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Clustering a Body of Evidence Based on Conflict Measures

Andrey Bronevich[a] **and Alexander Lepskiy**[a]

[a]Department of Mathematics, Faculty of Economic Sciences,
National Research University Higher School of Economics,
Myasnitskaya 20, Moscow, 101000, Russia,
brone@mail.ru, alex.lepskiy@gmail.com

Abstract

In real applications, sometimes it is necessary to evaluate inner or external conflict of pieces of evidence. However, these numerical values cannot give us explanations why this conflict occurs. Thus, we need deeper analysis of available information. In the paper, we propose the clusterization of a given evidence on pieces of evidence in a way that we try to achieve the highest conflict among pieces of evidence and the smallest inner conflict within pieces of evidences based on several functionals that help us to evaluate inner and external conflict.

Keywords: Belief functions, Conflict measure, Clustering.

1 Introduction

A long time ago, it was observed [6] that if we try to evaluate conflict between sources of information described by belief functions based on Dempster's rule, we should take into account the inner conflict within belief functions. This conflict can be caused by the fact that every belief function can be also generated by merging of conflicting sources of information. The last idea was used in several papers [8, 9, 12], where authors tried to get a decomposition of evidence for evaluating inner or external conflict based on several aggregation rules.

In the paper, we propose tractable algorithms for finding an optimal decomposition of a body of evidence on pieces of information in a way that parts of information should have the smallest inner conflict and the highest conflict should be among them. For this purpose, we propose to use several functionals for evaluating the inner conflict. The external conflict can be evaluated using the assumption that the overall conflict in the body of evidence can be represented as a sum of inner and external conflict for a given decomposition.

The paper has the following structure. In Section 2, we give the basic definitions and constructions from the theory of belief functions used in the paper. In Section 3, we propose a system of axioms for functionals for evaluating the inner conflict and give some special examples of such functionals. In Section 4, we describe the optimization problem for finding an optimal decomposition of a body of evidence on parts with smallest conflict, propose several tractable algorithms to solve it, and show some remarkable properties of the proposed algorithms. In Section 5, we apply the proposed decompositions of a body of evidence on an example of analyzing political preferences of parties in Germany and their influence on popularity of such parties. The paper finishes with some conclusions and proposals for future research.

2 Belief Functions: Basic Constructions and Definitions

Let $X = \{x_1, ..., x_n\}$ be a finite reference set and let 2^X be the powerset of X. The set function Bel : $2^X \to [0,1]$ is called a belief function [5, 13] if there is a set function $m : 2^X \to [0,1]$ with $m(\emptyset) = 0$ and $\sum_{A\in 2^X} m(A) = 1$ called the basic belief assignment (bba) such that

$$Bel(A) = \sum_{B\subseteq A|B\in 2^X} m(B).$$

With the help of bba m we introduce also the plausibility function defined by

$$Pl(A) = \sum_{B\cap A\neq\emptyset|B\in 2^X} m(B).$$

It is well known that Bel and Pl are connected with the duality relation: $Pl = Bel^d$, where $Bel^d(A) = 1 - Bel(\bar{A})$ and $\bar{A}$ is the complement of A.

Let Bel be a belief function with the bba m, then $A \in 2^X$ is called a focal element for Bel if $m(A) > 0$. The set of all focal elements is called the body of evidence. A belief function is called categorical if its body of evidence has the only one focal element B. We denote such belief functions by $\eta_{\langle B \rangle}$ and obviously

$$\eta_{\langle B \rangle}(A) = \begin{cases} 1, & B \subseteq A, \\ 0, & \text{otherwise.} \end{cases}$$

Let μ, μ_1, μ_2 are set functions on 2^X, then we write:

1) $\mu = a\mu_1 + b\mu_2$ for $a, b \in \mathbb{R}$ if $\mu(A) = a\mu_1(A) + b\mu_2(A)$ for all $A \in 2^X$;

2) $\mu_1 \leq \mu_2$ if $\mu_1(A) \leq \mu_2(A)$ for all $A \in 2^X$.

