskiy SB RAS
Yakuts, Russia
suknyov@igds.ysn.ru

*Abstract –*The results of theoretical and experimental studies of the destruction of gypsum boards containing a circular hole and exposed to an unevenly distributed compressive load are presented. The samples made of high-strength gypsum brand GVVS-16 and construction gypsum brand G-5 were tested. The samples of high-strength gypsum were fragile, while specimens of gypsum showed a quasi-brittle nature of the destruction. To calculate the breaking load, it was proposed to use a modified non-local fracture criterion, which is the development of the average stress criterion, and contains a complex parameter characterizing the size of the pre-fracture zone and taking into account not only the structure of the material, but also the plastic properties of the material, the sample geometry and loading conditions. The results of the calculations are in good agreement with the experimental data obtained.

Keywords – gypsum; brittle fracture; quasi-brittle fracture; medium stress criterion; hole; scale effect Introduction.

Structurally inhomogeneous materials (concrete, gypsum, rocks) are characterized by low tensile strength compared with compressive strength. At the same time, even under the conditions of compressive loads, near the stress concentrators (inclusions, cavities), tensile stress zones can form and cracks form, which is dangerous for the normal operation of the structural element. In addition, structurally inhomogeneous materials demonstrate the dependence of strength properties on the loaded volume (scale effect), which is most pronounced in conditions of stress concentration, when the effective loaded volume is determined by the stress concentration zone, the size of which is small compared with the characteristic dimensions of the structural element.

Paper [1] considers the influence of boundary conditions on the destruction of brittle geomaterial in the zone of stress concentration under biaxial loading taking into account the scale effect. Based on experimental and theoretical studies, it was shown that for a fixed hole size, the effect of loading conditions on the critical (destructive) stress is well described by non-local criteria: medium stresses, stresses at a point and fictitious cracks. Nonlocal fracture criteria were developed in [2–14] and others. They are based on the idea of the formation in the material of a pre-fracture zone (fracture process zone), in which there is a local redistribution of stresses, while the main material is deformed elastically until fracture. A common feature of these criteria is the introduction of a new constant — the internal size of the material (intrinsic material length) d_0,

which characterizes its structure, and makes it possible to describe the scale effect under conditions of stress concentration and thereby expand the scope of application as compared to traditional criteria.

At the same time, as shown in paper [15], the redistribution of stresses within the size limits is not associated with the plastic properties of the material, but with the discreteness of its structure. Therefore, the scope of non-local criteria is a brittle fracture of materials with cuts. Non-local criteria can be applied to the case of quasi-brittle fracture, accompanied by the formation of a small-scale flow zone (pre-fracture zone) d, if its size is not much different from d_0, that is under the condition $d \approx d_0 = \text{const}.$

In this paper, the effect of the diameter of a circular hole on the destruction of geomaterial (gypsum) in the stress concentration zone under uneven distributed compression with the scale effect is studied theoretically and experimentally, and the analysis of the possibility to use nonlocal average stress criterion to describe a brittle and quasi-brittle fracture of gypsum material is carried out.

I. METHODS AND MATERIALS

The program of experimental studies included conducting two series of tests of gypsum samples with a circular aperture under the action of non-uniformly distributed compression. The load p was applied to the sample through rigid inserts placed between the sample and the loading plates. The inserts were placed in the center of the upper and lower edges of the sample (Fig. 1).

To carry out the tests, samples made for the first series of experiments (gypsum 1) from an aqueous solution of high-strength gypsum brand GVVS-16 containing calcium sulfate α-hemihydrate, and for the second series of experiments (gypsum 2) from an aqueous solution of gypsum brand G-5, containing calcium sulfateβ- hemihydrate. Due to structural peculiarities, α-modification of calcium sulfate hemihydrate is characterized by low water demand, which provides lower porosity and, consequently, higher strength characteristics of high-strength gypsum as compared to ordinary construction gypsum. For gypsum, 1 aqueous solution was prepared in a ratio (by weight) of 1 part of water to 2 parts of gypsum; for gypsum 2, in a ratio of 1 part of water to 1.5 parts of gypsum.

