

# Absolute Permeability and Distribution of Pore Throats of the Productive Strata of Western Siberia

Akhmetov R.T.

Department of Oil and Gas Field Exploration  
and Development

Ufa State Petroleum Technological University,  
Branch of the University in the City of Oktyabrsky  
Oktyabrsky, Republic of Bashkortostan, Russia

Kuleshova L.S.

Department of Oil and Gas Field Exploration  
and Development

Ufa State Petroleum Technological University,  
Branch of the University in the City of Oktyabrsky  
Oktyabrsky, Republic of Bashkortostan, Russia  
markl212@mail.ru

Mukhametshin V.V.

Department of Oil and Gas & Oil Field Development and Operation

Ufa State Petroleum Technological University

Ufa, Republic of Bashkortostan, Russia

vv@of.ugntu.ru

**Abstract** — The paper deals with the issues of constructing the distribution of pore throats by size for productive strata of Western Siberia at a known value of the coefficient of absolute permeability. It is shown that for this purpose it is possible to use the Brooks- Corey model, which allows approximating the entire set of capillary curves obtained in the laboratory conditions for a particular productive stratum. It is noted that it is possible to estimate parameters of the approximation model with the accuracy sufficient for practical purposes, at known value of absolute permeability of a layer. We obtained analytical expressions that allow us to move from the parameters of the approximation model to the parameters of the distribution of pore channels and sizes. The examples of distribution density for different values of absolute permeability are presented.

**Keywords** — pore throats; absolute permeability; density of pore throats distribution; capillarimetric studies; approximation models.

## I. INTRODUCTION

It is known that the distribution of the pore throats of the reservoir by size can be constructed according to the data of capillarimetric studies of core samples from the productive stratum.

The capillarimetric method of studying the structure of the hollow space is based on the statement that the water is squeezed out of the pores of a certain size when exposed to a water-saturated sample of a certain external pressure (e.g. air in a capillarimeter or in centrifuge).

Since the hollow space of rocks is represented by a family of capillaries of different sizes, in the beginning, the water is squeezed out of the capillaries of the largest cross-section at the lowest pressures acting on the sample, and then, as the capillary pressure increases, a wider coverage of the capillary family is carried out, down to the smallest ones filled with residual water. The result of the experiment to study capillary properties of the

samples is an empirical dependence of water saturation of core samples on capillary pressure.

Based on this dependence, we determine the distribution of pores by size and the proportion of pores in the filtering of pores of different sizes.

The information on the density of pore throats distribution is particularly important when predicting the relative permeability of the reservoir beds, as oil and water flow occurs through various filtration throats during waterflooding. In hydrophilic formations, for example, oil flows through the largest channels, and water flows predominantly through small channels.

## II. METHODS AND MATERIALS

In the given work it is shown that distribution of pore throats of reservoirs of Western Siberia is completely defined by parameters of the approximation model of the capillary curves offered by researchers of Brooks and Corey.

This paper uses the data of capillarimetric studies of core samples from the productive stratum BV<sub>6</sub> of Las-Yeganskoye field.

The parameters of the approximation model and moments of pore throats size distribution are received by means of statistical processing of the data of experimental research of capillary pressure curves.

## III. RESULTS

The size of the pore throats and their distribution is an important characteristic of the hollow space of reservoir rocks.

The function of distribution of pore throats by size not only gives an idea about the size of the throats and their number, but also allows to talk about the degree of heterogeneity of the hollow space.

In practice, to determine the distribution function of pore throats, capillarimetric studies of core samples are carried out.

To describe the structure of the hollow space, the type of pore throat function by sizes should be selected in such a way as to approximate the capillary pressure curves with sufficient accuracy.

According to foreign researchers, the capillary pressure curves obtained in the laboratory on a large collection of samples of reservoir rocks have the following features:

1. The dependence of the logarithm of capillary pressure on the logarithm of relative water saturation of the hollow space has a linear character.

It is important to note that the relative water saturation ( $\bar{K}_w$ ) represents the proportion of moving water in the effective volume of hollow space:

$$\bar{K}_w = \frac{K_w - K_{rw}}{1 - K_{rw}}, \quad (1)$$

Here  $K_w$  is total water saturation of the hollow space;  $K_{rw}$  is residual water saturation.

