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Asphalt Pavement Roughness Prediction Based on Gray GM(1,1|sin) Model

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ABSTRACT

Roughness is a comprehensive assessment indicator of pavement performance. Prediction of pavement roughness exhibits great difficulties by using traditional methods such as mechanistic-empirical method and regression method. Considering the fact that the value of international roughness index (IRI) varies in a fluctuant manner, in this paper, a new gray model based method is proposed to predict the roughness of pavement. The proposed method adopts GM(1,1|sin) model as the prediction model. In GM(1,1|sin) model, a sinusoidal term is added into GM(1,1) model, making it can fit fluctuant data more precisely than GM(1,1) model. A particle swarm optimization (PSO) algorithm is used to select the optimal parameter of GM(1,1|sin) model. Experimental results demonstrate its effectiveness of the proposed method. Furthermore, the proposed method only uses the history IRI data in prediction and leads to a large savings of collecting pavement condition data.

1. INTRODUCTION

Pavement performance refers to the serviceability provided by pavement for driving under natural environment conditions. It includes the surface structure characteristics, smoothness, damage condition and structural bearing capacity of pavement. The structural characteristics of pavement surface affect the driving safety, which can be evaluated by measuring the friction coefficient between tire and pavement. Pavement smoothness and damage condition are directly related to the speed and comfort of driving, and are the main factors affecting the quality of road driving. Systematic and long-term observation of pavement performance and establishment of pavement performance evaluation system are important components of pavement management system (PMS).

Pavement performance prediction is a primary objective for PMS. Accurate prediction can help decision makers reasonably allocate substantial manpower and capital resource to maintain the pavement and thus prolong its serviceability. Performance indicators are used to evaluate pavement condition and serviceability. There are several indicators, such as the present serviceability index (PSI), pavement condition index (PCI) and international roughness indictor (IRI), to assess pavement performance. Particularly, IRI is a key measure of pavement performance, which is adopted in the Mechanistic-Empirical Pavement Design Guide (MEPDG). IRI takes into account not only both ride quality and comfort of the pavement, but also serving as an indicator of the presence of collective distresses [1]. Prediction of IRI had received much attention from researchers. Traditionally, multiple linear regression (MLR) was used to predict IRI [2,3]. The regression model took traffic, material, geometric and climatic condition as model variable. However, the pavement performance is affected by many factors and it is difficult to select reasonable model variables to build the regression model. Due to its powerful nonlinearity approximation ability, artificial neural network (ANN) was also adopted to predict pavement performance. In Ref. [4], Attoh-Okine proposed to use ANN to predict roughness progression in flexible pavement. Latter, Robert and Attoh-Okine performed a comparative analysis of two ANNs in the prediction of pavement performance [5]. In Ref. [6], Choi *et al.* adopted back-propagation neural network to predict pavement roughness using long-term pavement performance (LTPP) data. In Ref. [1], a hybrid gene expression programming neural network was used to predict pavement roughness. It was found that ANN can obtain better prediction performance than MLR. Prediction model based on ANNs can be considered as nonlinear version of MLR, it also needs to select approximated model variables or factors. Another method for pavement performance prediction is the Markov probability decision process [7]. This method regards the pavement performance as a stochastic variable. Many research results show that pavement roughness prediction involves complex relationship and it is not clear to us [8]. On the other hand, build linear or nonlinear regression model need to collect large number of data about pavement characteristics. The cost of gathering pavement characteristic is very high. Therefore, predicting pavement performance using a smallest set of actual measured characteristics can save large cost for agencies responsible for the pavements [5].

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Gray theory (GT) is a mathematical tool dealing with uncertain system with partial known information. It is widely used in different prediction tasks. As stated before, what factors mostly related with the pavement performance is still not clear to us. It is possible to use GT to deal with the prediction of pavement performance. In Ref. [9], Li *et al.* studied the feasibility of using gray model to predict pavement smoothness. Jiang and Li used gray model to estimate IRI for the Indiana Department of Transportation (INDOT) [10]. In Ref. [8], Wang and Li proposed to use fuzzy and GT to predict pavement smoothness, in which the fuzzy linear regression was used to calculate the coefficients of gray model. However, the above GT based methods still use multiple variables to predict pavement roughness. They possesses the same problem associated with the previous methods.