Every belief function can be represented as a convex sum of categorical belief functions

$$Bel = \sum_{B \in 2^X} m(B)\eta_{\langle B \rangle},$$

where m is the bba of Bel. A belief function is a probability measure if its body of evidence consists of singletons. In the next, $M_{bel}(X)$ denotes the set of all possible belief functions on 2^X, and $M_{pr}(X)$ denotes the set of all probability measures on 2^X. If we do not identify the reference set X, or it can be identified by the context, then we write simply M_{bel} or M_{pr}.

Let $Bel \in M_{bel}(X)$. Then we define a non-empty set of probability measures

$$\mathbf{P}(Bel) = \{P \in M_{pr}(X) | P \geq Bel\},$$

called the credal set. Credal sets play a key role for defining conflict measures in the next section.

3 Measures of Conflict

One can find various measures of conflict in the theory of belief functions and there are several attempts to identify their axiomatics [2, 6]. Following [2] we should firstly identify the conditions when a measure of conflict is equal to zero. Let $U_C : M_{bel} \to [0, +\infty)$ be a functional for measuring conflict, then there are three options, which are considered for every possible reference set X:

1) $U_C(Bel) = 0$ for $Bel \in M_{bel}(X)$ iff $Bel = \eta_{\langle B \rangle}$ for some $B \in 2^X$;

2) $U_C(Bel) = 0$ for $Bel \in M_{bel}(X)$ iff the body of evidence $\mathcal{A}$ of Bel is a chain of sets, i.e. the elements of $\mathcal{A}$ can be indexed such that $\mathcal{A} = \{B_1, ..., B_m\}$ and $B_i \subseteq B_j$, when $i \leq j$;

3) $U_C(Bel) = 0$ for $Bel \in M_{bel}(X)$ with a body of evidence $\mathcal{A} = \{B_1, ..., B_m\}$ iff $\bigcap_{i=1}^m B_i \neq \emptyset$.

In our investigation we will assume that 3) is fulfilled. There are also three requirements that are desirable for U_C:

a) $U_C(Bel_1) \leq U_C(Bel_2)$ for $Bel_1, Bel_2 \in M_{bel}(X)$ if $Bel_1 \leq Bel_2$.

For formulating the next requirement, we introduce some definitions. Let $\varphi : X \to Y$ be a mapping between finite sets X and Y, $Bel \in M_{bel}(X)$, and Bel^{φ} is the belief function on 2^Y defined by $Bel^{\varphi}(B) = Bel(\varphi^{-1}(B))$, where $B \in 2^Y$ and $\varphi^{-1}(B) = \{x \in X | \varphi(x) \in B\}$. It is easy to see that the bba m^{φ} for Bel^{φ} can be computed by the same formula, i.e. if m is the bba for Bel, then $m^{\varphi}(B) = m(\varphi^{-1}(B))$ for $B \in 2^Y$.

b) Let $\varphi : X \to Y$ be a mapping between finite sets X and Y, then $U_C(Bel^{\varphi}) \leq U_C(Bel)$ for every $Bel \in M_{bel}(X)$. In addition, $U_C(Bel^{\varphi}) = U_C(Bel)$ if φ is an injection.

c) Let $Bel = aBel_1 + (1-a)Bel_2$, where $a \in [0, 1]$ and $Bel_1, Bel_2 \in M_{bel}(X)$, then $U_C(Bel) \geq aU_C(Bel_1) + (1-a)U_C(Bel_2)$.

Lemma 1. *Let a functional U_C satisfies requirements 3), a), b) and c), then for every $Bel \in M_{bel}(X)$, we have $U_C(Bel) \leq U_C(P_n)$, where the measure P_n defines the uniform probability distribution on $X = \{x_1, ..., x_n\}$, i.e. $P_n(\{x\}) = 1/n$ for every $x \in X$.*

Theorem 1. *Let the functional $\Phi : M_{pr} \to \mathbb{R}$ satisfies requirements 3), a), b) on the set M_{pr} of all possible probability measures. Then Φ might be extended to the set M_{bel} of all belief functions by*

$$U_C(Bel) = \inf \{\Phi(P) | P \in \mathbf{P}(Bel)\}.$$

Moreover, U_C satisfies conditions 3), a), and b).

