ATLANTIS PRESS
Atlantis Studies in Uncertainty Modelling, volume 1
11th Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT 2019)

Algorithm for Finding Domination Set in Intuitionistic Fuzzy Graph

A.Bozhenyuk[a] and M.Knyazeva[b] and I.Rozenberg[c]
[a]Department of Information and Analytical Systems of Safety, Southern Federal University, 1, Engels Str., Taganrog, Russia, 347922, avb002@yandex.ru
[b]Department of Information and Analytical Systems of Safety, Southern Federal University, 1, Engels Str., Taganrog, Russia, 347922, margarita.knyazeva@gmail.com
[c]Public corporation "Research and development institute of railway engineers", 27/1, Nizhegorodskaya Str., Moscow, Russia, 109029, i.yarosh@vniias.ru

Abstract

In this paper, the concept of minimal intuitionistic dominating vertex subset of intuitionistic fuzzy graph was considered, and on its basis the notion of a domination set as an invariant of the intuitionistic fuzzy graph was introduced. A method and an algorithm for finding all minimal intuitionistic dominating vertex subset and domination set were proposed. This method is the generalization of Maghout's method for fuzzy graphs. The example of finding the domination set of the intuitionistic fuzzy graph was considered as well.

Keywords: Intuitionistic fuzzy set, Intuitionistic fuzzy graph, Minimal intuitionistic dominating vertex subset, Domination set.

1 Introduction

Currently, science and technology are characterized by complex processes and phenomena for which complete information is not always available. For such cases, mathematical models of various types of systems containing elements of uncertainty have been developed. A large number of these models are based on the expansion of the usual set theory, namely, fuzzy sets. The concept of fuzzy sets was introduced by L. Zadeh [17] as a method of representing uncertainty and fuzziness. Since then, the theory of fuzzy sets has become an area of research in various disciplines.

In 1983, K. Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. He added a new component to the definition of a fuzzy set, which determines the degree of non - memership. Fuzzy sets give the membership degree of an element in a given set (where the degree of non-membership equals one minus the membership degree), while intuitionistic fuzzy sets give both a membership degree and a degree of non-membership that are more or less independent of each other. The only restriction is that the sum of these two degrees does not exceed 1. Intuitionistic fuzzy sets are fuzzy sets of a higher order. Their application makes the solution procedure more complicated, but if the complexity of calculations in time, volume of calculations, or memory can be neglected, then a better result can be achieved.

The theory of fuzzy graphs is finding an increasing number of applications for modeling real-time systems, where the level of information inherent in the system depends on different levels of accuracy. The original definition of a fuzzy graph [7] was based on fuzzy relations by L.Zadeh [18]. In [11], fuzzy analogs of several basic graphical concepts were presented. In [8, 14], the notion of fuzzy graph complement was defined and some operations on fuzzy graphs were studied. The concepts of intuitionistic fuzzy relations and intuitionistic fuzzy graphs were introduced in [12, 13] and some of their properties were investigated. In [6, 10, 15], the concepts of a dominating set, a regular independent set, a domination edge number of edges in intuitionistic fuzzy graphs were considered.

In this paper we introduce the concepts of minimal intuitionistic dominating vertex subsets and domination set in intuitionistic fuzzy graphs. These concepts are a generalization of the minimal dominating vertex subsets of a crisp graph [9] and the domination set of a fuzzy graph [4], respectively.

2 Basic Concepts and Definitions

Definition 1 [17]. Let X be a nonempty set. A fuzzy set drawn A from X is defined as $A = \{\langle \mu_A(x), x\rangle | x \in X\}$, where $\mu_A : X \to [0, 1]$ is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2 [2]. Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\},$$

where the functions $\mu_A(x), \nu_A(x) : X \to [0,1]$ define respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X, and

$$(\forall x \in X)[\mu_A(x) + \nu_A(x) \le 1].$$

Furthermore, value $\pi_A(x) = 1-\mu_A(x)-\nu_A(x)$ is called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of x to the intuitionistic fuzzy set.

The intuitionistic fuzzy relation R on the set $X \times Y$ is an intuitionistic fuzzy set of the form:

$$R = \{\langle (x,y), \mu_R(x,y), \nu_R(x,y)\rangle | (x,y) \in X \times Y\},$$

here $\mu_R : X \times Y \to [0,1]$ and $\nu_R : X \times Y \to [0,1]$.

