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Research on Teaching Design Based on BOPPPS Model

—Taking 'Gauss formula' for example

Lijun Mu*

School of Mathematics and Physics

Qingdao University of Science and Technology

Qingdao, China

***Abstract*—The design method of classroom teaching based on BOPPPS is explored by taking "Gauss formula" in Higher Mathematics for example. Starting from the cultivation on students' consciousness of question and investigation, the interactive teaching guided by the prior knowledge is strengthened based on layers of questions and reasoning, students are guided to finally find Gauss formula by themselves by the mean of previous knowledge so as to realize the innovation. The student-oriented modular teaching thought is expressed in the teaching design in order to improve the effectiveness of classroom teaching.**

Keywords—BOPPPS; Gauss Formula; Teaching Design; Discovery

I. INTRODUCTION

Gauss formula is an important theorem connecting surface integral and triple integral in multi-function integral calculus of *Higher Mathematics*, which establishes the bridge relationship between surface integral and triple integral and plays an important role as a bridge in the theoretical research of physics [1]. The formula is directly given in the teaching method and it's certified with the help of numerical integration transforming relationship, which expresses the abstraction and precision of mathematics; however, the difficulties that students possibly encounter and the research process on the new knowledge are not expressed during the teaching process, so the students are forced to accept such complicated formula and the formula can only be used by memory method, which is obviously not favorable for the improvement of learning ability, and the students' learning interest and self-innovation ability is hard to cultivate. The new design is performed combined with the information teaching platform "Rain Classroom" through the teaching process of bridge-in, outcome, pre-test, participation, post-test, summary and thought based on the "students-orientation idea" as well as modularized teaching design mode BOPPPS with the aim of mobilizing the learning enthusiasm of all the students [2,3]; students are guided to explore new questions and solutions based on students' previous knowledge so as to ultimately find the Gauss formula, then the strict explanation and certification is given to complete the careful confirmation. It's shown in the teaching effect that the students' participation is obviously improved. The teaching design link of BOPPPS concept is given.

II. 'GAUSS FORMULA' TEACHING DESIGN BASED ON BOPPPS MODEL

A. Review and Retrospect, Activation on Previous Knowledge, Questions& Answers, Bridge-in of Life Phenomenon—Bridge-in and Pre-Assessment

Retrospect on all of the integral types: definite integral, multiple integral, line integral and surface integral. Retrospect on the guiding concept and strategy during the establishment process of integral concept: microelement analysis method (reduce the large scale, variable replaced by constant, near to the sum and take the limit), the essence lies in "infinitesimal is obtained through variable replaced by constant in order to get the integral from the limit". Make preparations for the knowledge and thought.

Retrospect on the calculation method of various integrals: the unknown questions are transferred to the known knowledge: the double integral is transferred to definite integral; triple integral is transferred to double integral; the line integral is transferred to definite integral; the surface integral is transferred to double integral; the key points for integral transfer lie in the confirmation of integral areas and the relationship confirmation among the variables, various calculation formula can be used to check if the students have understand it. Make preparations for the calculation and methodology.

Students are guided to observe the discipline: the Newton Leibniz Formula is presented: the definite integral can be expressed by the function value of interval point; Green Formula: the double integral can be expressed by the line integral at the boundary curve; then the relationship between the inner and boundary is revealed. So the space boundary of triple integral is closed surface, we can't help asking if it can be expressed by the surface integral at the boundary? Analyze task, successfully transfer and introduce the following text.

Natural phenomenon in life (see Fig.1): Sun has just risen with glorious shining; vast sky in the night with beautiful fireworks; spring with surging flow and scattered into the lake. The phenomenon scattering to the surroundings in the life may be bright or dark, may be quick or slow, so the questions that the intensity measurement is scattered to the outside are produced and considered: what's related to scattering intensity?

Fig. 1. Natural phenomenon in life

Answers: it's related to the flow from the centrosome, and it's also relevant to the volume of the centrosome. But the key point lies in the flow calculation. The mathematical model solution is established by taking solar radiation as the example (the common sense in life that's easy to understand). The learning enthusiasm is inspired by the introduction of life phenomenon, meanwhile students' observation and thoughts can be produced for sake of making preparations for the successful entry of formula and breaking through the difficulties. The intuitive relationship map is established to refine the thought; the questions shall be naturally introduced so as to imperatively realize the task transfer.