Theorem 1 allows us to extend a measure of conflict from the set of all probability measures to the set of all belief functions.

The next problem is how U_C can be defined on M_{pr}. Assume that $X = \{x_1, ..., x_n\}$. Then every probability measure $P \in M_{pr}(X)$ is uniquely defined by the vector $(P(\{x_1\}, ..., P(\{x_n\})$, i.e. we can consider functions $f_n : \Omega_n \to [0, +\infty)$, $n = 1, 2, ...$, that correspond to the functional U_C such that

1) $\Omega_n = \{(t_1, ..., t_n) \in \mathbb{R}^n | \sum_{i=1}^n t_i = 1, \, t_i \geq 0, \, i = 1, ..., n\}$;

2) $U_C(P) = f_n(P(\{x_1\}, ..., P(\{x_n\})$.

The properties of such functions are given in the following theorem.

Theorem 2. *A system of functions $f_n : \Omega_n \to [0, +\infty)$, $n = 1, 2, ...$, defines a measure U_C of conflict on M_{pr} with properties 3), a),b) and c) iff* *(i)* $f_n(t_1,...,t_n) = 0$ *if* $t_1 = 1$ *and* $f_n(t_1,...,t_n) > 0$ *if* $t_1 \in (0,1)$*;*

(ii) $f_{n+1}(t_1,...,t_n,0) = f_n(t_1,...,t_n)$ *for every* $(t_1,...,t_n) \in \Omega_n$*;*

(iii) $f_n(t_{\varphi(1)},...,t_{\varphi(n)}) = f_n(t_1,...,t_n)$ *for every one-to-one mapping* $\varphi : \{1,...,n\} \to \{1,...,n\}$*;*

(iv) $f_{n-1}(t_1+t_2,t_3,...,t_n) \le f_n(t_1,t_2,t_3,...,t_n)$ *for every* $(t_1,...,t_n) \in \Omega_n$*;*

(v) f_n *is a concave function on* Ω_n*, i.e.* $f_n(a\mathbf{t}_1 + (1-a)\mathbf{t}_2) \ge af_n(\mathbf{t}_1)+(1-a)f_n(\mathbf{t}_2)$ *for every* $a \in [0,1]$ *and every* $\mathbf{t}_1, \mathbf{t}_2 \in \Omega_n$*.*

Example 1. Let $f_n(t_1,...,t_n) = \min\{1-t_1,...,1-t_n\}$. Clearly, in this case $f_n(1,0,...,0) = 1$ and $f_n(t_1,...,t_n) > 0$ if $t_1 \in (0,1)$, i.e. (i) is fulfilled. It is easy to check that (ii)-(iv) are also satisfied. Let us show that (v) holds. Consider an arbitrary $\mathbf{t}_1 = (t_1^{(1)},...,t_n^{(1)})$ and $\mathbf{t}_2 = (t_1^{(2)},...,t_n^{(2)})$ in Ω_n, then

$$f_n(a\mathbf{t}_1 + (1-a)\mathbf{t}_2) = \min\{a(1-t_1^{(1)}) + (1-a)(1-t_1^{(2)}),...\}, \quad (1)$$

$$af_n(\mathbf{t}_1) + (1-a)f_n(\mathbf{t}_2) = \min\{a(1-t_1^{(1)}),...,a(1-t_n^{(1)})\}+ \min\{(1-a)(1-t_1^{(2)}),...,(1-a)(1-t_n^{(2)})\}.$$