In this paper, a new GT based method is proposed to predict the roughness of pavement. The proposed method only needs some initial IRI data and results in large savings in gathering pavement characteristics. Considering the fact that the actual IRI data is fluctuant, and is not strict increasing or decreasing, the gray GM(1,1|sin) model is adopted. To select the optimal parameters of GM(1,1|sin) model, the particle swarm optimization (PSO) algorithm is used. The rest of this paper is organized as follows: In section 1, the gray GM(1,1|sin) model is introduced.

2. GM(1,1|sin) MODEL

Let $x^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right)^T$ be original data sequence with n samples. The accumulated generating operation (AGO) applied to $x^{(0)}$ is defined as

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i). \tag{1}$$

The sequence $x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\}$ is called 1-th order AGO sequence of original sequence $x^{(0)}$. Let

$$z^{(1)}(k) = \frac{x^{(1)}(k-1) + x^{(1)}(k)}{2}. \tag{2}$$

Sequence $z^{(1)} = \left(z^{(1)}(1), z^{(1)}(2), \cdots, z^{(1)}(n)\right)^T$ is called the mean generated sequence.

Definition 1. *Equation*

$$x^{(0)}(k) + az^{(1)}(k) = b_1 \sin pk + b_2 \tag{3}$$

is called gray GM(1,1|sin) model.

Theorem 1. *Let $x^{(0)}$ be the original non-negative sequence, and $x^{(1)}$ be the first order AGO sequence, $z^{(1)}$ be the mean generated sequence. Denote $\theta = [a, b_1, b_2]^T$ be the parameters of gray model (3), and*

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & \sin 2p & 1 \\ -z^{(1)}(3) & \sin 3p & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & \sin np & 1 \end{bmatrix}, \tag{4}$$

then the estimated parameters of gray model (3) satisfies $\hat{\theta} = (B^TB)^{-1}B^TY$.

Proof. Substituting the data into equation $x^{(0)}(k) + az^{(1)}(k) = b_1 \sin pk + b_2$ yields

$$\begin{aligned} x^{(0)}(2) + az^{(1)}(2) &= b_1 \sin 2p + b_2 \\ x^{(0)}(3) + az^{(1)}(3) &= b_1 \sin 3p + b_2 \\ &\vdots \\ x^{(n)}(n) + az^{(1)}(n) &= b_1 \sin np + b_2. \end{aligned}$$

Writing the above equations into matrix form, one has $Y = B\theta$. Replacing $x^{(0)}(k)$ with $-az^{(1)}(k) + b_1 \sin kp + b_2$, one can get the error sequence

$$\varepsilon = Y - B\hat{\theta}. \tag{5}$$

Let $s = \varepsilon^T\varepsilon = \left(Y - B\hat{\theta}\right)^T\left(Y - B\hat{\theta}\right)$, the best estimation of a, b_1 and b_2 satisfies the Eq. (6) (see the next page). Writing the above equations into matrix form, one can get

$$\begin{cases} \dfrac{\partial s}{\partial a} = 2\displaystyle\sum_{k=2}^{n}\left(x^{(0)}(k) + az^{(1)}(k) - b_1 \sin kp - b_2\right) z^{(1)}(k) = 0 \\ \dfrac{\partial s}{\partial b_1} = -2\displaystyle\sum_{k=2}^{n}\left(x^{(0)}(k) + az^{(1)}(k) - b_1 \sin kp - b_2\right) \sin pk = 0 \\ \dfrac{\partial s}{\partial b_2} = -2\displaystyle\sum_{k=2}^{n}\left(x^{(0)}(k) + az^{(1)}(k) - b_1 \sin kp - b_2\right) = 0 \end{cases} \tag{6}$$