Relative water saturation varies from one at 100% water saturation to zero at maximum saturation of the hollow space at non-wetting phase.

The linear relationship  $\lg P_k = f(\lg \bar{K}_w)$  is broken only at the values  $\bar{K}_w$ , which are close to one. It follows from the above that in a wide range of changes in water saturation relationship  $P_k = f(\bar{K}_w)$  has a degree character.

2. The slope angle of the graph  $P_k = f(\bar{K}_w)$  in the logarithmic coordinate system characterizes the degree of heterogeneity of the hollow space (dispersion of the distribution). The smaller the angle between the straight line and the pressure axis, the greater the heterogeneity of the rock and vice versa.

At zero dispersion (ideally homogeneous structure) the capillary pressure curve is perpendicular to the capillary pressure axis.

3. In case of the equal heterogeneity of the hollow space structure, the change in the average size of the pore throats causes a curve shift in the direction of the capillary pressure axis.

The smaller the average radius, the higher the capillary pressure curve and vice versa.

In addition, the distribution of pores by size is often asymmetrical and, it is obviously defined only for positive values of pore throats radii.

Analysis shows that all of the above conditions correspond to the gamma distribution defined by the following formula for density distribution  $V(\rho)$ :

$$V(\rho) = \frac{\lambda^\alpha}{\Gamma(\alpha)} \rho^{\alpha-1} e^{-\lambda\rho}, \quad (2)$$

where  $\alpha$  and  $\lambda$  are the distribution parameters,  $\Gamma(\alpha)$  is a gamma function.

It is known that this distribution well describes different permeability combinations, besides, this function is universal, it covers an extremely wide range of heterogeneity values: from zero to infinitely large. This distribution has a relatively simple form and is convenient for many mathematical operations.

The average radius ( $\bar{\rho}$ ) and dispersion (D) of the gamma distribution are expressed through its parameters by simple formulas

$$\bar{\rho} = \frac{\alpha}{\lambda}; \quad D = \frac{\alpha}{\lambda^2}; \quad (3)$$

Gamma distribution coefficient of variation square  $W^2 = \alpha$ . The parameter  $\alpha$  mainly determines the left curve, and the parameter  $\lambda$  covers the right curve of the distribution.

The maximum distribution density corresponds to the radius

$$\rho_0 = \frac{\alpha - 1}{\lambda}. \quad (4)$$

Figure 1 shows three calculated graphs of gamma distribution density corresponding to reservoirs with the same average pore throats radius but differing in dispersion, i.e. in the degree of heterogeneity.

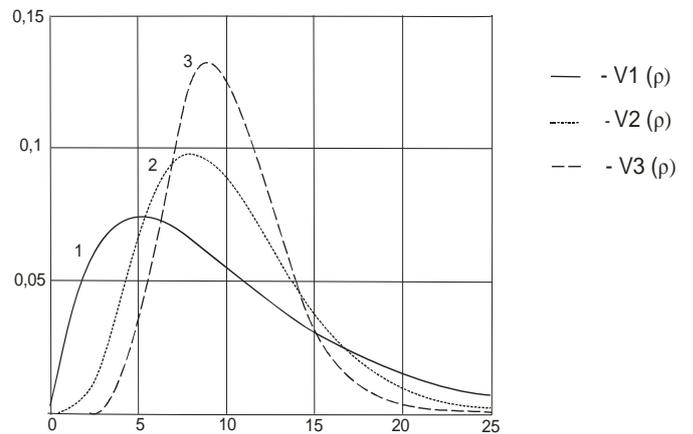


Fig. 1. Gamma distribution differential curves: — -  $D=50 \mu\text{m}^2$ ;  $R_{av}=10 \mu\text{m}$ ;  $\alpha=2$ ; ..... -  $D=20 \mu\text{m}^2$ ;  $R_{av}=10 \mu\text{m}$ ;  $\alpha=5$ ; - - - -  $D=10 \mu\text{m}^2$ ;  $R_{av}=10 \mu\text{m}$ ;  $\alpha=10$

Comparison of calculated differential gamma distribution curves with pore throats distribution curves of real reservoir rocks shows that they are almost identical.

The paper [2] presents the results of visual analysis of microphotographs of the pore volume of sandstones.