Assume that the minimum in (1) is achieved for some $k \in \{1,...,n\}$. Then, obviously,

$$\min\{a(1-t_1^{(1)}),...,a(1-t_n^{(1)})\} \le a(1-t_k^{(1)}),$$
$$\min\{(1-a)(1-t_1^{(2)}),...,(1-a)(1-t_n^{(2)})\} \le (1-a)(1-t_k^{(2)}),$$

i.e. $f_n(a\mathbf{t}_1+(1-a)\mathbf{t}_2) \ge af_n(\mathbf{t}_1)+(1-a)f_n(\mathbf{t}_2)$, and this system of functions obeys properties (i)-(v). If we extend this measure of conflict to M_{bel}, then it can be expressed through the plausibility function $Pl = Bel^d$ as $U_C(Bel) = 1 - \max_{x \in X} Pl(\{x\})$. This measure has been possibly firstly introduced by M. Daniel [3].

Some other examples of conflict measures can be found using the following proposition.

Proposition 1. [1] *Let* $g : [0,1] \to [0,+\infty)$ *be a concave function with the following properties:*

a) $g(0) = g(1) = 0$*;*

b) $g(t)$ *is strictly decreasing at* $t = 1$*. Then the system of functions*

$$f_n(t_1,...,t_n) = \sum_{i=1}^{n} g(t_i), \quad (t_1,...,t_n) \in \Omega_n, \quad (2)$$

obeys properties (i)-(v) from Theorem 2.

Example 2. If $g(t) = -t\ln t$, then

$$f_n(t_1,...,t_n) = -\sum_{i=1}^{n} t_i \ln t_i$$

defines the Shannon entropy. The corresponding conflict measure on M_{bel} is called the minimal Shannon entropy [1, 7]. One can check the concavity of g by taking the second derivative $g''(t) = -(1/t) < 0$ for $t \in (0,1]$.

Example 3. Assume that we evaluate the conflict between probability measures using Dempster's rule of aggregation. Let $P_1, P_2 \in M_{pr}(X)$ and $X = \{x_1,...,x_n\}$, Then the conflict is defined through the following functional

$$k(P_1,P_2) = 1 - \sum_{i=1}^{n} P_1(\{x_i\})P_2(\{x_i\}).$$

The functional

$$k(P,P) = 1 - \sum_{i=1}^{n} P(\{x_i\})P(\{x_i\}) = \sum_{i=1}^{n} P(\{x_i\})(1-P(\{x_i\}))$$

can be used for evaluating the inner conflict in the probability measure P on 2^X. We can describe this functional using the function

$$f_n(t_1,...,t_n) = \sum_{i=1}^{n} t_i(1-t_i),$$

i.e. f_n is represented through the function $g(t) = t(1-t)$ as in Proposition 1. It easy to check that such a g obeys properties formulated in Proposition 1.

4 Clustering a Body of Evidence Using Conflict Measures

Let Bel be a belief function on 2^X. Then the problem is to disaggregate Bel on parts $Bel_1,...,Bel_k \in M_{bel}$ such that $Bel = \sum_{i=1}^{k} a_i Bel_i$, where $\sum_{i=1}^{k} a_i = 1$, $a_i \ge 0$, such that the conflict within belief functions Bel_i should be minimal, and the external conflict among them should be maximal. The internal conflict can be evaluated by $\sum_{i=1}^{k} a_i U_C(Bel_i)$ and the external conflict by $U_C(Bel) - \sum_{i=1}^{k} a_i U_C(Bel_i)$. We see that due to properties of considered conflict measures the amount of external conflict is always a non-negative value. Notice that the given problem makes sense if we introduce some restrictions on the choice of Bel_i and on the number k of clusters. For example, we can assume that $\sum_{i=1}^{k} a_i U_C(Bel_i) \le \varepsilon$ and the number k

[1]Similar non-Shannon entropies are investigated in [11].

of clusters should be minimal. We can also identify the way of generating Bel_i. Assume also that Bel has the body of evidence $\mathcal{A}$ and belief functions Bel_i, $i = 1, ..., k$, correspond to a partition $\{\mathcal{A}_1, ..., \mathcal{A}_k\}$ of $\mathcal{A}$ such that $a_i Bel_i = \sum_{B\in\mathcal{A}_i} m(B)\eta_{\langle B\rangle}$. Let us solve the given problem when we put $\varepsilon = 0$, i.e. we demand that every extracted part of information should be non-conflicting.

