

# Strategy Selection Study of Out-of-stock Substitution and Probabilistic Selling

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**Abstract.** With the development of the economy and the improvement of productivity, many products have entered the buyer's market from the seller's market. To maximize the appeal of consumers with diverse needs, retailers offer a range of similar and alternative products for consumers to choose from. For retailers, the increase in product categories has expanded the market size and increased sales revenue, but it has also increased the difficulty of retailer purchasing decisions and inventory management. The uncertain demand for multiple products and the substitution of products to meet demand make it even more difficult for retailers to match inventory and demand and improve inventory efficiency. In this case, retailers can consider two strategies to manage the demand uncertainty of this multi-product, namely out-of-stock replacement and probabilistic sales. In view of the fact that out-of-stock substitution and probabilistic sales strategies can manage the uncertainty of multi-product demand through demand substitution, this paper will build an extended newsboy model under two strategies, and compare and select these two strategies.

**Keywords:** Demand Uncertainty; Out-of-stock Substitution; Probabilistic Selling; Strategic Choice.

## 1. Introduction

Due to the continuous development of the economy, the continuous improvement of the level of productivity, and the enormous enrichment of materials, the supply and demand of the market has undergone significant changes. Today, most of the products have entered the buyer's market by the seller's market. There are a large number of products with the same or similar functions on the market, and at the same time, consumers' demand for products is becoming more diverse. To maximize the appeal of consumers with diverse needs, retailers offer a range of similar and alternative products for consumers to choose from. For example, in the clothing market, H&M, Zara, etc. provide customers with shirts of the same style but different colors; in the food market, Wumei, Wal-Mart, etc. provide customers with different flavors of beverages or biscuits, etc.

The rich products not only meet the diversified needs of consumers, but also expand the retailer market and increase the retailer's sales profits. But at the same time, the increase in product categories has increased the difficulty of retailer purchasing decisions and inventory management. Specifically, on the one hand, the increase in product categories has led to greater retailer inventory levels, longer inventory cycles, higher safety stocks, which has increased retailer inventory costs; on the other hand, supply and demand caused by uncertainty in product demand Mismatches will directly reduce retailer profits, and the uncertainties in demand for multiple products and the substitution of products to meet demand make it more difficult for retailers to match inventory and demand and improve inventory efficiency.

In this case, retailers can consider two strategies to manage the demand uncertainty of this multi-product, namely out-of-stock replacement and probabilistic sales. Out-of-stock replacement strategies encourage consumers to choose to purchase another acceptable and available product in the face of product out-of-stocks, thereby reducing the mismatch between retailer inventory and consumer demand [1]. The probabilistic sales strategy uses the retailer's existing products to create a probabilistic product with certain attributes hidden in a certain proportion [2-4]. By encouraging consumers to abandon the purchase of certain products, they choose to purchase low-priced but random probabilistic products to reduce inventory and A mismatch between requirements. Both strategies can make consumers in the market with weaker product preferences abandon their pursuit of a certain product, and instead accept another product offered by the retailer [5].

In view of the shortage of substitutes and probabilistic sales strategies, the demand uncertainty of multi-products can be managed through demand substitution. This paper will compare and select these two strategies. Specifically, this article mainly answers the following two questions: (1) Which sales strategy should retailers choose? Out of stock replacement or probabilistic sales? (2) How much inventory should the retailer order for the product?

## 2. Model

This article considers retailers selling two similar products, indexed  $j = 1, 2$  (only has one different attribute, such as color or style), the cost of both products is  $c, c > 0$  and the price of both products is  $p, p > c$ . We assume that the market size  $\Psi$  is normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . Let  $f(x)$  and  $F(x)$  be the probability density function and cumulative density function of  $\Psi$ , respectively. When selling both products, retailers can adopt out-of-stock substitution and probabilistic selling strategies. Since the retailer does not know which product is more popular before the sales season, the retailer will order the same inventory for Product 1 and Product 2, i.e.  $Q_1 = Q_2 = Q$ . Before the start of the sales season, the retailer decides which sales strategy to adopt and determines the inventory of Product 1 and Product 2 based on the selected strategy. The detailed description of the symbols used in this article is shown in Table 1.

Table 1. Notation used throughout the paper

$j = 1, 2$	Index for the product 1 and 2, respectively
$c$	Unit purchasing cost of product 1 and 2
$p$	Selling price of product 1 and 2
$Q$	Inventory level (stocking quantity) of product 1 and 2 (decision variable)
$X$	Consumer position on the Hotelling line (random variable), $X \sim U(0, 1/2)$
$V_j$	Consumer valuation of the product $j, j \in \{1, 2\}$
$U_j$	The utility of the consumer to buy the product $j, j \in \{1, 2\}$
$\Psi$	market size, $\Psi \sim N(\mu, \sigma^2)$
$F(\cdot), f(\cdot)$	c. d. f. and p. d. f. of $\Psi$
$D_j^p$	Primary demand of product $j, j \in \{1, 2\}$
$D_j$	demand of product $j, j \in \{1, 2\}$
$D_0$	Probabilistic product demand
$\Pi$	Expected profit

### 2.1 Out-of-stock Substitution

Under the out-of-stock substitution, when the product is out of stock, some consumers will choose to purchase another product, and the retailer considers the impact of out-of-stock substitution when making inventory pricing decisions. At this point, the product demand consists of two parts: the basic needs of consumers and the alternative needs [1]. Below we derive the expression of product demand through the consumer selection process.

