

# Stressed Deformed Condition of Base Foundation in the Model of Hard-Linear-Deformable Soil Ground

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**Abstract** –The article justifies the need to take into account the structural strength of soils in the calculations of the base foundations yielding, as well as limiting the depth of the compressible thickness of the base by the value of the structural soil strength. The concept of the depth of the compressible thickness of the base, according to the authors of the article, is a necessary condition for the creation of a model of a linear deformable soil base. A hard-linear deformable model of the soil environment is proposed, within the framework of which the boundary-value problem is solved. Analytic functions of a complex variable for the Kolosov- Muskhelishvili equations are found. Based on the model of a hard-linear deformable soil, the analytical studies of the effect of the structural strength of soils on the value of base foundation have are conducted, and the calculation procedure is given. For example, selected typical soils of the Rostov region with characteristic physic and mechanical properties are studied. In the course of the solution of the problem, the dependence of precipitation of the base foundation on the structural strength of the soil was obtained. The obtained results are analyzed, conclusions are made.

**Keywords** – base yielding; structural strength; compressible depth; hard-linear deformable soil model.

## I. INTRODUCTION

Calculating the yielding, the impact of the tape-embedded foundation on the base is replaced by a strip load with a preload (Fig. 1). When choosing a linear deformable soil model, national regulatory documents allow using linear-elastic solutions to find stresses.

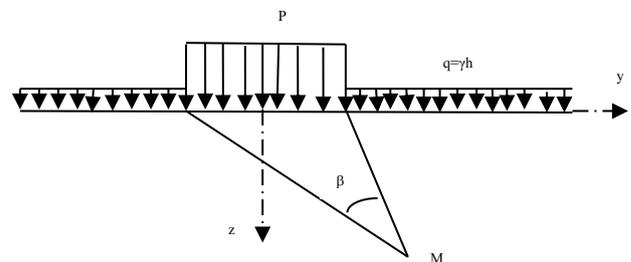


Fig. 1. Linear-elastic additional stresses at point M depend only on the angle of visibility

The main stresses from the strip load with the preload are determined by the existing formulas:

$$\begin{cases} \sigma_3 = -\frac{P-q}{\pi}(\beta + \sin\beta) - q \\ \sigma_1 = -\frac{P-q}{\pi}(\beta - \sin\beta) - q \end{cases} \quad (1)$$

The formulas (1) show that compressive stresses are considered to be negative. In order (1) to fully describe the stress state of the base, it is necessary to point out that the greatest absolute voltage is directed along the bisector of the angle of visibility.

From Hooke law it follows that:

$$\varepsilon_3 = \frac{1}{E}[\sigma_3 - \mu(\sigma_1 + \sigma_2)] \quad (2)$$

Given that the expression (2) can be rewritten as follows

$$\varepsilon_3 = \frac{1+\mu}{E} [(1-\mu)\sigma_3 - \mu\sigma_1]. \quad (3)$$

Substituting the main stresses from (1) into (3), we get

$$\varepsilon_3 = \frac{1+\mu}{E} \left\{ \frac{p-q}{\pi} [(2\mu-1)\beta + \sin\beta] + (2\mu-1)q \right\}. \quad (4)$$

On the axis of OZ symmetry, therefore, finding the yielding of the base is reduced to the integration of the vertical deformation along z.

Since the deformations from the preload q (from the burial) “have already occurred”, the vertical deformations should be calculated using the formula

$$\varepsilon_z = \frac{1+\mu}{E} \left\{ \frac{p-q}{\pi} [(2\mu-1)\beta + \sin\beta] \right\} \quad (5)$$

However, the integral of the deformations diverges. (5)

$$\int_0^{+\infty} \varepsilon_z dz \quad (6)$$

Hence, the concept of depth of the compressible thickness is a necessary condition for constructing a model of a linear deformable base. The value of yielding strongly depends on the purpose of the depth of the compressible strata. [1,2,3]

Moreover, by arbitrarily choosing the depth of the compressible stratum, a yielding of any size can be obtained.

Note 1.

In the standard method of layer-by-layer summation, it is assumed that each layer of soil is under compression and its deformation is determined by the simplified formula

$$\varepsilon_z = \frac{0.8}{E} \sigma_z. \quad (7)$$

However, even for such deformations, integral (6) diverges. Consequently, in the normative method it is impossible to manage without the concept “depth of the compressible sequence”.

Note 2.

The sums of layered yielding in the regulatory method are integral sums aspiring to the integral

$$\int_0^H \varepsilon_z dz, \quad (8)$$

where H – the depth of compressible thickness.

## II. METHODS AND MATERIALS

In the guidelines, the lower limit of the compressible thickness is chosen at such a depth, where the additional stresses from the strip load are 0.2 of the household stresses in the ground.

The authors of the article believe that it is physically more reasonable to choose the boundary of the compressible zone based on the value of the structural strength of the soil (if, of course, the soil has structural strength).

In other words, the base area, where the stresses do not exceed the values of structural strength, should be considered hard. Thus, the authors obtain the model of a hard-linear deformable soil (Fig. 2).

