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Creating a Local Quasigeoid Model for the Territory of Vietnam Using the Global Model EGM2008

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***Abstract* – The article presents the results of studies on the development of methods for obtaining normal heights in Vietnam using the global model EGM2008 and software for processing GNSS measurements. Errors between normal heights calculated from the global geoid model EGM2008 and heights determined from GNSS measurements and geometric leveling to the territory of Vietnam are determined. The method of localization of the global geoid model EGM2008 for the territory of Vietnam is proposed, such a geoid model is practically formed as an EGM2008_ TN file. Test tests of the developed technique and the localized geoid model EGM2008__TN were carried out, showing the possibility of its application for obtaining normal heights from GNSS measurements in Vietnam with leveling accuracy of class IV.**

Keywords – geoid; quasigeoid; geometric leveling; satellite leveling; normal height; modeling.

I. INTRODUCTION

GNSS technologies make it possible to determine geodetic heights for points on the earth's surface with high accuracy. However, in production, normal heights are used more often, therefore, to go from geodesic to normal heights with an accuracy of geometric leveling, it is necessary to perform a number of transformations based on knowledge of the anomaly of height-difference of the surface of a quasigeoid and ellipsoid. Determining normal heights from satellite leveling favorably differs from geometric leveling in difficult conditions, for example, meteorological, in mountainous marshy areas, under various obstacles, etc. In addition, even without the results of ground-based geometric leveling, we can only determine the height of any point with the help of GNSS measurements, provided the model of height anomalies is available with the required accuracy. In this regard, the actual task is to determine the normal heights using GNSS-technology without laying leveling moves.

There are various global models of geoid EGM2008, EIGEN-6C4, GECO, GAO-2012. The highest degree of decomposition of the geopotential (2190-degree) is the model of the National Geospatial-Intelligence Agency (NGA) National Geospatial-Intelligence Agency (NGA) EGM2008 with a grid size of 1x1 or 2.5x2.5 minutes. Due to the high level of detail, it is EGM2008 that is widely used in production. However, in mountainous areas, it is necessary to refine this model, since in some cases the result obtained from the results of satellite leveling using the model of anomalies of altitudes does not coincide with the results of a more reliable geometric leveling.

II. METHODS AND MATERIALS

To create a local quasi-geoid model, more accurate than the global model (EGM2008), it is necessary to use a set of points with known normal altitudes, at which satellite observations were made and geodetic heights were determined. [2, 5-7, 15]. In this case, the constructed model of the local quasigeoid (surface) is applicable only for the local territory.

The heights are interconnected by an expression (fig. 1):

$$H = H^g + N = H^\gamma + \zeta, \tag{1}$$

where: H is the geodesic height, Hg is the orthometric (absolute) height (correlated with the geoid), H is the normal height (correlated with the quasigeoid), aн is the height anomaly, N is the height of the geoid.

According to [13], at present there are two methods for constructing a model of anomaly height: residual and wave. The most common and simpler is the residual method based on interpolating the height anomaly according to the EGM2008 model. It has a higher accuracy [4, 8–11, 13, 14], especially at low density of GNSS-leveling points. The residual method gives the best model of anomalies of heights on a local scale (in a small area). This method can be considered as a method for adjusting the EGM2008 model in accordance with the actual values of the height anomalies.

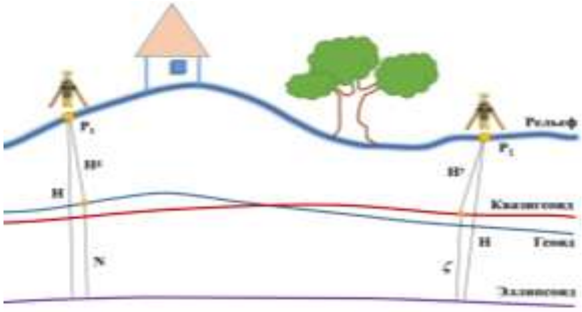

Fig. 1. Diagram of heights

Knowing the exact model of the height anomaly ζ, it is possible to transform the geodetic height determined by GNSS technology to the normal height. To achieve this goal, a model of anomalies of heights необходима with an accuracy of 1 cm is required, to determine the height with a leveling accuracy of class III or IV [12,16-17].