The samples were square plates measuring 200x200 mm and 40 mm thick (gypsum 1) and 36 mm thick (gypsum 2). After fabrication, the samples were dried in air for 30–40 days.
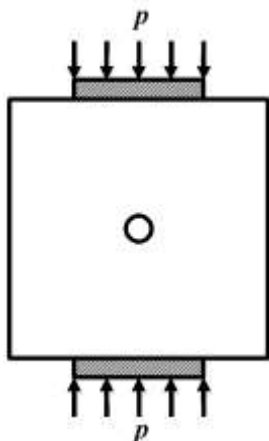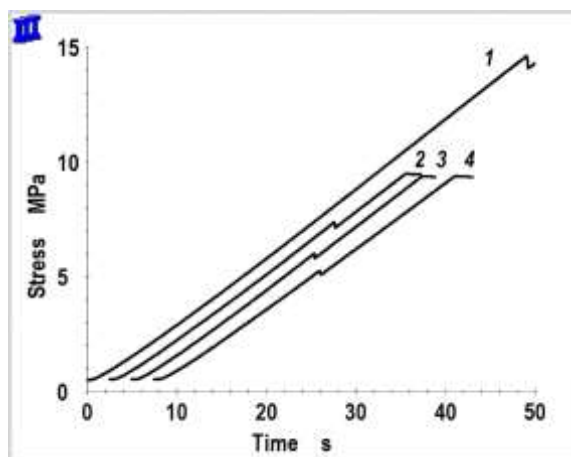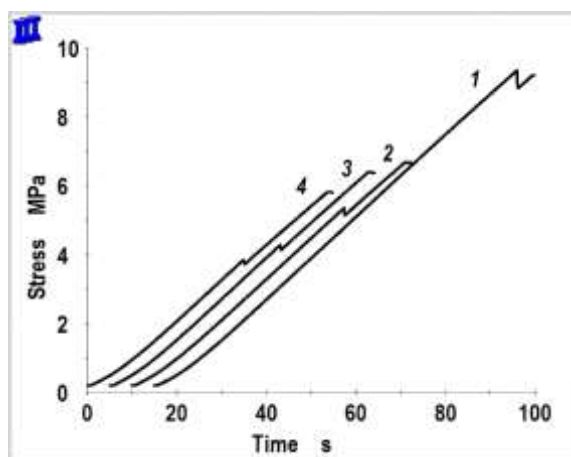

Fig. 1.   Sample loading scheme.

a

б

Fig. 2.   Deformation diagrams of specimens from gypsum 1 (a) and gypsum 2 (b) with holes of different diameters (*1* – 1 mm; *2* – 5 mm; *3* – 10 mm; *4* – 15 mm).

Before testing, circular holes of various diameters from 1 mm to 20 mm were drilled in the center of the samples. It was manufactured and tested on 5 samples with holes of each diameter. The loading of the samples was made through inserts with a size of 120 mm. In the process of testing samples in the areas of concentration of tensile stresses on the contour of the hole, the formation of tear cracks was observed, which was of a sudden nature and was accompanied by a characteristic click. In samples with a hole with a diameter of 5 mm to 20 mm, the cracks instantly spread to a distance of about 50 mm along the line of application of a compressive load, with further loading their growth stabilized. In samples with a hole with a diameter of 1 mm and 2 mm, cracks instantly spread over almost the entire vertical section of the sample. The formation of cracks was also accompanied by local unloading of the sample, which was reflected in the deformation diagram in the form of the appearance of a tooth. Samples with a hole of the smallest diameter of 1 mm were subjected to the greatest unloading. In fig. 2, *a, b* are shown characteristic diagrams of deformation of samples with holes of different diameters in the form of screenshots of testXpert program windows with test results. The critical load at the moment of cracking was determined by the top of the tooth in the diagram.

II.   THEORETICAL APPROACH

The calculation of the critical pressure was made according to usual and modified average stress criterion [15].

In paper [1], it was shown that as a result of loading the sample according to the scheme shown in Fig. 1, in the central part (outside the zone of influence of the hole), a fairly uniform biaxial stress state is realized: tensile forces $\alpha\sigma$ along the horizontal axis and compression forces along the vertical axis of the sample (Fig. 3).
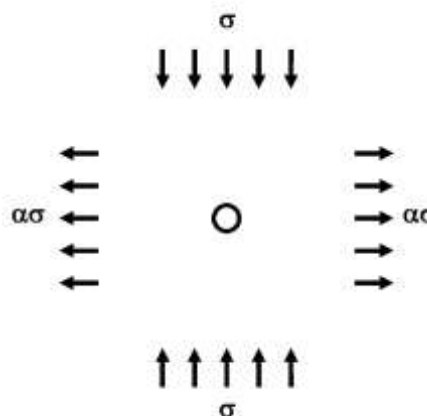

Fig. 3.   Circular bore with biaxial loading.