According to A.E. Scheidegger, microphotographs give very good results when determining porosity and refer to the direct optical method of studying porometric characteristics.

The authors note that any equipotential section of the pore space can be considered as a multi-linked area that can be divided into many single-linked areas. After single-linked areas have been identified in microphotographs, the hydraulic radius value is calculated for each area as the ratio of the area of the single-linked area to its perimeter.

Further, statistical processing was carried out, which consisted of determining the average value of the hydraulic radius and choosing the probability law of its distribution.

As the analysis of the results of statistical processing has shown, both statistical distributions are well consistent with the known theoretical gamma distribution. The compliance assessment of the theoretical and experimental distribution with A.N. Kolmogorov test has shown that there is an exceptional convergence in both cases. The authors come to the similar conclusion and note that gamma distribution is the type of theoretical distribution that most fully describes the degree and specificity of microinhomogeneity of the porous medium.

As a result, in most cases, the gamma distribution allows us to describe the density of the pore throats of real reservoir rocks with an accuracy sufficient for practical purposes.

To analyze the structure of the hollow space of the productive stratum, it is necessary to have generalized analytical links between the reservoir features and the results of capillary core research. There is a large number of models approximating the entire set of capillary curves [1-5]. However, the Brooks-Corey model is the most widely used; it allows to get the best possible convergence with the experimental data.

$$K_w = K_{rw} + (1 - K_{rw}) \cdot \left( \frac{P_H}{P_K} \right)^\alpha \quad (5)$$

where  $K_w$  is water saturation;  $K_{rw}$  is residual water saturation;  $P_H$  is original (upstream) capillary pressure;  $P_K$  is capillary pressure;  $\alpha$  is a curve factor (steepness) of capillary curves.

The formula should be rewritten as follows:

$$\frac{K_w - K_{rw}}{1 - K_{rw}} = \left( \frac{P_H}{P_K} \right)^\alpha \quad (6)$$

If we have two values of current water saturation for the corresponding values of capillary pressure within the plateau-like area, then the values of parameters  $\alpha$  и  $P_H$  can be calculated by formula (6).

The analysis of the data of capillary research of the BV<sub>6</sub> Las-Yeganskoye field formation has shown that the values of current water saturation of capillary curves at fixed capillary pressure values correlate well with logarithm of absolute permeability coefficient according to the formula:

$$K_w = \ln \left( \frac{m}{K_{pc}^n} \right) \quad (7)$$

where  $m$  and  $n$  are the coefficients that depend only on capillary pressure;  $K_{pc}$  is the absolute permeability coefficient.

Thus, at known value of absolute permeability of rock it is possible to estimate current water saturation for fixed values of capillary pressure with high accuracy.

Further, by substituting the current water saturation values for specific capillary pressure values in expression (6), one can estimate the parameters of the Brooks-Corey model:  $\alpha$  and  $P_c$ .

From the Brooks-Corey model, we should move on to the distribution of pore throats by size. For this purpose, it is necessary to replace the ratio of capillary pressures in the formula (5) with the corresponding ratio of pore throats radii, using the Laplace formula:

$$K_w = K_{rw} + (1 - K_{rw}) \cdot \left( \frac{r}{r_m} \right)^\alpha \quad (8)$$

where  $r$  is a capillary radius of the corresponding pressure  $P_{cp}$ ;  $r_m$  is a maximum radius, corresponding to the original (upstream) capillary pressure  $P_c$ .

Note that formula (6) indicates that

$$G(r) = \left( \frac{r}{r_m} \right)^\alpha, \quad (9)$$

where  $G(r)$  is an integral function of pore throats size distribution of the effective part of the hollow space.

Thus, we have two competing densities of distribution of pore throats of the reservoir rock. Both densities are defined by two parameters. Now it is necessary to find out what is the difference between these distributions.

For this purpose, we should compare the central distribution moments (mean radius and dispersion).

We should calculate the central moments of distribution obtained on the basis of the Brooks-Corey model and equate to them to the corresponding moments of gamma distribution.

$$\frac{\alpha_k}{\alpha_k + 1} \cdot r_m = \frac{\alpha}{\lambda} \quad (10)$$

$$\frac{\alpha_k}{(\alpha_k + 1)^2 \cdot (\alpha_k + 2)} \cdot r_m^2 = \frac{\alpha}{\lambda^2}$$

The result is a system of two equations.