The wave method is the construction of a local model of height anomalies without improving the model EGM2008. The local model of elevation anomalies is based on interpolation for calculating the value of ζ at an arbitrary point over a set of GNSS points with known normal heights and leveling the heights of the GNSS points leveling. The wave method is often used when there is a fairly high density of GNSS measurement points at which normal heights are known.

The EGM2008 model has now been significantly improved, and it can be used in conjunction with GNSS measurement data to build a local model of elevation anomalies for the territories of Vietnam.

Equalization algorithm combining the geodesic height, normal height and height anomaly

For network points, using the values of geodetic height, normal height, and height anomaly, we can calculate the vector of deviations of $\Delta\zeta_i$ using the formula:

$$\Delta\zeta_i = H_i - h_i - \zeta_i^{2008} = \zeta_i^{ГНСС/Нив} - \zeta_i^{2008} \qquad (2)$$

and create equation equations for equalization:

$$\Delta\zeta_i = f(\phi_i, \lambda_i) + v_i = a_i^T x_i + v_i \qquad (3)$$

Elements of matrix A and unknowns, dependent on the model of parameters, are chosen to describe the difference between the three elements of height. In equations (2) and (3), H_i, h_i, ζ_i^{2008} - respectively, the geodesic height, normal height, and height anomaly (obtained from model EGM2008) correspond to point i, and i, и $H_i - h_i = \zeta_i^{ГНСС/Нив} -$ is the height anomaly of GNSS leveling.

To assess the accuracy, 5 types of models are possible: including 4 parameters, 5 parameters, a polynomial of 1 degree, a polynomial of 2 degree and a polynomial of 3 degree [3,14].

In general

$$a_i^T x_i = \sum_{m=0}^{M}\sum_{n=0}^{N} x_q (\varphi_i - \varphi_0)^n (\lambda_i - \varphi_0)^m \cos^m \varphi_i \qquad (4)$$

In matrix form, the system of measurement equations and its solution are written as follows:

$$L = Ax + V \qquad (5)$$

$$x = -(A^T PA)^{-1} A^T PL = -(A^T A)^{-1} A^T L \qquad (6)$$

where P is the weight matrix (the inverse dispersion matrix is the covariance matrix of measurements). Suppose that measurements and their corresponding errors do not correlate between heights and between themselves in the same measurement system, that is, they are independent. On the other hand, we also assume that the heights are measured with the same accuracy, then the weight matrix becomes the identity matrix.

In addition to the above model, geodesy also widely uses the interpolation method based on spline functions [1, 3]. Spline interpolation is commonly used in the wave method to build a local model of anomalies.

Consider both cases with and without the use of the global model EGM2008. The anomaly of the height of a point is interpolated by points using the Spline function, using the following formula

$$\Delta\zeta\left[P(x,y)\right] = \sum_{i=1}^{n} a_i r_{PP_i}^2 \ln(r_{PP_i}^2) + \tau_1 + \tau_2 x + \tau_3 y, \qquad (7)$$

where: $r_{PP_i} = \sqrt{(x - x_i)^2 + (y - y_i)^2}$, $a_i (i = 1 \div n); \tau_1, \tau_2, \tau_3 -$ desired parameters of the system of equations:

$$\left[\begin{array}{cccc|ccc} 0 & g_{1,2} & \dots & g_{1,n} & 1 & x_1 & y_1 \\ g_{2,1} & 0 & \dots & g_{2,n} & 1 & x_2 & y_2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ g_{n,1} & g_{n,2} & \dots & 0 & 1 & x_n & y_n \\ \hline 1 & 1 & \dots & 1 & 0 & 0 & 0 \\ x_1 & x_2 & \dots & x_n & 0 & 0 & 0 \\ y_1 & y_2 & \dots & y_n & 0 & 0 & 0 \end{array}\right] \times \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} \Delta\zeta_1 \\ \Delta\zeta_2 \\ \dots \\ \Delta\zeta_n \\ 0 \\ 0 \\ 0 \end{bmatrix}, \qquad (8)$$

where

$$g_{i,j} = g_{j,i} = \begin{cases} r_{P_iP_j}^2 \ln(r_{P_iP_j}^2); (i \neq j) \\ 0; (i = j) \end{cases}. \qquad (9)$$