The values σ и α were calculated by the finite element method in the center of the samples loaded through inserts of a given size and not containing holes. The load applied to the sample was modeled by moving an absolutely rigid insert. In the area of sample contact with the insert, a non-slip condition was imposed. For the inserts used in the experiments, the value of σ was 0.764 p, $\alpha = 0.187$.

The average stress criterion criterion is:

$$\langle \sigma_e \rangle_d < \sigma_0 ,$$

where σ_0 – material tensile strength; $\langle \sigma_e \rangle_d$ – the averaged over a distance d over a dangerous cross section, the value of the equivalent stress characterizing the internal stress state of a deformable body. For brittle materials, the averaging size d is assumed to be the constant of the material characterizing its structure: $d = d_0 = \text{const}$.

In accordance with the criterion of average stresses, the critical stress for a sample with a circular aperture of radius a, subjected to biaxial loading (Fig. 3), is determined by the formula [1]:

$$\sigma_c = 2\sigma_0 \left[(1+\gamma)\gamma^{-3} + \alpha(1+\gamma^{-1})(2+\gamma^{-2}) \right]^{-1} , \quad (1)$$

where $\gamma = 1 + d/a$. When the value of the parameter $\gamma = 1$ formula (1) gives the calculation of the critical stress according to the traditional criterion of destruction.

To describe the quasi-brittle fracture, the size of the averaging will be determined by the formula [15]:

$$d = d_0 + \beta L_e , \quad (2)$$

where $L_e = \dfrac{\sigma_e}{|\text{grad}\,\sigma_e|}$ – size of stress concentration zone, β – dimensionless parameter characterizing the plasticity of the material. For fragile materials $\beta = 0$, for ductile materials $\beta \gg 1$. When $\beta \sim 1$, the material is characterized by moderate plastic properties. The first term in expression (2) characterizes the actual structure of the material, and the second reflects the contribution of inelastic deformations. Thus, the plastic properties of the material begin to manifest themselves when $d > d_0$ and appear the stronger, the larger d is in relation to d_0. If $d = d_0$, we talk about brittle fracture, if $d > d_0$, we talk about quasi-brittle fracture, which in the limit $d \gg d_0$ becomes viscous fracture.

Using the well-known solution of the Kirsch problem, we obtain the following estimate for the size of the stress concentration zone [16]: $L_e = a\dfrac{1+3\alpha}{5+7\alpha}$. Therefore, the expression for the parameter γ in formula (1) is as follows:

$$\gamma = 1 + \frac{d_0}{a} + \beta\frac{1+3\alpha}{5+7\alpha} . \quad (3)$$

In accordance with formula (1) and taking into account the estimates made for σ and α, we write the expression for the critical pressure in a sample with a circular hole:

$$p_c = 2\chi C_0 \left[0.764(1+\gamma)\gamma^{-3} + 0.143(1+\gamma^{-1})(2+\gamma^{-2}) \right]^{-1} , \quad (4)$$

where $\chi = \sigma_0 / C_0$; C_0 – material compressive strength. The parameter γ is defined by formula (3), when $\alpha = 0.187$.

Asymptotic (when $a \to \infty$) value of critical pressure (when $a \to \infty$):

$$T_s = T_0 \frac{2(1+3\alpha)}{(1+\gamma_s)\gamma_s^{-3} + \alpha(1+\gamma_s^{-1})(2+\gamma_s^{-2})} , \quad (5)$$

when $\gamma_s = 1 + \beta\dfrac{1+3\alpha}{5+7\alpha}$, $T_0 = 0.838\chi C_0$ – asymptotic critical pressure for brittle material.