Algorithm 1.

1. To find a subset $B \subseteq X$ with the smallest cardinality such that $Pl(B) = 1$.

2. Assume that $B = \{y_1, ..., y_k\}$, then

$$\mathcal{A}_1 = \{A \in \mathcal{A} | y_1 \in A\},$$
$$\mathcal{A}_2 = \{A \in \mathcal{A}\backslash\mathcal{A}_1 | y_2 \in A\},$$
$$\vdots$$
$$\mathcal{A}_k = \{A \in \mathcal{A}\backslash(\mathcal{A}_1 \cup ... \cup \mathcal{A}_{k-1} | y_k \in A\}.$$

It easy to see that Algorithm 1 finds the required representation but there are some suspicions that this clusterization does not exactly reflect the structure of given data.

Another approach consists in the following. Assume that we have a partition $\{\mathcal{A}_1, ..., \mathcal{A}_k\}$ of $\mathcal{A}$ such that $y_1 \in \bigcap_{A_i\in\mathcal{A}_1} A_i$,..., $y_k \in \bigcap_{A_i\in\mathcal{A}_k} A_i$. Then we say that this clusterization keeps the structure of given data if the belief function $Bel_B = \sum\limits_{A_i\in\mathcal{A}} m(A_i)\eta_{\langle B\cap A_i\rangle}$ has the same inner conflict as Bel. Because $A_i \cap B \subseteq C$ is equivalent to $A_i \subseteq C \cup \bar{B}$, $\eta_{\langle B\cap A_i\rangle}(C) = \eta_{\langle A_i\rangle}(C \cup \bar{B})$ or $Bel_B(C) = Bel(C\cup\bar{B})$ for all $C \in 2^X$. If we denote $Pl_B = Bel_B^d$, then obviously $Pl_B(C) = Pl(C \cap B)$ for all $C \in 2^X$.

Proposition 2. *Let $P \in M_{pr}$ be a solution of the optimization problem for finding $U_C(Bel)$, i.e. $P \in \mathbf{P}(Bel)$ and $U_C(P) = U_C(Bel)$. Then for $B = \{x \in X | P(\{x\}) > 0\}$ we have $U_C(Bel_B) = U_C(Bel)$.*

The above conclusions allows us to propose the following algorithm that keeps the structure of given data.

Algorithm 2.

1. To find $P \in \mathbf{P}(Bel)$ such that $U_C(P) = U_C(Bel)$.

2. Let $\{y_1, ..., y_k\} = \{x \in X | P(\{x\}) > 0\}$ and y_i, $i = 1, \ldots, k$, are numbered so that $P(\{y_1\}) \geq P(\{y_2\}) \geq ... \geq P(\{y_k\})$. Then we construct the partition $\{\mathcal{A}_1, ..., \mathcal{A}_k\}$ of $\mathcal{A}$ as

$$\mathcal{A}_1 = \{A \in \mathcal{A} | y_1 \in A\},$$
$$\mathcal{A}_2 = \{A \in \mathcal{A}\backslash\mathcal{A}_1 | y_2 \in A\},$$
$$\vdots$$
$$\mathcal{A}_k = \{A \in \mathcal{A}\backslash(\mathcal{A}_1 \cup ... \cup \mathcal{A}_{k-1} | y_k \in A\}.$$

One can find the justification of Algorithm 2 in the next results.