The intuitionistic fuzzy relation R satisfies the condition:

$$(\forall x, y \in X \times Y)[\mu_R(x,y) + \nu_R(x,y) \le 1].$$

Definition 3. Let p and q be intuitionistic fuzzy variables that have the form: $p = (\mu(p), \nu(p))$, $q = (\mu(q), \nu(q))$, here $\mu(p)+\nu(p) \le 1$, and $\mu(q)+\nu(q) \le 1$. Then the operations & and $\vee$ are defined as [3]:

$$p\&q = (min(\mu(p), \mu(q)), max(\nu(p), \nu(q))), \quad (1)$$
$$p \vee q = (max(\mu(p), \mu(q)), min(\nu(p), \nu(q))). \quad (2)$$

We assume that $p < q$ if $\mu(p) < \mu(q)$ and $\nu(p) > \nu(p)$.

Definition 4 [7]. A fuzzy graph is a triplet $\tilde{G} = (V, \sigma, \mu)$, where V is finite and non-empty vertex set, $\sigma : V \to [0,1]$ is a fuzzy subset of V, and $\mu : V \times V \to [0,1]$ is fuzzy relation on $X \times X$ such that:

$$(\forall x, y \in V)[\mu(x,y) \le min(\sigma(x), \sigma(y))].$$

This definition considers a fuzzy graph as a collection of fuzzy vertices and fuzzy edges. Another version of a fuzzy graph was proposed in [5, 16] as a set of crisp vertices and fuzzy edges:

Definition 5 [16]. A fuzzy graph is a pair $\tilde{G} = (V, R)$, where V is a crisp set of vertices and R is a fuzzy relation on V, in which the elements (edges) connecting the vertices V, have the membership function $\mu_R : V \times V \to [0,1]$.

Such fuzzy graph in [5] was called a fuzzy graph of the *first kind.*

Definition 6 [12, 13]. An intuitionistic fuzzy graph is a pair $\tilde{G} = (A, B)$, where $A = \langle V, \mu_A, \nu_A\rangle$ is an intuitionistic fuzzy set on the set of vertices V, and $B = \langle V \times V, \mu_B, \nu_B\rangle$ is an intuitionistic fuzzy relation such that:

$$\begin{aligned} \mu_B(x,y) &\le min(\mu_A(x), \mu_A(y)), \\ \nu_B(x,y) &\le max(\nu_A(x), \nu_A(y)). \end{aligned} \quad (3)$$

and the following condition is fulfilled:

$$(\forall x, y \in V)[0 \le \mu_B(x,y) + \nu_B(x,y) \le 1].$$

It should be noted that Definition 5 is an extension of a fuzzy graph in the sense of Definition 3, in which the vertices and edges of the graph are considered not as fuzzy, but as intuitionistic sets. In the case of using a fuzzy graph in the sense of Definition 4, such a definition of an intuitionistic fuzzy graph does not make sense, since in the latter case the values $\mu_A(x) = \mu_A(y) = 1$, $\nu_A(x) = \nu_A(y) = 0$, and therefore, the quantity $\nu_B(x,y) = 0$. In this regard, we introduce the following definition:

Definition 7. An intuitionistic fuzzy graph of the first kind is a pair $\tilde{G} = (V, U)$, where V is a crisp set of vertices, $U = \langle V \times V, \mu, \nu\rangle$ is intuitionistic fuzzy relation (intuitionistic fuzzy edges) such that:

$$(\forall x, y \in V)[0 \le \mu(x,y) + \nu(x,y) \le 1].$$

3 Domination Set of Intuitionistic Fuzzy Graph

Let $\tilde{G} = (V, U)$ be an intuitionistic fuzzy graph of the first kind. Let $p(x,y) = (\mu(x,y), \nu(x,y))$ be an intuitionistic fuzzy variable that determines the degree of adjacency and the degree of non-adjacency from vertex x to vertex y. Let X is an arbitrary subset of the vertices set V. For each vertex $y \in V \setminus X$, we define the volume:

$$p_X(y) = \bigvee_{x \in X} p(x,y). \quad (4)$$

Definition 8. We call the set X an *intuitionistic dominating vertex set for vertex y* with the intuitionistic degree of domination $p_X(y)$.