III. RESULTS

In fig. 4, *a* presents experimental data (points) on the magnitude of the load at the time of cracks formation in the hole contour, depending on its diameter l, obtained on samples from gypsum 1, and the results of calculating the critical pressure (curve) using formula (4) when $\beta = 0$. The size d_0 was 0.6 mm and was comparable to the size of the largest pores. The dashed straight line is calculated according to the traditional approach. Fig. 4, *b* shows the experimental data (points) and the results of calculating the critical pressure for gypsum 2 with values $\beta = 0$ (curve 1) and $\beta = 0.6$ (curve 2). The size d_0 was 1.0 mm. In accordance with formula (5), the voltage T_s in the first case is equal to T_0 (dashed straight line), in the second case $T_s = 1.3T_0$ (continuous straight line).
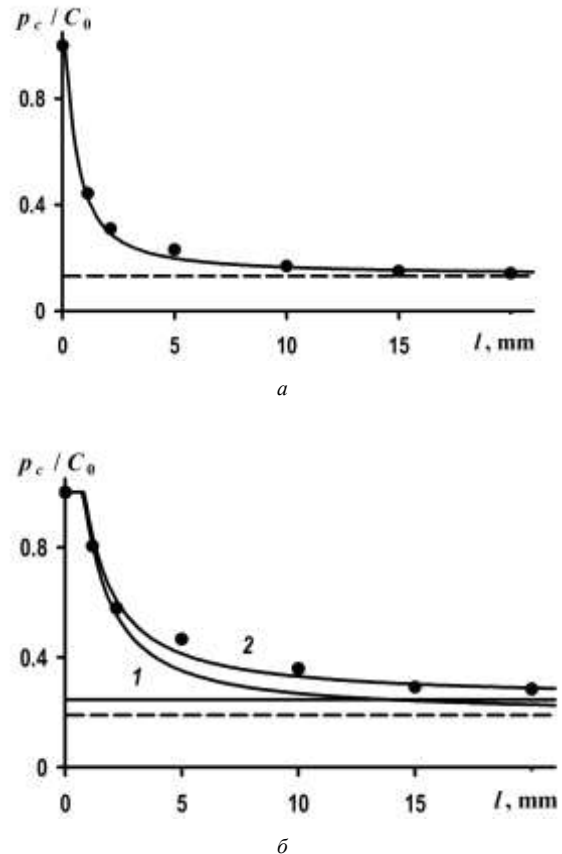

Fig. 4. Dependence of critical pressure on hole diameter for gypsum 1 (a) and gypsum 2 (b).

Fig. 4, *a, б* shows the significant scale effect, that is the effect of the hole diameter on the local strength of the material. With its decrease, the critical pressure increases, reaching the compressive strength, with increasing, asymptotically approaches the stress T_0 for gypsum 1 and the stress T_s for gypsum 2. This behavior is well described by the modified average stress criterion, in which the averaging size d is determined by the formula (2).

IV. DISCUSSION

As it can be seen from fig. 4, the destruction of specimens from gypsum 1, characterized by the sudden formation of a hole on the contour and rapid spreading of cracks along the compression axis, can be described within the framework of the usual criterion of average stresses. The experimental data confirm the asymptotic tendency of critical pressure predicted by a non-local criterion to a value calculated in accordance with the traditional approach for an elastic body. All this allows us to characterize the destruction of this material as brittle.

At the same time, the use of the criterion to describe the experimental data obtained on samples from gypsum 2 makes it possible to obtain satisfactory estimates of the critical pressure only for small (1–2 mm) hole diameters. The results of calculations made for large hole diameters give underestimated values of critical pressure. The experimental data obtained indicate that with an increase in the hole diameter, the critical pressure asymptotically tends to a value that exceeds by 30% the value calculated for an elastic body. In this case, as in the first case, the destruction of specimens from gypsum 2 is characterized by the sudden formation of a hole on the contour and the rapid propagation of tearing cracks along the compression axis. It characterizes the destruction of this material in the investigated range of hole diameters as quasi-brittle.

As it can be seen from fig. 4, this behavior of the critical pressure during the destruction of samples from gypsum 2 is well described by a modified average stress criterion. In this criterion, the "structural" parameter (size of the pre-fracture zone) d is represented as the sum of two terms, the first of which characterizes the structure of the material itself, and the second reflects the formation of an inelastic deformation zone and depends on the plastic properties of the material, the sample geometry and loading conditions (edge conditions). The results of calculations of the critical pressure, made by the formula (4), are in good agreement with the experimental data. Note that in samples with holes of small (1–2 mm) diameter, the zone of inelastic deformations is also small and does not have a noticeable effect on the results of calculations performed according to the usual and modified criteria.