Using the above mathematical models, it is possible to calculate the anomalies of the heights □ζ points along which to go to normal heights according to the results of GNSS positioning:

$$h = H - (\zeta^{2008} + \Delta\zeta) \qquad (10)$$

III. RESULTS

The geodetic network created on the basis of GNSS technology consists of 180 points (GNSS leveling) used for the considered (test) territory of the Central Plateau of Vietnam. The measurements were performed by dual-frequency receivers for at least 6 hours each reception. The GNSS leveling points are distributed relatively uniformly across the provinces from Quang Nam to Lam Dong, Bin Fuok, Ninh Thuan (Fig. 2). In the study, 163 points were used, 17 remaining points were control points for testing the methodology.

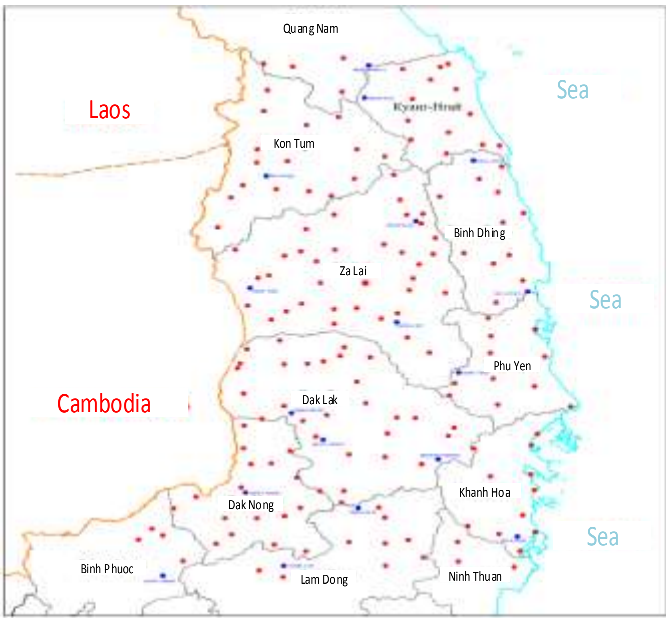

Fig. 2. GNSS leveling network (Central Plateau of Vietnam)

TABLE I. COORDINATES OF POINTS (PART)

№	Имя	B(0)	L(0)	H (м)	h (м)	$\zeta^{THCC/нив}$ (м)	ζ^{2008} (м)	$\Delta\zeta$ (м)
1	III(QK-LT)8	11.693324	107.79628	862.805	861.031	1.774	1.156	0.618
2	III(LT-DT)5	11.740386	107.66505	674.824	674.06	0.764	0.233	0.531
3	I(VL-HT)181	11.748384	109.07071	25.893	20.293	5.600	4.736	0.864
…	…	…	…	…	…	…	…	…
162	I(DN-BMT)28	15.307949	107.73007	552.322	559.686	-7.364	- 8.215	0.851
163	III(BHA-HD)9	15.341790	108.17678	90.384	97.62	-7.236	- 7.928	0.692

The Alltrans EGM2008 Caculator software is designed to calculate the height anomaly of the EGM2008. On this basis, a model of height anomalies with a cell size of 2.5 '× 2.5' is obtained. The program was used to calculate the height anomalies of the above points of the GNSS leveling system (163 points).

The results are shown in the following figures and tables. In Table 2, the mean square error (CCP) and the standard deviation are calculated using the following formula:

$$\mu = \pm\sqrt{\frac{[\Delta_i \Delta_i]}{n}}; \sigma = \pm\sqrt{\frac{[(\Delta_i - \bar{\Delta}\,]}{n-1}}, \qquad (11)$$

where: μ – mean square error; - σ standard deviation; Difference of calculations before and after using the data of the i-th grid point of GNSS leveling; $\bar{\Delta}$ - average deviation; *n* is the number of points.