Definition 9. We call the set X an *intuitionistic dominating vertex set* for graph $\tilde{G}$ with the intuitionistic degree of domination:

$$\beta(X) = \underset{y \in V \setminus X}{\&} p_x(y) = \underset{y \in V \setminus X}{\&} \bigvee_{x \in X} p(x,y). \quad (5)$$

The operations & and $\vee$ are defined by (1) and (2) in expressions (4) and (5). Assuming that $(\forall y \in V)[p(y,y) = (1,0)]$, expression (5) can be rewritten as:

$$\beta(X) = \underset{y \in V \setminus X}{\&} p_x(y) = \underset{y \in V}{\&} \bigvee_{x \in X} p(x,y). \quad (6)$$

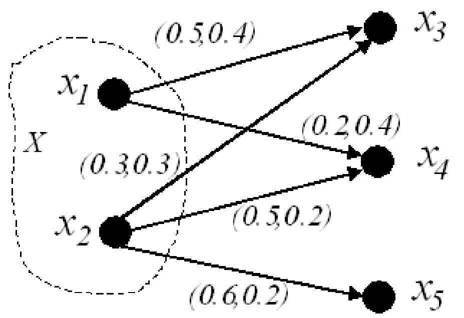

Figure 1: Intuitionistic fuzzy graph, $X = \{x_1, x_2\}$

Intuitionistic degree of domination $\beta(X) = (\mu(X), \nu(X))$ means that there is some vertex in subset $X \subseteq V$ that is adjacent to any other vertex of the graph with degree at least $\mu(X)$, and there is some vertex that is not adjacent to any vertex of the graph with degree to not more than $\nu(X)$.

Example 1. For the intuistic fuzzy graph $\tilde{G} = (V, U)$, and the subset $X = \{x_1, x_2\}$, shown in Figure 1, we define the values $p_X(x_3) = (0.5, 0.3)$, $p_X(x_4) = (0.5, 0.2)$, $p_X(x_5) = (0.6, 0.2)$. Consequently, intuitionistic degree of domination $\beta(X) = (0.5, 0.3)$. *Note 1.* If the graph $\tilde{G}$ is a crisp graph, then the value $p(x, y) = (1, 0)$, if the vertex y is adjacent to the vertex x, and $p(x, y) = (0, 1)$ otherwise.

Note 2. If the graph $\tilde{G}$ is a crisp graph, then the value $\beta(X) = (1, 0)$, if subset $X \subseteq V$ is dominating subset of crisp graph [9], and $\beta(X) = (0, 1)$ otherwise.

Definition 10. We call the subset $X \subseteq V$ a *minimal intuitionistic dominating vertex subset* with the degree $\beta(X)$, if the condition $\beta(X') < \beta(X)$ is true for any subset $X' \subseteq X$.

Example 2. For the intuitionistic fuzzy graph presented in Figure 1, the minimal intuitionistic dominating vertex subsets are $X_1 = \{x_2\}$ with $\beta(X_1) = (0.3, 0.3)$, and $X_2 = \{x_1, x_2\}$ with $\beta(X_2) = (0.5, 0.3)$.

Denote by $Y_k = \{X_{k1}, X_{k2}, ..., X_{kl}\}$ the family of all minimal intuitionistic dominating vertex subsets with k vertices and degrees of domination β_{k1}, β_{k2},..., β_{kl} respectively. Let's $\beta_k^0 = \beta_{k1} \vee \beta_{k2} \vee ... \vee \beta_{kl}$. It means that there is a minimal intuitionistic dominating vertex subset with k vertices with a degree of domination β_k^0 in the graph $\tilde{G}$ and there is no other intuitionistic dominating vertex subset with k vertices whose degree of domination would be greater than β_k^0.

Definition 11. An intuitionistic fuzzy set

$$\tilde{D} = \{\langle\beta_1^0/1\rangle, \langle\beta_2^0/2\rangle, ..., \langle\beta_n^0/n\rangle\}.$$

is called a *domination set* of graph $\tilde{G}$.

The domination set is an invariant of intuitionistic fuzzy graph $\tilde{G}$, since it does not change during the structural transformations of the graph.

Example 3. For the intuitionistic fuzzy graph presented in Figure 1, the domination set is:

$$\tilde{D} = \{\langle(0.3, 0.3)/1\rangle, \langle(0.5, 0.3)/2\rangle,$$

$$\langle(0.5, 0.3)/3\rangle, \langle(0.6, 0.2)/4\rangle, \langle(1, 0)/5\rangle, \}.$$

Note 3. The concept of domination set was introduced for the intuitionistic fuzzy graph of the first kind. However, taking into account inequalities (3), it also holds for the intuitionistic fuzzy graph of a general form (in the sense of Definition 6).