Writing the above equations into matrix form, one can get

$$\Phi\hat{\theta} = \Gamma \tag{7}$$

where

$$\Phi = \begin{bmatrix} \sum_{k=2}^{n}\left[z^{(1)}(k)\right]^2 & -\sum_{k=2}^{n}\left[z^{(1)}(k)\right]\sin pk & -\sum_{k=2}^{n}\left[z^{(1)}(k)\right] \\ -\sum_{k=2}^{n}\left[z^{(1)}(k)\right]\sin pk & \sum_{k=2}^{n}\left(\sin pk\right)^2 & \sum_{k=2}^{n}\left(\sin pk\right) \\ -\sum_{k=2}^{n}\left[z^{(1)}(k)\right] & \sum_{k=2}^{n}\left(\sin pk\right) & \sum_{k=2}^{n} 1 \end{bmatrix}, \tag{8}$$

and

$$\Gamma = \begin{bmatrix} \sum_{k=2}^{n} x^{(0)}(k)\, z^{(1)}(k) \\ \sum_{k=2}^{n} x^{(0)}(k) \sin pk \\ \sum_{k=2}^{n} x^{(0)}(k) \end{bmatrix} \tag{9}$$

It can be verified that $\Phi = B^TB$ and $\Gamma = B^TY$. Therefore, $B^TB\hat{\theta} = B^TY$. It is easy to get $\hat{\theta} = (B^TB)^{-1}B^TY$. This completes the proof.

Definition 2. *Let $x^{(0)}$ be a non-negative sequence, and $x^{(1)}$ is the first order AGO sequence of $x^{(0)}$. $z^{(1)}$ is the mean generated sequence, $\theta = \left(B^TB\right)^{-1}B^TY$. The following equation*

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b_1 \sin pt + b_2, \tag{10}$$

is called the whiten equation of GM(1,1|sin) model.

Theorem 2. *Let B, Y, θ as stated in Theorem 1, and* $\theta = (B^TB)^{-1}B^TY$. *Then, the solution or time response of whiten Eq. (10) is*

$$x^{(1)}(t+1) = \left(x^{(1)}(1) + \frac{b_1 p}{a^2+p^2} - \frac{b_2}{a}\right)e^{-at} + \frac{b_1}{a^2+p^2}\left(a\sin pt - p\cos pt\right) + \frac{b_2}{a} \tag{11}$$

Proof. The homogeneous equation of Eq. (10) is

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = 0, \tag{12}$$

the general solution of the homogeneous equation is

$$x(t) = e^{-at}. \tag{13}$$

Assuming the particular solution of Eq. (10) is

$$x(t) = e^{-at}h(x).$$

Substituting it into Eq. (10), one has

$$\left[e^{-at}h(x)\right]' + ae^{-at}h(x) = b_1\sin pt + b_2 \tag{14}$$

From Eq. (14), it follows that

$$h(x) = \int e^{at}\left(b_1\sin pt + b_2\right)dt \tag{15}$$

Calculating the integral Eq. (15), one can get

$$h(x) = \frac{b_1}{a^2+p^2}e^{at}\left(a\sin pt - p\cos pt\right) + \frac{b_2}{a}e^{at} + ce^{at}, \tag{16}$$

where c is undetermined coefficient. So, the general solution of the whiten Eq. (10) is

$$\begin{aligned} x^{(1)}(t+1) &= e^{-at}h(x) \\ &= \frac{b_1}{a^2+p^2}\left(a\sin pt - p\cos .pt\right) + \frac{b_2}{a} + ce^{-at}. \end{aligned} \tag{17}$$

When $t = 0$, $x^{(1)}(1) = -\frac{b_1 p}{a^2+p^2} + \frac{b_2}{a} + c$, after some manipulations, the undetermined coefficient is obtained as $c = x^{(1)}(1) + \frac{b_1 p}{a^2+p^2} - \frac{b_2}{a}$. Therefore, the solution of the whiten equation is

$$x^{(1)}(t+1) = \left(x^{(1)}(1) + \frac{b_1 p}{a^2+p^2} - \frac{b_2}{a}\right)e^{-at} + \frac{b_1}{a^2+p^2}\left(a\sin pt - p\cos pt\right) + \frac{b_2}{a}. \tag{18}$$

This completes the proof.