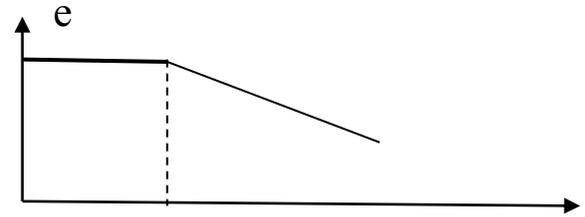


Fig. 2. Hard-linear deformable soil model

With active loading of the base from a hard-linear deformable soil with a strip load, the base remains undeformable until the structural strength function reaches a certain value, i.e.

$$f_{str}(\sigma_1, \sigma_3) = p_{str}. \quad (9)$$

In this case, in an undeformable base, there exists a free stress field satisfying the conditions on the surface and the condition

Including the stress field (1)

Further increase in the load will lead to the appearance of an area in which the ground will be linearly deformed (Fig. 3). At the boundary of this area, the displacements are zero. Since, according to equation (1), the main stresses are constant along a circle defined by a given viewing angle, then the boundary of the deformation region will be one of such circles.

The study of the condition for the violation of structural strength (9) is a difficult and ambiguous task, the solution of which depends on various hypotheses.

For a hard-linear deformable soil, it is possible to find the visibility angle, which determines the area of deformation, from the equation

$$\sigma_z = P_{str} \quad (10)$$

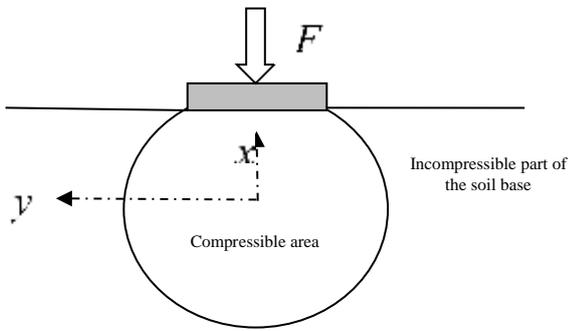


Fig. 3. On the formulation of the problem within the model of a hard linear deformable soil

On the border of areas of different hardness, the condition of continuity of displacements is set.

Deformations in the ground flow through time. The deformations within the selected model will be inside a circle passing through the edges of the foundation at the time of application of the load at the base - an elastic field, in which, under any condition of structural strength.

If the deformations from stresses that do not cause a violation of structural strength can be neglected, then based on the model of a hard linear deformable soil, it is possible to conduct analytical studies of the effect of the structural strength of soils on the value of yielding.

The question of the form of the condition of structural strength destruction  $f_{cmp}(\sigma_1, \sigma_2) = 0$  requires the same study like ordinary strength conditions. This article uses the condition  $|\sigma_3| = p_{cmp}$ .

**Стр - стр**

In the studied case, the plane deformation in a compressible region is determined by two analytical functions of the complex variable  $\varphi(z)$ ,  $\psi(z)$ .

Using the system of basic equations of the flat theory of elasticity, G.V. Kolosov obtained formulas (11) and (12). [4] According to these formulas, voltage components are expressed through functions  $\varphi(z)$ ,  $\psi(z)$ . [5]

$$\sigma_x + \sigma_y = 2\{\varphi'(z) + \overline{\varphi'(z)}\} = 4 \operatorname{Re} \varphi'(z), \quad (11)$$

$$\sigma_y - \sigma_x + i\tau_{xy} = 2\{z\varphi''(z) + \psi'(z)\} \quad (12)$$

The analytical functions themselves are determined from a boundary value problem, which can be of two kinds. If the effect of the foundation on the base is replaced by a strip load, then a mixed task is obtained, since at the boundary the compressible and incompressible parts of the base of the movement are zero. If along the base of the foundation the vertical displacement  $\delta$  is set, then the second main boundary problem of the flat theory of elasticity is obtained (13), which will be considered in the future.

$$\chi\varphi(t) - t\overline{\varphi'(t)} - \overline{\psi(t)} = 2\mu(u + iv), \quad (13)$$

where  $t$  – boundary point of the compressible area;  $u$ ,  $v$  – specified moving components on the border;

$\mu = \frac{E}{2(1+\nu)}$ ,  $\chi = 3 - 4\nu$ , where  $E$  – modulus of soil deformation;

$\nu$  – soil Poisson ratio. [6, 7]

The authors denote several values (Figure 4).