Property 1. For intuitionistic dominating set the following proposition is true:

$$(0, 1) \leq \beta_1^0 \leq \beta_2^0 \leq, ..., \leq \beta_n^0 = (1, 0).$$

4 Method and Algorithm for Finding Domination Set

We will consider the method of finding a family of all minimal intuitionistic dominating vertex subsets. The given method is similar to Maghout's method for the definition of all minimal fuzzy dominating vertex sets [4] for fuzzy graphs.

Let us assume that set X_β is an intuitionistic dominating vertex subset of the graph $\tilde{G}$ with the degree of domination $\beta = (\mu_\beta, \nu_\beta)$. Then for an arbitrary vertex $x_i \in V$, one of the following conditions must be true.

a) $x_i \in X_\beta$;

b) if $x_i \notin X_\beta$, then there is a vertex x_j so that it belongs to set X_β, while the vertex x_j is the adjacent to vertex x_i with the degree $(\mu(x_j, x_i), \nu(x_j, x_i)) \geq \beta$.

In other words, the following statement is true:

$$\begin{aligned}&(\forall x_i \in V)[x_i \in X_\beta \vee (\exists x_j \in X_\beta | \\ &\mu(x_j, x_i) \geq \mu_\beta \& \nu(x_j, x_i) \leq \nu_\beta)].\end{aligned} \tag{7}$$

To each vertex $x_i \in V$ we assign Boolean variable p_i that takes value 1, if $x_i \in X_\beta$ and 0 otherwise. We assign the intuitionistic variable $\xi_{ji} = (\mu_\beta, \nu_\beta)$ for the proposition $(\mu(x_j, x_i), \nu(x_j, x_i)) \geq (\mu_\beta, \nu_\beta)$. Passing from the quantifier form of proposition (7) to the form in terms of logical operations, we obtain a true logical proposition:

$$\Phi_D = \underset{i=\overline{1,n}}{\&}(p_i \vee \bigvee_{j=\overline{1,n}}(p_j \& \xi_{ji})).$$

Here, $n = |V|$. Supposing $\xi_{ii} = (1, 0)$ and considering that the equality $p_i \vee \bigvee_j (p_j \& \xi_{ji} = \bigvee_j (p_j \& \xi_{ji}$ is true for

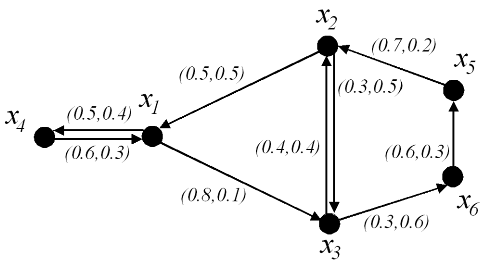

Figure 2: Intuitionistic fuzzy graph $\tilde{G}$

any vertex x_j, we finally obtain:

$$\Phi_D = \underset{i=\overline{1,n}}{\&} \Big(\bigvee_{j=\overline{1,n}} (p_j \& \xi_{ji}) \Big). \qquad (8)$$

We open the parentheses in the expression (8) and reduce the similar terms by following rules:

$$\begin{gathered} a \vee a\&b = a; a\&b \vee a\&\bar{b} = a; \\ (\xi_1 \geq \xi_2) \rightarrow (\xi_1 \& a \vee \xi_2 \& a \& b = \xi_1 \& a). \end{gathered} \qquad (9)$$

Here, $a, b \in \{0, 1\}$ and $\xi_1, \xi_2 \in [(0,1),(1,0)]$.

Then the expression (8) may be presented as

$$\Phi_D = \bigvee_{i=\overline{1,l}} (p_{1i} \& p_{2i} \& \ldots \& p_{ki} \& \beta_i). \qquad (10)$$

We may prove next property:

Property 2. Each disjunctive member in the expression (8) gives a minimum intuitionistic dominating vertex subset with the degree β_i.

The following algorithm of foundation of intuitionistic dominating set may be proposed on the base of Property 2:

- We write proposition (8) for given intuitionistic fuzzy graph $\tilde{G}$;

- We simplify proposition (8) by proposition (9) and present it as proposition (10);

- We define all minimum intuitionistic dominating vertex subsets, which correspond to the disjunctive members of proposition (10);

- We define the domination set of graph $\tilde{G}$.

5 Numerical Example

Let's consider an example of finding domination set of graph $\tilde{G}$ shown in Figure 2.