TABLE II. STATISTICAL RESULTS OF DEVIATIONS OF GNSS-LEVELING ANOMALIES AND MODEL EGM2008

Mathematical model	Max. (м)	Min. (м)	Average (м)	RMS error (м)	Standart deviation (м)
Only EGM2008 model	1.186	0.169	0.818	±0.841	±0.196
4 parameters model	0.365	-0.614	0.000	±0.187	±0.187
5 parameters model	0.336	-0.640	0.000	±0.186	±0.187
Model 1 degree polynomial	0.356	-0.602	0.000	±0.193	±0.194
Model 2 degree polynomial	0.361	-0.523	0.000	±0.178	±0.178
Model 3 degree polynomial	0.428	-0.403	0.000	±0.143	±0.143

From table 2 it follows that a suitable model in this case is a model of a third degree polynomial. In addition to the above, models of a higher degree were investigated. The results of the study showed that in other models the deviations do not decrease, and in some cases even give worse results than the third degree polynomial model.

Based on the aforementioned GNSS leveling network, a local model of the height anomaly in the territory under consideration was created using the wave method using spline interpolation.

In the case without using the EGM2008 model, we use the height anomaly calculated using satellite leveling data using spline functions (8) and (10). The calculation was performed for 163 points of the height anomaly, and then a comparison was made and the accuracy was assessed using 17 control points.

In the case of using the EGM2008 model, the free term is not a directly calculated anomaly. It is the difference in altitude anomalies between the value calculated from GNSS measurements and the anomaly determined by the EGM2008 model ($l=H-h-\zeta^{2008}= \zeta^{THC/Hu\text{в}}-\zeta^{2008}$). The results are shown in table 2.

TABLE III. INTERPOLATION RESULTS USING THE SPLINE FUNCTION (($\zeta^{SLPLINE\_1}$) WITHOUT EMG2008 MODEL DATA (WAVE METHOD)

№	Name	*H*	*h*	$\zeta^{THCC/Niv} = H - h$	ζ^{Spline_1}	$\Delta\zeta = \zeta^{THCC/Hu\text{в}} - \zeta^{Spline\_1}$
1	I(BMT-APD)12	907.678	907.976	-0.298	-0.298	-0.032
2	I(BMT-APD)35	294.751	296.987	-2.236	-2.236	-0.076
3	III(QK-LT)5	899.929	898.242	1.687	1.687	-0.096
...	...	...	...	...	...	...
16	III(HD-BHB)3-1	156.62	163.256	-6.636	-6.636	0.011
17	III(AL-DT)1	33.897	36.866	-2.969	-2.969	0.019
						Min = – 0.143 Max = +0.144

where: ζ^{Spline_1}– interpolation height anomaly based on spline function without using EMG2008; $\zeta^{THCC/Niv}$ – anomaly height GNSS-leveling; ζ^{2008} - EMG2008 model height anomaly.

TABLE IV. INTERPOLATION RESULTS USING SPLINE FUNCTIONS (ζ TN) AND EMG2008 MODELS (WAVE METHOD)

№	Name	$\zeta^{ГНСС/Нив}$	ζ^{2008}	$\delta\zeta^{TN}$	$\zeta^{TN}=\zeta^{2008}+\delta\zeta^{TN}$	$\Delta\zeta=\zeta^{ГНСС/Нив}-\zeta^{TN}$
1	I(BMT-APD)12	-0.298	-1.16	0.847	-0.313	0.015
2	I(BMT-APD)35	-2.236	-2.777	0.622	-2.155	-0.081
3	III(QK-LT)5	1.687	1.043	0.645	1.688	-0.001
...	...	...	...	...	...	...
16	III(HD-BHB)3-1	-6.636	-7.299	0.714	-6.585	-0.051
17	III(AL-DT)1	-2.969	-3.723	0.777	-2.946	-0.023
					Min = Max =	-0.0.12 +0.089

where: ζ^{TN} – interpolation elevation anomaly based on spline function using EMG2008; $\delta\zeta^{TN}$ – height anomaly correction based on spline function for EMG2008 model height anomaly.