Then we should square both parts of the first equation and divide them into the second equation.

The result is as follows:

$$\alpha = \alpha_k \cdot (\alpha_k + 2) \tag{11}$$

Thus, according to formula (11) the curvature  $\alpha$  of gamma distribution clearly correlates with the curvature  $\alpha_k$  of the Brooks-Corey distribution. Then we should divide the first equation of the system (10) into the second one.

The result is as follows:

$$\lambda = \frac{(\alpha_k + 1) \cdot (\alpha_k + 2)}{r_m} \tag{12}$$

Further, according to the first equation of the system we have

$$r_m = \frac{\alpha_k + 1}{\alpha_k} \cdot \bar{r}_k \tag{13}$$

where  $\bar{r}_k$  is an expectancy of the Brooks-Corey distribution.

If we substitute the expression for  $r_m$  in the formula (12), we will get:

$$\lambda = \frac{\alpha_k \cdot (\alpha_k + 2)}{\bar{r}_k} \tag{14}$$

On the other hand, according to the formula

$$\lambda = \frac{\alpha}{r},$$

hence we finally get

$$\alpha \equiv \alpha_k \cdot (\alpha_k + 2) \quad \bar{r} \equiv \bar{r}_k. \tag{15}$$

Thus, at the level of moments of the first and second order the analyzed distributions are identical, at the same time, their mathematical expectations coincide, and the parameters of curvature are unambiguously interrelated according to the expression (11).

The table of comparison of curvature coefficients is presented below as an example.

TABLE I. THE COMPARISON OF CURVATURE COEFFICIENTS

$\alpha_k$	0.41	0.5	1.0	2.0	3.0
$\alpha$	1.0	1.25	3.0	8	15

The analysis shows that the third order moments of the considered distributions do not coincide anymore. This is due to the fact that the Brooks-Corey distribution abruptly cuts off and turns to zero at the maximum radius value, while the right curve of the gamma distribution graph comes out to the asymptote. The integral distribution curves act the same way.

It is important to note that the integral distribution curves of the pore throats sizes of real reservoir rocks, obtained in laboratory conditions at large radius values are also asymptotically close to one.

In this sense, the gamma function describes more accurately the pore throats distribution density of real reservoirs.

However, the gamma distribution integral is not expressed in elementary functions. This creates inconveniences and serious difficulties in the analysis.

At the same time, the distribution obtained on the basis of the Brooks-Corey model is simple and is therefore preferable for analytical calculations.

We should move on to the density of distribution. For this purpose, we transform the formula (6) to the following form:

$$K_w = K_{rw} + (1 - K_{rw}) \cdot \int_0^r \frac{\alpha}{r_m^\alpha} \cdot r^{\alpha-1} dr \tag{15}$$

Thus, for the density of pore throats distribution by size  $g(r)$ , we obtain the following formula [6-8]:

$$g(r) = \frac{\alpha}{r_m} \left( \frac{r}{r_m} \right)^{\alpha-1} \tag{16}$$

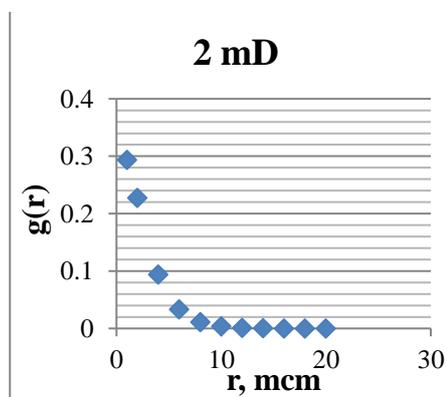


Fig. 2. Distribution density of pore throats at the absolute permeability value of 2 mD

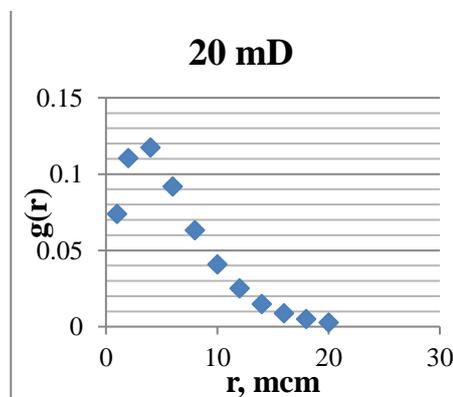


Fig. 3. Distribution density of pore throats at the absolute permeability value of 20 mD