V. CONCLUSIONS

The destruction of a gypsum material containing a stress concentrator (aperture) was studied theoretically and experimentally with non-uniformly distributed compression, and the analysis of the possibility of using a nonlocal average stress criterion to estimate the breaking load was carried out. As a result of laboratory tests, it was established that samples made of high-strength plaster of the brand GVVS-16 were fragile, and

in this case the critical load can be calculated by the usual criterion of average stresses. Samples made from building plaster grade G-5, demonstrated the quasi-brittle nature of the destruction. In this case, the application of the usual criterion of average stresses does not enable to obtain satisfactory estimates of the breaking load. Therefore, to calculate the breaking load, it was proposed to use a modified non-local fracture criterion, which is the development of the average stress criterion, and contains a complex parameter characterizing the size of the pre-fracture zone and taking into account not only the structure of the material, but also the plastic properties of the material, the sample geometry and loading conditions. The results of calculations according to a modified criterion are in good agreement with the experimental data obtained.

References

[1] S.V. Suknev, "Fracture of brittle geomaterial with a circular hole under biaxial loading", J. Appl. Mech. Tech. Phys., vol. 56, no. 6, pp. 1078–1083, 2015.

[2] V.V. Novozhilov, "On a necessary and sufficient criterion for brittle strength", J. Appl. Math. Mech., vol. 33, no. 2, pp. 201–210, 1969.

[3] J.M. Whitney, R.J. Nuismer, "Stress fracture criteria for laminated composites containing stress concentrations", J. Compos. Mater., vol. 8, no. 4, pp. 253–265, 1974.

[4] S.K. Maiti, R.A. Smith, "Comparison of the criteria for mixed mode brittle fracture based on the preinstability stress-strain field. Part I: Slit and elliptical cracks under uniaxial tensile loading", Int. J. Fract., vol. 23, no. 4, pp. 281–295, 1983.

[5] A. Seweryn, Z. Mroz, "A non-local stress failure condition for structural elements under multiaxial loading", Eng. Fract. Mech., vol. 51, no. 6, pp. 955–973, 1995.

[6] S.E. Mikhailov, "A functional approach to non-local strength condition and fracture criteria", Eng. Fract. Mech., vol. 52, no. 4, pp. 731–754, 1995.

[7] A.V. Dyskin, "Crack growth criteria incorporating non-singular stresses: Size effect in apparent fracture toughness", Int. J. Fract., vol. 83, no. 2, pp. 191–206, 1997.

[8] J. Toribio, "A fracture criterion for high-strength steel notched bars", Eng. Fract. Mech., vol. 57, no. 4, pp. 391–404, 1997.

[9] D. Leguillon, "Strength or toughness? A criterion for crack onset at a notch", Eur. J. Mech. A/Solids., vol. 21, no. 1, pp. 61–72, 2002.

[10] D. Taylor, P. Cornetti, N. Pugno, "The fracture mechanics of finite crack extension", Eng. Fract. Mech., vol. 72, no. 7, pp. 1021–1038, 2005.

[11] D. Taylor, The theory of critical distances: a new perspective in fracture mechanics, Oxford: Elsevier, 2007, 284 p.

[12] B. Lecampion, "Modeling size effects associated with tensile fracture initiation from a wellbore", Int. J. Rock Mech. Min. Sci., vol. 56, pp. 67–76, 2012.

[13] A.R. Torabi, E. Pirhadi, "Stress-based criteria for brittle fracture in key-hole notches under mixed mode loading", Eur. J. Mech. A/Solids., vol. 49, pp. 1–12, 2015.

[14] A. Sapora, A.R. Torabi, S. Etesam, P. Cornetti, "Finite Fracture Mechanics crack initiation from a circular hole", Fatigue Fract. Eng. Mater. Struct., vol. 41, no. 7, pp. 1627–1636, 2018.

[15] S. Suknev, "Nonlocal criteria for brittle and quasi-brittle fracture of geomaterials and rocks", E3S Web Conf., vol. 56, 02003, 2018.

[16] S.V. Suknev, "Experimental substantiation of nonlocal failure criteria for geomaterial plates with a circular hole under nonequicomponent compression" J. Min. Sci., vol. 51, no. 3, pp. 435–441, 2015.