Tables 3 and 4 show that even in the case when the EGM2008 model is not used, the accuracy of the interpolation method based on spline functions is much better than that of mathematical models. Based on the above analysis, it can be concluded that it is highly advisable to use the spline function to interpolate height anomalies at points in the local model, which significantly increases the accuracy of determining the normal height.

Building a local quasigeoid model using the EGM 2008 model

With reference to our data (163 points in the Central Plateau), we have the coordinates of $B_1 = 11^0 41'$, $L_1 = 107^0 00'$; northeast - $B_2 = 15^0 21'$, $L_2 = 109^0 25'$. The region size is $\Delta B = 3{,}67^0$, $\Delta L = 2{,}52^0$ with a cell size of 2.5'x 2,5', the number of grid points (N) will be 5251 pcs. (N = 89x59). At each grid point (i) we have 3 values: latitude B_i, longitude L_i and height anomaly (from model EGM2008) ζ_i (i = 1,2 ..., 5251).

The calculation of corrections for grid points is performed using the Spline function (8, 10). Height anomalies in the global model EGM2008 are calculated by the formula: $\zeta^{TN} = \zeta^{2008} + \delta\zeta$, where ζ^{2008} is the anomalous height obtained from the EGM2008 model, $\delta\zeta$ – is the correction of the anomaly obtained from wave interpolation (based on the spline function), ζ^{TN} – is the height anomaly of the production model). The calculation of the height anomalies is shown in the block diagram (Fig. 3).

After calculating the height anomaly at the network points (ζ^{TN}), the quasigeoid model of the central plateau region (and the South Central Coast) has been refined.

Models of the quasigeoid of the Central Plateau and the South Central Coast are created according to the model EGM2008, latitude from 11041 'to 15021', longitude from 107000 'to 109025', the area is about 80,000 km2. There is a model in the grid view, the cell size is 2.5 '× 2.5' (including 5251 points). The data file is described in table 6.

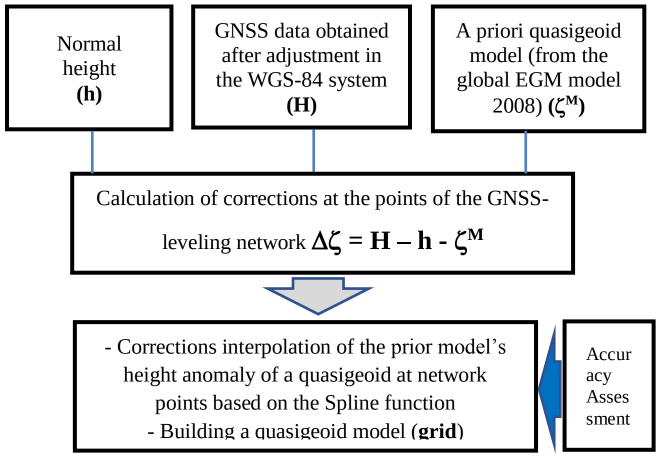

Fig. 3. The scheme for constructing a local quasigeoid model from the data of the global model EGM 2008

TABLE V. HEIGHT ANOMALY AND CORRECTIONS FOR GRID POINTS (MODEL CELLS) (FRAGMENT)

№	Name	$B(^0)$	$L(^0)$	ζ^{2008}(м)	$\delta\zeta$ (м)	ζ^{TN} (м)
1	1	15.3500	107.0000	-12.550	0.711	-11.839
2	2	15.3500	107.0417	-12.132	0.726	-11.406
3	3	15.3500	107.0833	-11.715	0.742	-10.973
…	…	…	…	…	…	…
5250	5250	11.6833	109.3750	5.370	1.314	6.684
5251	5251	11.6833	109.4167	5.519	1.364	6.883
				Min = -12.55 Max = 5.519	Min = 0.031 Max = 1.364	Min = -11.839 Max = 6.883

The cell size in the local model of the quasigeoid EGM2008_TN is 2.5’ × 2.5’, the radius of the circle is from 6 km to 11 km. With such a radius, the number of selected points will be from 4 to 13 cell points. Points in this radius will be used as reference points for interpolating GNSS measurement points.