B. Mutual Research—Objective

According to the function of Gauss formula in higher mathematics system and the application in physics, the classroom teaching objective is confirmed as: the integrand of triple integral is searched from the physical significance so as to confirm the workable conditions and formula content of Gauss formula, the Gauss formula can be strictly certified by the calculation transfer method so as to define the physical significance of flux and divergence; the Gauss formula can be used to realize the integral transfer so as to reach the purpose of simplified integral calculation. The key points of teaching are to understand the content of Gauss formula and use it to perform calculation. The difficulty in teaching is the certification of Gauss formula and physical significance of flux and divergence.

C. Discovery and Retrospect on the Previous Knowledge—Participation

Firstly, the mathematization of specific questions is realized so as to establish reasonable mathematics model. The boundary surface of the closed region is the smooth surface, the velocity field of solar radiation is set as the vector, written as $\vec{A} = \boldsymbol{P}(x,y,z)\vec{i} + \boldsymbol{Q}(x,y,z)\vec{j} + \boldsymbol{R}(x,y,z)\vec{k}$, the same vector is calculated through two aspects of physical significance on the 2nd surface integral and the flow expression from the infinitesimal method and the produced flow so as to obtain the flow expression of the closed region:

$$\Phi = \iint_{\Sigma} \boldsymbol{P}dydz + \boldsymbol{Q}dxdz + \boldsymbol{R}dxdy$$

According to the energy conservation law, the flow from the sun (this is just why it's extracted from the outside of the curved surface) is the one produced by it, and the produced flow is the sum of flow at each point, so the flow produced from the sun can be obtained by means of infinitesimal method; firstly, the flow of element produced from the volume of element is confirmed as $\mathbf{d\Phi}$, then the triple integral is calculated to get another expression of flow:

$$\Phi = \iiint_{\Omega} d\Phi$$

From this we can obtain the relationship between the surface integral and triple integral, so the question is transferred to obtain the expression of flow element so as to realize the breakthrough on the difficulties and put emphasis on the key points.

Looking for the expression of flow element $\mathbf{d\Phi}$, the volume element is assumed as small cuboid, the six sides are paralleled to the coordinate surface, the produced flow is obtained along with six sides, the expression of flow element at the left/right side can be explained to serve as an example; similarly, the expression of front/back and up/down can be obtained. The velocity vertical to the left/right velocity field is $Q(x,y,z)$, the area is $\Delta x\Delta z$, and only y changes along with the direction of y axis, that's:

$$\Delta\boldsymbol{\Phi}_{left/right} = |\boldsymbol{Q}(x, y+\Delta y, z) - \boldsymbol{Q}(x,y,z)|\Delta x\Delta z$$

If the partial derivative for independent variable y exists in function $Q(x,y,z)$ and it's continuous, it can be obtained from Lagrange Mean Value Theorem and continuous function:

$$\Delta\boldsymbol{\Phi}_{left/right} \to \frac{\partial \boldsymbol{Q}}{\partial y}(x,y,z)\Delta x\Delta y\Delta z$$

So its expression of left/right flow element is:

$$\mathbf{d}\boldsymbol{\Phi}_{left/right} = \frac{\partial \boldsymbol{Q}}{\partial y}\mathbf{d}V$$

Similarly, we can get:

$$\mathbf{d}\boldsymbol{\Phi}_{front/back} = \frac{\partial \boldsymbol{P}}{\partial x}\mathbf{d}V, \mathbf{d}\boldsymbol{\Phi}_{up/down} = \frac{\partial \boldsymbol{R}}{\partial z}\mathbf{d}V$$

So the triple integral is calculated to get another expression of flow:

$$\Phi = \iiint_{\Omega} (\frac{\partial \boldsymbol{P}}{\partial x} + \frac{\partial \boldsymbol{Q}}{\partial y} + \frac{\partial \boldsymbol{R}}{\partial z})dV$$

Therefore, the expressions of two flows are the same, the theorem of Gauss formula can be obtained according to the deduction process.

Theorem (Gauss formula): Let Ω be the closed region which is surrounded by the scattered smooth closed curved surface Σ, the direction is subject to the outside, the function P, Q and R exist continuous partial derivative in Ω, then

$$\iint_{\Sigma} \boldsymbol{P}dydz + \boldsymbol{Q}dxdz + \boldsymbol{R}dxdy = \iiint_{\Omega} (\frac{\partial \boldsymbol{P}}{\partial x} + \frac{\partial \boldsymbol{Q}}{\partial y} + \frac{\partial \boldsymbol{R}}{\partial z})dV.$$

Establish students' learning confidence, inspire enthusiasm and ignite the hope by guiding students to complete the discovery of Gauss formula. Then the thought on certification and analogical Green formula is introduced through the explanation on the history, the surface integral is transferred to

the double integral about the *xy* through the calculation rule of "fist projectting, second substituting and third fixed sign", the triple integral is transfer to the double integral of *xy* according to the "projection method", it's certified that partial surface integral equals to partial triple integral, and the changing of projection coordinate surface can demonstrate the rest parts. The specific process is neglected here. Students realize the certification on the Gauss formula due to their enthusiasm on the formula discovery with the help of the known integral calculation method, at the same time, not only the previous knowledge is reviewed, but also its application can be found so as to reach the dual objectives that the previous and current knowledge can be reviewed.