From Theorem 2, we can get the time response sequence of gray GM(1,1|sin) model is

$$x^{(1)}(k+1) = \left(x^{(0)}(1) + \frac{b_1 p}{a^2+p^2} - \frac{b_2}{a}\right)e^{-ak}v \quad v + \frac{b_1}{a^2+p^2}\left(a\sin pk - p\cos pk\right) + \frac{b_2}{a} \tag{19}$$

Also, one can get the predicted value of original sequence as

$$\begin{aligned} \hat{x}^{(0)}(k+1) &= x^{(1)}(k+1) - x^{(1)}(k) \\ &= \left(1 - e^{a}\right)\left(x^{(0)}(1) + \frac{b_1 p}{a^2+p^2} - \frac{b_2}{a}\right)e^{-ak} \\ &\quad + \frac{b_1}{a^2+p^2}\left(a\sin pk - p\cos pk\right) \\ &\quad - \frac{b_1}{a^2+p^2}\left(a\sin p(k-1) - p\cos p(k-1)\right). \end{aligned} \tag{20}$$

3. PARAMETER OPTIMIZATION OF GM(1,1|sin) MODEL

In GM(1,1|sin) model, the parameter p can not be determined from Theorem 1. However, the parameter p has great effect on the prediction accuracy. Therefore, it should be carefully selected. In this paper, the parameter p is optimally selected by minimizing a criterion function. Mean absolute percentage error (MAPE) is a common criterion of judging the merits of a model. In this paper, the parameter p is determined by minimizing MAPE, i.e.,

$$\begin{aligned} p^* &= \arg\min \text{MAPE}\left(p\right) \\ &= \frac{1}{n-1}\sum_{k=2}^{n}\frac{|\hat{x}^{(0)}(k) - x^{(0)}(k)|}{x^{(0)}(k)} \times 100\%. \end{aligned} \tag{21}$$

Since the MAPE is not a analytic function of parameter p, one can not use traditional optimization method to find the optimal value of p. In this paper, PSO algorithm is adopted to find the optimal value of p.

PSO is a population based heuristic search algorithm, which was proposed by Kennedy and Eberhart in 1995 [11]. In PSO, each individual is called a particle, which represents a candidate solution of an optimization problem. In d-dimensional search space, each particle has a position $X_i = (x_{i1}, x_{i2}, \cdots, x_{id})$ and a velocity $V_i = (v_{i1}, v_{i2}, \cdots, v_{id})$, where i denote the i-th particle in the swarm. In the process of searching, all particles fly through the search space, and adjust its velocity and position to find a better solution (position) according to its own experience and experience of neighboring particles iteratively. Let $P_i(t) = \left(p_{i1}, p_{i2}, \cdots, p_{id}\right)$ denotes the best position found by particle i within t iteration steps, $P_g(t) = \left(p_{g1}, p_{g2}, \cdots, p_{gd}\right)$ denotes the best position among all particles in the swarm so far. Particles update their positions and velocities as shown in the following formulas:

$$\begin{aligned} V_i(t+1) &= wV_i(t) + c_1r_1(P_i(t) - X_i(t)) \\ &\quad + c_2r_2\left(P_g(t) - X_i(t)\right) \end{aligned} \tag{22}$$

$$X_i(t+1) = X_i(t) + V_i(t+1), i = 1, 2, \cdots, n, \tag{23}$$

where n denotes the number of particles in the swarm, $V_i(t)$ and $X_i(t)$ represent the velocity and position of particle i in the solution space at t-th iteration step respectively; r_1 and r_2 are two random numbers uniformly distributed in the range $[0, 1]$; c_1 and c_2 are acceleration constant, usually $c_1 = c_2 = 2.0$; w is the inertia weight. Generally, the value of each component in V_i can be clamped to the range $[V_{min}, V_{max}]$ to control excessive roaming of particles outside the search space. Each particle flies toward a new position according to Eqs. (22) and (23). In this way, all particles of the swarm find their new position and apply these new position to update their individual best position $P_i(t)$ and global best position $P_g(t)$ of the swarm. This process is repeated until a user-defined stopping criterion, usually is maximum iteration number t_{max}, is reached. Usually, the inertia weight w varies with a linearly decreasing manner as

$$w = w_{max} - \frac{w_{max} - w_{min}}{t_{max}} \times t, \tag{24}$$

where w_{max} and w_{min} denote the maximum and minimum of inertia weight.

4. THE PROCESS OF IRI PREDICTION USING GM(1,1|sin) MODEL

In this section, the detailed process of using GM(1,1|sin) model to predict IRI is summarized as follows:

Step 1 Preparing the original sequence of IRI $x^{(0)}$.

Step 2 Generating the first order AGO sequence $x^{(1)}$ and mean generated sequence $z^{(1)}$.