Proposition 3. *Let $Bel \in M_{bel}$ and $P \in \mathbf{P}(Bel)$ with $U_C(P) = U_C(Bel)$. Let us denote $\{y_1, ..., y_k\} = \{x \in X | P(\{x\}) > 0\}$ and let $\mathcal{A}$ be the body of evidence of Bel. Then there is a partition $\{\mathcal{A}_1, ..., \mathcal{A}_k\}$ of $\mathcal{A}$ such that $y_1 \in \bigcap_{A_i\in\mathcal{A}_1} A_i$,,$y_k \in \bigcap_{A_i\in\mathcal{A}_k} A_i$ and*

$$Bel = \sum_{i=1}^{k} P(\{y_i\}) Bel_i,$$

where $P(\{y_i\})Bel_i = \sum_{B\in\mathcal{A}_i} m(B)\eta_{\langle B\rangle}$, $i = 1, ..., k$.

We call a function $g : [0, 1] \to [0, +\infty)$ from Proposition 1 strictly concave if $g(x+\Delta x+\Delta y) - g(x+\Delta x) - g(x + \Delta y) + g(x) < 0$ for every $x, x + \Delta x, x + \Delta y, x + \Delta x + \Delta y \in [0, 1]$ and $\Delta x, \Delta y > 0$. If g is twice differentiable on [0,1], then g is strictly concave if $g''(x) < 0$ for any $x \in [0, 1]$.

Proposition 4. *Let the functional U_C on M_{bel} be constructed using the system of functions f_n from Proposition 1, and let g be a strictly concave function. Consider an arbitrary $Bel \in M_{bel}$ with a body of evidence $\mathcal{A}$ and assume that $U_C(Bel) = U_C(P)$, where $P \in \mathbf{P}(Bel)$. Consider the representation $Bel = \sum\limits_{i=1}^{k} P(\{y_i\})Bel_i$ from Proposition 3 and assume that $P(\{y_1\}) \geq P(\{y_2\}) \geq ... \geq P(\{y_k\})$. Then the corresponding partition of $\mathcal{A}$ can be constructed by*

$$\mathcal{A}_1 = \{A \in \mathcal{A} | y_1 \in A\},$$
$$\mathcal{A}_2 = \{A \in \mathcal{A}\backslash\mathcal{A}_1 | y_2 \in A\},$$
$$\vdots$$
$$\mathcal{A}_k = \{A \in \mathcal{A}\backslash(\mathcal{A}_1 \cup ... \cup \mathcal{A}_{k-1} | y_k \in A\}.$$

5 Ranging Questions Based on the Results of Elections in Germany Using Conflict Measures

We will demonstrate how the algorithm works on statistical data characterizing elections in Germany. The study of the influence of party positions on the voting result is an important problem in political sciences [16]. In some countries, there are services designed to assess the positions of parties before elections. For this purpose, voters choose the list of valuable questions and parties should answer them before elections. We use the data given by a service in Germany [18] collected before elections to Bundestag in 2013 [17]. To avoid computational problems, we take only 8 questions (the most valuable from the total number of 38). Every party on each question can answer 'yes' or 'no'. The importance of questions can be also evaluated after knowing the results of elections. We present these results using belief functions that help us to model the importance of groups of questions. The methodology of such evaluation is described in [10], and based on the assumption that the number of expected true answers is proportional to the number of voices given to the parties. Thus, our belief function is constructed on the set of questions $X = \{x_1, ..., x_8\}$, and by the computations we can see that the body of evidence consists of 255 focal elements and the mass function on them is small but its values vary from one focal element to another. Thus, we need some computational method to analyze these data. We have computed the Shapley value [14]

$$V = (0.11, 0.175, 0.152, 0.11, 0.121, 0.11, 0.11, 0.11),$$

also called the pignistic transformation in the transferable belief model (TBM) [15], for our belief function, and according to it, questions 2, 3, and 5 are the most valuable, but the difference of importance of questions seems to be according to this criterion is not so high. Whereas, the proposed approach based on clustering a body of evidence can reflect the conflict among pieces of information and in a view of the our example can reveal the degree of the heterogeneity of society w.r.t. the list of questions. For this purpose, we have applied Algorithm 2 to our belief function with the following parameters: the conflict measure U_C is defined as in Theorem 1 with Φ from Example 3, i.e. $g(t) = t(1-t)$. We found that the value of $U_C(Bel)$ is achieved on a probability measure $P_0 \in \mathbf{P}(Bel)$, whose values are defined by a vector