The adjacent matrix of intuitionistic fuzzy graph $\tilde{G}$ looks like this:

$$R_X = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} (1,0) & (0,1) & (0.8,0.1) & (0.5,0.4) & (0,1) & (0,1) \\ (0.5,0.5) & (1,0) & (0.3,0.5) & (0,1) & (0,1) & (0,1) \\ (0,1) & (0.4,0.4) & (1,0) & (0,1) & (0,1) & (0.3,0.6) \\ (0.6,0.3) & (0,1) & (0,1) & (1,0) & (0,1) & (0,1) \\ (0,1) & (0.7,0.2) & (0,1) & (0,1) & (1,0) & (0,1) \\ (0,1) & (0,1) & (0,1) & (0,1) & (0.6,0.3) & (1,0) \end{pmatrix} \end{array}$$

The corresponding expression (8) for this graph has the following form:

$\Phi_D = [(1,0)p_1 \vee (0.5,0.5)p_2 \vee (0.6,0.3)p_4]\&$
$\&[(1,0)p_2 \vee (0.4,0.4)p_3 \vee (0.7,0.2)p_5]\&$
$\&[(0.8,0.1)p_1 \vee (0.3,0.5)p_2 \vee (1,0)p_3]\&$
$\&[(0.5,0.4)p_1 \vee (1,0)p_4]\&[(1,0)p_5 \vee (0.6,0.3)p_6]\&$
$\&[(0.3,0.6)p_3 \vee (1,0)p_6]$.

Multiplying parenthesis 1 and 2, parenthesis 3 and 4, parenthesis 5 and 6, and using rules (9) we obtain:

$\Phi_D = [(1,0)p_1p_2 \vee (0.4,0.4)p_1p_3 \vee (0.7,0.2)p_1p_5] \vee$
$\vee (0.5,0.5)p_2 \vee (0.6,0.3)p_2p_4 \vee (0.4,0.4)p_3p_4 \vee$
$\vee [(0.6,0.3)p_4p_5]\&$
$\&[(0.5,0.4)p_1 \vee (0.8,0.1)p_1p_4 \vee (0.3,0.5)p_2p_4] \vee$
$\vee (1,0)p_3p_4]\&$
$\&[(0.3,0.6)p_3p_5 \vee (1,0)p_5p_6 \vee (0.6,0.3)p_6]$.

Multiplying parenthesis 2 and 3, we obtain:

$\Phi_D = [(1,0)p_1p_2 \vee (0.4,0.4)p_1p_3 \vee (0.7,0.2)p_1p_5] \vee$
$\vee (0.5,0.5)p_2 \vee (0.6,0.3)p_2p_4 \vee (0.4,0.4)p_3p_4 \vee$
$\vee [(0.6,0.3)p_4p_5]\&$
$\&[(0.3,0.6)p_1p_3p_5 \vee (0.5,0.4)p_1p_6 \vee (0.3,0.5)p_2p_4p_6 \vee$
$\vee (0.8,0.1)p_1p_4p_5p_6 \vee (0.6,0.3)p_1p_4p_6 \vee (0.6,0.3)p_3p_4p_6$
$\vee (1,0)p_3p_4p_5p_6]$.

Multiplying the resulting parenthesis, we finally get:

$\Phi_D = \underline{(0.5,0.4)p_1p_2p_6} \vee (0.3,0.6)p_1p_3p_5 \vee$
$(0.4,0.4)p_1p_3p_6 \vee (0.5,0.4)p_1p_5p_6 \vee (0.5,0.5)p_2p_4p_6 \vee$
$\vee (0.4,0.4)p_3p_4p_6 \vee (0.6,0.3)p_1p_2p_4p_6 \vee$
$\vee \underline{(0.7,0.2)p_1p_4p_5p_6} \vee (0.6,0.3)p_2p_3p_4p_6 \vee$
$\vee (0.6,0.3)p_1p_4p_5p_6 \vee (0.6,0.3)p_3p_4p_5p_6 \vee$
$\vee \underline{(0.8,0.1)p_1p_2p_4p_5p_6} \vee \underline{(1,0)p_1p_2p_3p_4p_5p_6}$.

It follows from the last equality that graph $\tilde{G}$ has 13 minimal intuitionistic dominating vertex subsets, and domination set is defined as:

$\tilde{D} = \{\langle(0.5,0.4)/3\rangle, \langle(0.7,0.2)/4\rangle, \langle(0.8,0.1)/5\rangle,$
$\langle(1.0,0.0)/6\rangle\}$.

This domination set shows, in particular, that there is a subset in the graph ($X = \{x_1, x_4, x_5, x_6\}$), consisting of 4 vertices such that all other vertices of the graph ($D \backslash X = \{x_2, x_3\}$) are adjacent to at least one vertex of the subset X with degree at least 0.7, and not adjacent with degree at most 0.2.