Step 3 Randomly initializing a set of candidates parameters for p in a certain range.

Step 4 For each p, obtain the parameter of Eq. (3) according to Theorem 1 and calculate the predicted value according to Eqs. (19) and (20).

Step 5 Updating the velocity and position for each particles.

Step 6 Updating the global and local best of the particle swarm.

Step 7 Repeat Steps 4 and 6 until the best p is found.

Step 8 Using the best p, obtain the parameters of GM(1,1|sin) model Eq. (3) according to Theorem 1, and calculated the predicted value according to Eqs. (19) and (20).

5. RESULTS

To demonstrate the effectiveness of the proposed method, a prediction experiment is conducted. The data of IRI is taken from Ref. [5], which comes from PMS of Kansas Department of Transportation (KDOT). In KDOT, roughness is used as a primary indicator of a pavement's distress state and performance level. The IRI data used in this paper is listed in Table 1.

In this paper, the GM(1,1), GM(1,1|sin) and fractional GM(1,1) [12] (FGM(1,1)) model are all implemented to predict IRI and their prediction results are compared each other. The fractional order r of FGM(1,1)model are also optimally selected using PSO as our proposed method. In experiments, the parameters of PSO is set as $c_1 = c_2 = 2$, $w_{\max} = 0.9$, $w_{\min} = 0.4$, the maximum iteration number is 1000. After iterations, the optimal value of p found by PSO is 1.11417. To objectively evaluate the prediction performance, the prediction error, absolute percentage error (APE), MAPE, root mean squared error (RMSE) are adopted. These indices are defined as

$$\text{Error} = \hat{x}(k) - x(k). \tag{25}$$

where $\hat{x}(k)$ is the prediction value and $x(k)$ is the true value.

$$\text{APE} = \frac{|\hat{x}(k) - x(k)|}{x(k)} \times 100\%, \tag{26}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{k=1}^{n} |\hat{x}(k) - x(k)|^2}. \tag{27}$$

Table 2 lists the prediction results of the GM(1,1), FGM(1,1) and GM(1,1|sin) model. To see the prediction results more intuitively, Fig. 1 shows the true value, predicted value of GM(1,1) and GM(1,1|sin) model. The RMSE and MAPE are shown in Fig. 2. From Table 2, it can be seen that the prediction of GM(1,1|sin) model are more accurate than that of GM(1,1) and GM(1,1|sin) model. Although the MAPE of GM(1,1|sin) is slightly higher than FGM(1,1) model, the GM(1,1|sin) model can follow the fluctuant characteristics of original data, but the GM(1,1) and FGM(1,1) model can not. The reason behind this is that GM(1,1|sin) introduces a sinusoidal term, making it has fluctuant property.

Table 1 The value of IRI.

Number	1	2	3	4	5	6	7	8	9	10
IRI	191	147	85	107	108	74	77	50	77	97

IRI, international roughness indictor.

Table 2 Prediction results of GM(1,1), FGM(1,1) and GM(1,1|sin) model.

Number	Actual	GM(1,1)			FGM(1,1)			GM(1,1|sin)		
		Prediction	Error	APE	Prediction	Error	APE	Prediction	Error	APE
1	191	-	-	-	-	-	-	-	-	-
2	147	121.0217	25.9783	17.6723	148.4951	−1.4943	1.0165	111.6680	35.3320	24.0354
3	85	112.1668	−27.1668	31.9609	104.1544	−19.1548	22.5351	104.8937	−19.8937	23.4044
4	107	103.9597	3.0403	2.8414	90.3297	16.6699	15.5793	108.6216	−1.6216	1.5155
5	108	96.3532	11.6468	10.7841	83.607	24.3927	22.5858	108.0000	0.0000	0.0000
6	74	89.3032	−15.3032	20.6800	80.0007	−6.0009	8.1093	94.1164	−20.1164	27.1844
7	77	82.7690	−5.7690	7.4922	78.1114	−1.1115	1.4435	74.4305	2.5695	3.3370
8	50	76.7130	−26.7130	53.4259	7.012	−27.3011	54.6022	63.9297	−13.9297	27.8593
9	77	71.1000	5.9000	7.6623	77.235	−0.2332	0.3028	67.4232	9.5768	12.4374
10	97	65.8978	31.1022	32.0642	77.7127	19.2877	19.8842	73.8282	23.1718	23.8884

APE, absolute percentage error.