Evaluating the accuracy of interpolation results

The model has been tested on 17 satellite network points that are not used in modeling. First of all, we need to investigate and select the interpolation algorithm for height anomalies from EGM2008_TN.

According to the data obtained by 17 points, we calculate the differences in height between these points, and then compare them with the tolerances for leveling class III, IV and technical leveling.

In practice, when measuring heights using GNSS technology, the network should have at least one point with a normal height. For this point, using the geoid model, it is necessary to determine the height anomaly Δζ, then calculate the difference Δh using the formula:

$$\Delta h = \Delta H - \Delta\zeta , \qquad (12)$$

where ΔH – the difference of geodetic heights at the point under consideration.

Denote $\Delta H_{i,k}$ as the difference of geodetic heights between two points; ; $\Delta h_{i,k}$ as the difference between normal heights, and $\Delta\zeta_{i,k}$ as the height anomaly determined by the global model (EGM2008) between two points. Calculate the difference between the normal height of 2 points:

$$\delta_{i,k} = \Delta H_{i,k} - \Delta h_{i,k} - \Delta\zeta_{i,k} \qquad (13)$$

Taking into account the weight of measurements of the standard deviation on 1 km can be calculated by the formula (17)

$$m_{км} = \pm\sqrt{\frac{[P\delta_i\delta_i]}{m}} \qquad (14)$$

Weight is calculated by the formula:

$$P_{i,k} = \frac{1}{D_{i,k}} , \qquad (15)$$

where $D_{i,k}$ – distance between two points in kilometers, m – the number of pairs of points to study. In this case, m should not be less than 20, and the distance between points $D_{i,k}$ should have different values. The results are shown in table 6.

TABLE VI. EVALUATION OF THE ACCURACY OF DETERMINING NORMAL HEIGHTS USING THE EGM2008_TN MODEL

№	Factors	EGM 2008	weight average	1st degree polynom.	2nd degree polynom.	Spline-function
1	Number of points	136	136	136	136	136
2	Non-compliance with technical leveling tolerance	9 (7%)	0 (0%)	0 (0%)	0 (0%)	0 (0%)
2	Compliance with technical leveling tolerance	127 (93%)	136 (100%)	136 (100%)	136 (100%)	136 (100%)
3	Compliance with leveling tolerance class IV	108 (79%)	136 (100%)	136 (100%)	136 (100%)	136 (100%)
4	Compliance with leveling tolerance class III	54 (40%)	94(69%)	103 (76%)	108(79%)	115 (85%)
5	СКО (м) – $m_{км}$	0.0244	0.0119	0.0102	0.0096	0.0086

From the obtained interpolation results for 17 control points, it can be seen that all four interpolation methods give acceptable results. In accordance with Table 7, in all four interpolation methods, about 80% of the routes have permissible accuracy for grade III leveling, including for mountainous areas. 100% accuracy for technical leveling is naturally assured. In practice, it is advisable to use the “Weight-average” interpolation method.

IV. CONCLUSION

The developed procedure for calibrating the height anomaly EGM2008 is designed for use on a large scale, having many points in the network. The calculation method is relatively simple and improves the accuracy of determining the height from the GNSS measurement results.

The network points (GNSS points) should coincide with the leveling points of class III and above. Points should be distributed relatively uniformly and fairly densely.

The local model of the quasigeoid EGM2008_TN, built on the basis of the local model EGM2008, has greater accuracy and can be used in various hard-to-reach areas. According to the results of an assessment of accuracy based on 17 test points, the EGM08_TN model showed an accuracy of 100%, corresponding to the leveling of class IV, including for mountainous areas. 85% - corresponds to class III. The error in determining the normal height from GNSS measurements was reduced from 0.0244 m / km to 0.0086 m / km.

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