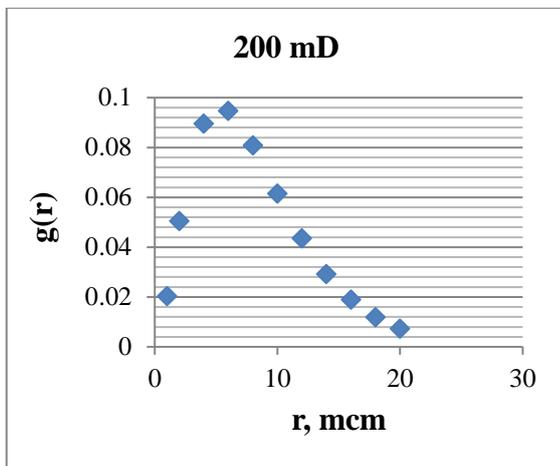


Fig. 4. Distribution density of pore throats at the absolute permeability value of 200 mD

#### IV. CONCLUSION

The absolute permeability of the reservoir bed allows us to estimate the parameters of the Brooks-Corey approximation model.

The parameters of the approximation model of capillary curves are unambiguously related to the density of distribution of pore throats by size. This fact makes it possible to estimate the distribution of pore throats by size by the value of the absolute permeability.

#### References

- [1] R.H. Brooks, A.T. Corey, "Hydraulic Properties of Porous Media", Colorado State University Hydrology, no. 3, 1964.
- [2] S.J. Adams, R.J. Van den Oord, Capillary Pressure and Saturation-Height Functions, Report EP 93-0001 SIPM BV, 1993.
- [3] D. Tiab, E.C. Donaldson, Petrophysics: Theory and Practice of Measuring Reservoir Rock and Fluid Transport Properties. Houston, TX: Gulf Publishing, 1999, 608 p.
- [4] Inc. Baker Hughes, Introduction to Wireline Log Analysis (Baker Atlas), 1995.
- [5] Schumberger. Log Interpretation Principles/Applications, 1989.
- [6] R.T. Akhmetov, V.V. Mukhametshin, "Range of application of the Brooks-Corey model for approximation of capillary curves in reservoirs of Western Siberia", Advances in Engineering Research (AER), vol. 157, pp. 5–8, 2018 [Proceedings of the International Conference "Actual Issues of Mechanical Engineering" (AIME 2018)]. DOI: 10.2991/aime-18.2018.2
- [7] R.T. Akhmetov, A.V. Andreev, V.V. Mukhametshin, "Residual oil saturation and the displacement factor prediction methodology based on geophysical studies data to evaluate efficiency of nanotechnologies application", Nanotechnologies in Construction, vol. 9, no. 5, pp. 116–133, 2017. DOI: 10.15828/2075-8545-2017-9-5-116-133.
- [8] R.T. Akhmetov, V.V. Mukhametshin, A.V. Andreev, Sh.Kh. Sultanov, "Some Testing Results of Productive Strata Wettability Index Forecasting Technique", SOCAR Proceedings, no. 4, pp. 83–87, 2017. DOI: 10.5510/OGP20170400334.
- [9] R.F. Yakupov, V.Sh. Mukhametshin, K.T. Tynchero, "Filtration model of oil coning in a bottom water-drive reservoir", Periodico Tche Quimica, vol. 15, no. 30, pp. 725–733, 2018.
- [10] Yu.V. Zeigman, V.Sh. Mukhametshin, V.V. Sergeev, F.S. Kinzyabaev, "Experimental study of viscosity properties of emulsion system with SiO<sub>2</sub> nanoparticles", Nanotechnologies in Construction, vol. 9, no. 2, pp. 16–38, 2018. DOI: 10.15828/2075-8545-2017-9-2-16-38.
- [11] V.V. Mukhametshin, "Rationale for trends in increasing oil reserves depletion in Western Siberia cretaceous deposits based on targets identification", Bulletin of the Tomsk Polytechnic University, Geo Assets Engineering, vol. 329, no. 5, pp. 117–124, 2018.