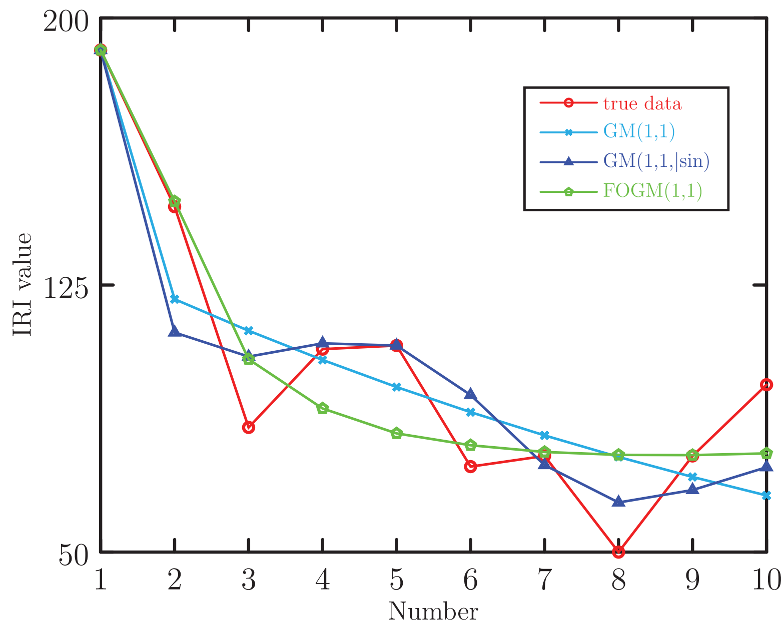

Figure 1 Comparison of predicted value of GM(1,1), FGM(1,1) and GM(1,1|sin) model.

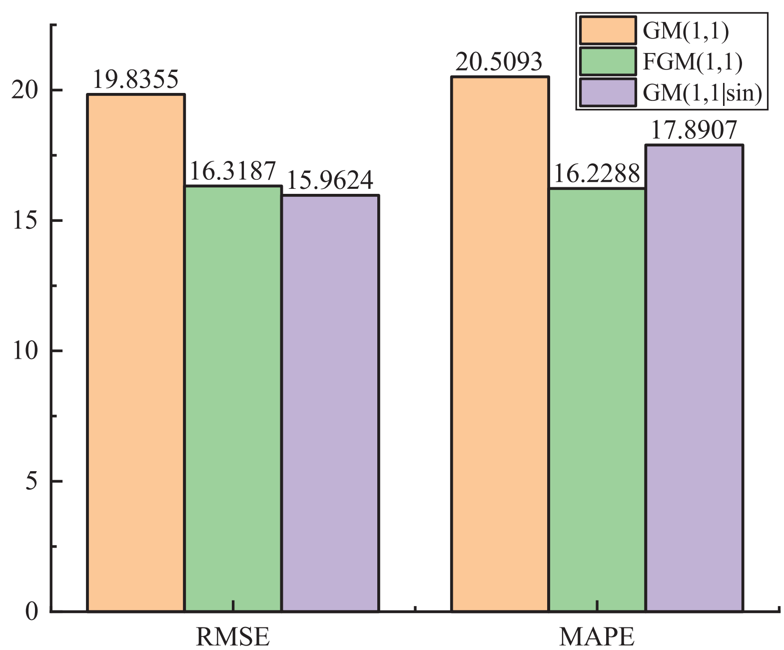

Figure 2 Comparison of root mean squared error (RMSE) and mean absolute percentage error (MAPE) of GM(1,1), FGM(1,1) and GM(1,1|sin) model.

6. CONCLUSIONS

In this paper, a new method based on gray theory is proposed for the purpose of predicting pavement roughness. Considering the IRI data is fluctuant, a sinusoidal term is added into gray GM(1,1) model to enhance the prediction accuracy. The parameter of sinusoidal term is optimally determined using PSO algorithm. Results show that the proposed method improve the prediction accuracy.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHORS' CONTRIBUTIONS

Chunming Ji conceives the idea of the paper, and Xiuli Zhang implements the algorithm. Xiuli Zhang wrote the original manuscript and Chunming Ji revised it.

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