Consumer selection process:

Different consumers have different valuations of products, and consumers independently choose to purchase products to maximize their utility. Real consumer valuations are unknown, but can be characterized by random variables. In this paper, we extend the valuation assumption adopted by Xie and Fay (2015), that is, the random variable  $X$  is used to indicate the position of the consumer on the Hotelling line [3]. This assumption means that one product is popular and the other product is unpopular. In other words, if the popular product is not out of stock, the unpopular product has no demand. For convenience, we use Product 1 for popular products and Product 2 for unpopular products. However, it should be noted that we are not clear which product is product 1, and which product is product 2.

Under this assumption, consumers' valuations for products 1 and 2 are:

$$v_1 = 1 - x, v_2 = x \quad (1)$$

The net utilities of consumers purchasing products 1 and 2 are:

$$u_1 = v_1 - p = 1 - x - p, u_2 = v_2 - p = x - p \quad (2)$$

The conditions for consumers to purchase Product 1 are:

$$\begin{cases} u_1 > 0 \\ u_1 > u_2 \end{cases} \Rightarrow x < 1 - p \quad (3)$$

The conditions for consumers to purchase Product 2 are:

$$\begin{cases} u_2 > 0 \\ u_2 > u_1 \end{cases} \Rightarrow \emptyset \quad (4)$$

Therefore, without considering the shortage of goods, the primary demand of product is:

$$D_1^p = \begin{cases} \Psi & p \leq 1/2 \\ 2(1-p)\Psi & p > 1/2 \end{cases}, D_2^p = 0 \quad (5)$$

Consider the out-of-stock replacement. If the product 1 is out of stock, only the consumer who purchases the product 2 can get the positive effect. Therefore, the demand for products under stock-out replacement:

$$D_1 = \begin{cases} \Psi & p \leq 1/2 \\ 2(1-p)\Psi & p > 1/2 \end{cases}, D_2 = \begin{cases} [(1-2p)\Psi - (Q-2p\Psi)^+]^+ & p \leq 1/2 \\ 0 & p > 1/2 \end{cases} \quad (6)$$

Retailer profit function is:

$$\max_{p,Q} \Pi = pE[\min(D_1, Q)] + pE[\min(D_2, Q)] - 2cQ \quad (7)$$

## 2.2 Probabilistic Selling

When a retailer adopts a probabilistic selling strategy, the retailer provides a probabilistic product to the consumer. Because retailers don't know which products are more popular with consumers beforehand, retailers will set probabilistic products in a one-to-one ratio. Consumer valuation of probabilistic products is  $\frac{1}{2}(1-x) + \frac{1}{2}x = \frac{1}{2}$ . In order to extract the maximum consumer surplus, the probability product price should be set to  $\frac{1}{2}$ . At this point, it is determined that the selling price of product 1 and product 2 must be greater than  $\frac{1}{2}$ , otherwise the consumer will not purchase the probabilistic product.

Same as before, the demand of product 1 and 2 under probabilistic selling are:

$$D_1 = 2(1-p)\Psi, D_2 = 0 \quad (8)$$

The demand of probabilistic product is:

$$D_0 = (2p-1)\Psi \quad (9)$$

Retailer profit function is:

$$\max_{p,Q} \Pi = pE[\min(D_1, Q)] + \frac{1}{2}E[\min(D_0, Q)] - 2cQ \quad (10)$$

### 3. Model Calculation

#### 3.1 Optimal Stock Level

The first derivative of  $Q$  is obtained for the profit function under the two strategies, and the first derivative is zero, and the optimal inventory level of different pricing intervals under different strategies is obtained. Table 2 below summarizes the expressions that satisfy the optimal inventory levels.

Table 2. Expression of optimal stock level

Price range	Out-of-stock Substitution	Probabilistic Selling
$p \leq \frac{1}{2}$	$F(2Q) = \frac{p-c}{p}$	—
$\frac{1}{4} < p \leq \frac{1}{2}$	$F\left(\frac{Q}{1-2p}\right) + F\left(\frac{Q}{2p}\right) = \frac{2(p-c)}{p}$	—
$p > \frac{1}{2}$	$F\left(\frac{Q}{2-2p}\right) = \frac{p-2c}{p}$	$pF\left(\frac{Q}{2-2p}\right) + \frac{1}{2}F\left(\frac{Q}{2p-1}\right) = p + \frac{1}{2} - 2c$

#### 3.2 Study Calculation

Analysis can be obtained, for the fixed price  $p$ , we can always determine the optimal inventory of the product corresponding to the different strategies under the price, and then get the corresponding profit. When  $p \leq \frac{1}{2}$ , retailers can only adopt a stock-out alternative strategy. When  $p > \frac{1}{2}$ , the retailer needs to compare the two strategies. Below we use a specific example to show the above results.

Let  $\mu = 100, \sigma = 30, c = 0.2, p = 0.7$ , calculate the optimal inventory and profit of the retailer under the two strategies through matlab software, and make inventory selection. The results are shown in Table 3 below.

Table 3. Expression of optimal stock level

Strategic	$Q^*$	$\Pi^*$	Strategic Choice
Out-of-stock Substitution	56.7598	13.0555	√
Probabilistic Selling	70.4014	3.4673	

### 4. Summary

This paper analyzes the out-of-stock replacement strategy and the optimal inventory level of the product under the probabilistic sales strategy. It points out that in the case of a given product selling price, the merchant realizes the strategy selection by calculating the profit and determines the feasibility of the product inventory.

Future research can use the price of the product as an endogenous variable to make a joint decision on inventory pricing, and then conduct a comparative study of the two strategies. In addition, from the perspective of consumer behavioral economics, we can study the influence of consumers' aversion psychology on retailer's strategic choices or the influence of consumers' limited choices on retailer's strategic choices.

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