6 Conclusion

In this paper we considered the concepts of fuzzy minimal intuitionistic dominating vertex subsets and domination set of intuitionistic fuzzy graph. The method and algorithm of finding families of all fuzzy minimal dominating vertex subsets and intuitionistic dominating set were considered. This method is the generalization of Maghout's method for fuzzy graph. This generalization consists of the definitions of the fuzzy dominating vertex set with the degree of domination. The definitions allow to estimate the any fuzzy graph with the position of existence of fuzzy invariants. It should be noted that the considered method is a method of complete ordered search, since such tasks are reduced to coverage problems, i.e. these tasks are NP-compete tasks. However, the proposed method can be effective for intuitionistic fuzzy graphs with inhomogeneous structure and not large dimensionality. In addition, the considered method and algorithm can be used to find other invariants, in particular, externally stable set, intuitionistic base and antibase of the intuitionistic fuzzy graphs.

Acknowledgement

This work has been supported by the Ministry of Education and Science of the Russian Federation under Project "Methods and means of decision making on base of dynamic geographic information models" (Project part, State task 2.918.2017), and the Russian Foundation for Basic Research, Project #18-01-00023a.

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets, in: VII ITKR's Session, Sofia, Central Science and Technical Library, Bulgarian Academy of Sciences, 1697/84, 1983.

[2] K. T. Atanassov, Intuitionistic fuzzy sets: theory and application, Springer, Physica-Verlag Heidelberg, Germany, 1999.

[3] K. Atanassov, G. Gargov, Elements of intuitionistic fuzzy logic. Part I, Fuzzy Sets and Systems 95 (1998) 39–52.

[4] L. S. Bershtein, A. V. Bozhenuk, Maghout Method for Determination of Fuzzy Independent, Dominating Vertex Sets and Fuzzy Graph Kernels, Int. J. General Systems 30(1) (2001) 45–52.

[5] L. Bershtein, A. Bozhenyuk, Fuzzy Graphs and Fuzzy Hypergraphs, in: J. Dopico, J. de la Calle, A. Sierra (Eds.), Encyclopedia of Artificial Intelligence, Information SCI, Hershey, New York, 2008, pp. 704–709.

[6] M. G. Karunambigai, S. Sivasankar, K. Palanivel, Different types of Domination in Intuitionistic Fuzzy Graph, Annals of Pure and Applied Mathematics 14(1) (2017) 87–101.

[7] A. Kaufmann, Introduction a la theorie des sous-ensemles flous, Masson, Paris, 1977.

[8] J. N. Mordeson, P. S. Nair, Fuzzy Graphs and Fuzzy Hypergraphs, Springer, Heidelberg, Germany, 2000.

[9] O. Ore, Theory of Graphs, American Mathematical Society, Providence, USA, 1962.

[10] R. Parvathi, G. Thamizhendhi, Domination in intuitionistic fuzzy graphs, Notes on Intuitionistic Fuzzy Sets 16 (2010) 39–49.

[11] A. Rosenfeld, Fuzzy graphs, in: L. A. Zadeh, K. S. Fu, M. Shimura (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York, USA, 1975, pp. 77–95.

[12] A. Shannon, K. T. Atanassov, A first step to a theory of the intuitionistic fuzzy graphs, in: D. Lakov (Ed.), Proceeding of the FUBEST, Sofia, Bulgaria, 1994, pp. 59–61.

[13] A. Shannon, K. T. Atanassov, Intuitionistic fuzzy graphs from α-, β- and (α,β)-levels, Notes on Intuitionistic Fuzzy Sets 1(1) (1995) 32—35.

[14] M. S. Sunitha, A. V. Kumar, Complement of a fuzzy graph, Indian Journal of Pure and Applied Mathematics 33(9) (2002) 1451–1464.

[15] S. Velammal, Edge Domination in Intuitionistic Fuzzy Graphs, International Journal of Computational Science and Mathematics 4(2) (2012) 159—165.

[16] R. T. Yeh, S. Bang, Fuzzy Relations, Fuzzy Graphs, and Their Applications to Clustering Analysis, in: L. A. Zadeh, K. S. Fu, M. Shimura (Eds.), Fuzzy Sets and Their Applications, Academic Press, New York, 1975, pp. 125–149.

[17] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965) 338—353.

[18] L. A. Zadeh, Similarity relations and fuzzy orderings, Information Sciences 3(2) (1971) 177–200.