$$(0.014, 0.345, 0.005, 0.028, 0.54, 0.003, 0.055, 0.009),$$

and $U_C(Bel) = \Phi(P_0) = 0.586$. Thus, vouters have shown the highest consolidation in question 5, the rest of them in question 2, and etc. We have applied Algorithm 2 for constructing clusters. Clearly that the most representative clusters correspond to probabilities $P_0(x_2) = 0.345$ and $P_0(x_5) = 0.54$. It occurs that cluster 5 that correspond to x_5 contains 128 focal elements and cluster 2 contains 65 focal elements. The quality of clusterization can be evaluated by contour functions

$$Pl_i(\{x\}) = \sum_{x \in A | A \in \mathcal{A}_i} m(A),$$

and the Shapley values

$$V_i(x) = \frac{1}{a_i} \sum_{x \in A | A \in \mathcal{A}_i} m(A)/|A|,$$

where $x \in X$, $a_i = \sum_{x \in A | A \in \mathcal{A}_i} m(A)$, $i = 2, 5$. These functions are depicted on Fig. 1 and Fig. 2.

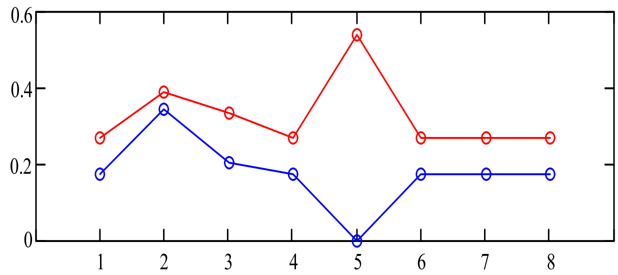

Figure 1: Contour functions of clusters: the red line is for cluster 5; the blue line is for cluster 2

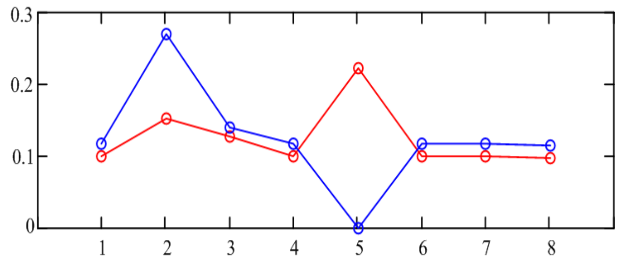

Figure 2: Shapley values of clusters: the red line is for cluster 5; the blue line is for cluster 2

We see that the most of focal elements are pinned to the questions 5 or 2. Question 5 is the question about increasing of the retirement age, and the question 2 is about subsidies to families whose children study in non-state institutions. In fact, the most of vouters proposed to include question 5 to the list (about 3.5 millions). Question 2 has been supported by 1 millions of vouters (this is the second place from the end), but its contribution to importance in groups of questions was sufficently high to form the repesentative cluster.

Fig. 2 shows that the contribution of question 2 in cluster 2 is higher than the contribution of question 5 to cluster 5 according to the Shapley values. This reflects the fact that the question of rising the retirement age is important for a larger number of voters than the question of subsidies to families with children. Therefore, it is included in a larger number of significant coalitions of questions.

6 Conclusion

The article proposes a method for clustering a complex body of evidence into simpler, in some sense, bodies of evidence. The proposed clustering is based on maximizing conflict among bodies of evidence and minimizing conflict withing bodies of evidence instead of a metric in classical clustering.

The following results were obtained:

- measures of conflict on belief functions has been proposed satisfying a number of desirable properties (axioms);
- the connection between introduced measures of conflict and clustering of body of evidence has been found;
- clustering of a body of evidence has been applied for evaluating the political heterogeneity of society based on answers to questions by political parties in Germany before the elections in 2013 and the results of these elections.

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