D. Formula Application and Examination on the Comprehensive Ability—Post-Assessment

We can obtain the flow expression from two aspects by analyzing the presented questions, one is the integral for closed surface and the other is triple integral. How to calculate the scattering intensity of the point in the centrosome? The average scattering intensity within small closed area shall be firstly considered based on the limit thought, then the closed area is compressed to the unsolved point. We reasonably choose the flow expression and triple integral so as to obtain the expression of the scattering intensity at certain point according to the mean value theorem of integrals and the definition of continuity, which is just the each component for vector that's corresponding to the sum of the partial derivative in independent variable, and it's defined as the scattering for the vector field.

What is the rest of application? The application of Gauss formula in the calculation of surface integral can be presented by means of analogical Green formula, pay attention to the application conditions: closed surface outside—construct the closed surface (easy borrowing and lending) to deepen impression by means of examples.

E. Summarizing and Purifying the Thoughts to Obtain Further Thought—Summary

"Unknown transfer" assists us to complete innovation during the discovery process of Gauss formula. The repeatedly applied "infinitesimal method", "substitution method" and "analogy method" in higher mathematics are also used in this process, and the innovation is realized based on the previous knowledge. This kind of method is not only suitable for the mathematical research, but also broadly applied for other scientific areas. Retrospect on the class content: the terms and conclusions of Gauss formula and divergence concept. The thinking method is summarized: "infinitesimal method", "unknown transfer" and "analogy method". The new thought is introduced: exploration on application. The key content in this chapter is summarized with students: the theorem conditions and conclusions of Gauss formula; retrospect on the discovery process, understand the teaching thought and method; finally the new thought is introduced so as to realize that principle of never too old to learn, questions are followed by questions. The human life is just a process to continuously solve questions, the lesson is the same.

F. Homework and Continuous Exploration—Post-Assessment

We make certain simplified consumption during the demonstration process of Gauss formula, and whether we can use the known knowledge to realize when the consumption is not workable. Get more understanding and grasp Gauss formula during the study so as to realize extrapolation. The knowledge transfer is realized through homework. 1. If the Gauss formula is still workable when the closed surface is the ring curve surface? 2. Establish the relationship of the first-class curved surface area and triple integral. 3. Please think about the previously studied Newton-Leibniz formula and Green formula so as to find the difference and try to present new questions. The Gauss formula is used to review the classical law, increase learning interest, express student's value and think about the actual questions so as to realize the knowledge expansion. 4. The Gauss formula is used to certify the Archimedean principle of buoyancy.

The after-class discussion is performed through the platform of Rain Classroom.

G. Teaching Reflection and Summary

The new questions are presented to attract students' attention based on the BOPPPS concept from the perspective of the previous knowledge, and cultural cultivation is endowed when the knowledge is studied. Diversified analysis is performed on the abstraction formula which is hard to understand combined with the physical phenomenon, the previous knowledge and thinking method is used to guide students' reasoning so as to strengthen students' participation and self-confidence; the final thinking questions are left to inspire students' in-depth consideration and perform innovation. The disadvantage lies in the weak preciseness, because there's no specific explanation for the content that's easy to observe due to the time limit. The cultivation of independent innovation ability is always strengthened in the teaching process [4].

Make efforts to cultivate students' mathematical thoughts imperceptibly when students are interested in it. Let the mathematical thoughts become a kind of habit, including unknown transfer, infinitesimal analysis, analogy, bold guess and careful inference, which are favorable for students to become comprehensive talents with independent innovation.

Based on five-star teaching design: focusing on questions--activating the previous knowledge--verifying new knowledge--using application proficiently---analogical and comprehensive design process, mixed with the edification of humanistic quality to realize students' whole process participating in the new knowledge, the questions and inspiration are taken as the main methods for the whole teaching process. It's promoted step by step, just like the detective novel that makes students can't get rid of it. The different expressions of flow are obtained from different angles so as to make students better understand the conclusions of Gauss formula and apply the previous knowledge to verify the new one. It establishes students' self-confidence, expresses students' self-value and gets close to the mathematical scholars.

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