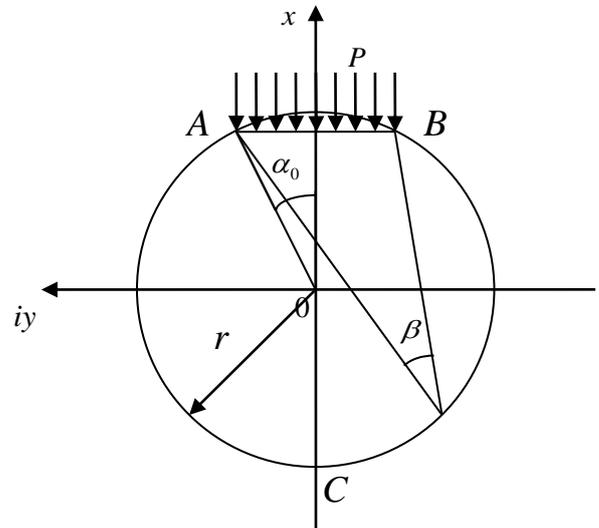


Fig. 4. Several values

The solution area is a circle of radius  $r$ . Chord  $AB$  – foundation base with width  $b$ , perpendicular to the axis  $Ox$ . Angle  $\alpha_0$  – fixed angle between the axis  $Ox$  and right  $OA$ . In addition  $\alpha_0 = \beta$ , where  $\beta$  – internal angle of circle, chorded on  $AB$ . The area under the chord is the studied area of the compressible stratum [8]. In order to simplify the chord  $AB$  may be replaced by  $\overset{\frown}{AB}$ .

On the arc  $\overset{\frown}{ACB}$  of movement  $u + iv \equiv 0$ . On the arc  $\overset{\frown}{AB}$  in virtue of the roughness of the base of the base  $v = 0$ , but due to the hardness of the foundation  $u = \delta$ . Within the interval  $-\pi < \alpha < \pi$  value  $u$  sets out to Fourier series:

$$u(t) = \frac{\alpha_0 \cdot \delta}{\pi} + \frac{\delta}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\alpha_0)}{n} \left[ \left(\frac{t}{r}\right)^n + \left(\frac{r}{t}\right)^n \right] \quad (14)$$

The second main boundary problem is defined. The functions are represented by power series the coefficients of which are determined using equation (14) and are equal to:

$$\varphi(t) = a_0 + \frac{\sin(\alpha_0)}{R(\chi-1)}t + \frac{h}{\pi} \sum_{n=2}^{\infty} \frac{\sin(n\alpha_0)}{n \cdot R^n \cdot \chi} t^n, \quad (15)$$

$$\psi(t) = b_0 - \sum_{n=1}^{\infty} \frac{h}{\pi \cdot R^n} \left( \frac{\sin(n\alpha_0)}{n} + \frac{\sin((n+2)\alpha_0)}{\chi} \right) t^n, \quad (16)$$

where  $h = 2\mu\delta$ .

From formulas (11), (12) the voltage component is expressed:

$$\sigma_x(t) = 2 \operatorname{Re} \varphi'(t) - \operatorname{Re}(\bar{t} \cdot \varphi''(t)) - \operatorname{Re} \psi'(t), \quad (17)$$

Substituting the functions (15) and (16) into the resulting formula, the final formula necessary for finding the yielding of the foundation is obtained.

The method of calculating the yielding of the foundation with regard to structural strength is based on formula (17). With its help the value of the average pressure under the base of the foundation is found (18):

$$\sigma_m = \frac{\int_{-\alpha_0}^{\alpha_0} \sigma_x(t) dt}{b}, \quad (18)$$

where  $b$  – foundation width. The radius of the study area and the angle  $\alpha_0$  is found on the formulas (19) and (20), taking into account  $\sigma_x = \sigma_{bld}$ .

$$r = \frac{b}{2 \cdot \sin(\beta)}, \quad (19)$$

$$\sigma_x = \frac{P}{\pi} (\beta + \sin(\beta)), \quad (20)$$

where  $b$  – foundation width;  $P$  – foundation base stress;  $\beta$  – visibility angle, while taking into account that  $\alpha_0 = \beta$ .

The value of yielding is the value at which the average pressure under the base of the foundation is as close as possible to the calculated resistance of the soil, but does not exceed it. [9, 10].

### III. RESULTS

For typical clayey soils of the Rostov region, the physic and mechanical properties of soils, structural strength and analytical calculations were determined on the basis of Civil Engineering Department of Platov South-Russian State Polytechnic University.

Calculation procedure:

1) Determination of calculated resistance R;

2) Assignment of p to R;

3) Selection of the yielding value that  $sr$  is equal to p;

4) If the yielding is less than permissible, then the calculation is completed, otherwise p decreases.

The results of calculations obtained using the MathCad package are shown in Figure 5:

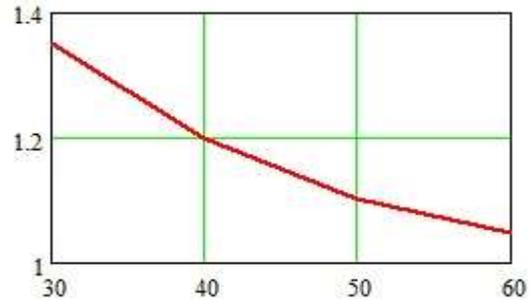


Fig. 5. Dependence of the yielding of the base of the clayey soil from the structural strength of the soil

### IV. CONCLUSION

The results of the calculations are close to the results of laboratory tests, which make it possible to judge the correctness of the obtained method. In the future, this method will be thoroughly tested in laboratory tests and, if necessary, refined. In addition, this method, after a more detailed study, can be applied to conditions of a heterogeneous medium.

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