Study on Endogenous Population Growth and Effectiveness of Fertility Policy
-- Theoretical Framework based on Dynamic General Equilibrium

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Abstract. Assessing Fertility Policy effectiveness by the theory of endogenous population growth will provide the theoretical references for universal two-child policy. We have established a dynamic general equilibrium mode includes family, business and government, 3 kinds of Fertility Policies about quantity, economy and child-rearing time are brought into the mode. The endogenous population growth mechanisms are deduced and the effectiveness of different Fertility Policy is assessed by the numerical simulation. The results have shown there are mutual constraint relationships between population growth and economic development. Economical Fertility Policies have the strongest effect to adjust fertility rate, but may be cause fertility accumulation; the effect of polices about child-rearing time occupies second place; the quantitative Fertility Policy effect is the weakest. The above conclusions have implications for optimization of current China's Fertility Policy.

Keywords: Endogenous Population Growth; Fertility Policy; Child-rearing Costs.

1. Introduction

With the long-term continuation of family planning policy, China's population has changed into a new growth mode of "low birth, low death and low growth". After 2009, the natural growth rate of population has dropped to less than 5‰[1], which has caused many problems, such as population aging, human capital shortage, increasing pressure on social security and public service supply. In view of the fact that the population base is huge and the fertility rate is low. However, survey data and statistical predictions show that the fertility rate has not risen significantly since the introduction of the policy of " two-child fertility policy for couples where either the husband or the wife is from a single-child family " or " the universal two-child policy ".[2]Since the 1970s, the significant adjustment of fertility policy has produced quite different policy effects, which has aroused people's thinking and controversy about the mechanism of population growth and the effectiveness of fertility policy. An in-depth analysis of the above issues is of great significance for deepening the theory of population growth, formulating population policies scientifically, and actively dealing with the problem of "population dividend" loss.

2. Review of Research

The study of the mechanism of population growth can be traced back to the 18th century Malthus Population Theory. [3] Subsequently, some derivative theories such as "population crisis theory", "moderate population theory", "population transition theory" and "zero population growth theory" were formed. .[4]The above theory reveals the law of population growth based on the matching relationship between population size and economic development, which has enlightening significance for explaining the mechanism of population growth. However, the hypothesis that population growth is exogenous to the economic system is unreasonable. Since the 1950s, scholars have tried to study the endogenous mechanism of economic development and income level affecting fertility.

Leibenstein (1954) put forward the cost-benefit theory of family reproductive behavior decision-making, believing that with the increase of family income, the income of child-bearing decreases and the cost increases. The family fertility tends to decrease. Becker (1960) incorporated fertility as an endogenous variable into the economic system and constructed a theoretical framework for the dynamic relationship between family fertility and income, thus it opened up the exploration of the mechanism of endogenous population growth. Barro and Becker (1989) deduced the classical theoretical explanation of negative correlation between technological progress, income increase and population growth in western countries by constructing an endogenous population growth model. Becker et al. (1990), Galor and Weil (2000), Zhuangziyin (2002), Xu Chaoyang and Lin Yifu (2009) continuously improved the theory of endogenous population growth.

Several major adjustments to China's fertility policy have aroused discussions among scholars on the effectiveness of the policy. Hong Guodong (1997), Guo Zhenwei and Chen Zaihua (2001) argue that strict family planning policies effectively control China's fertility and population growth. However, Zou Zhizhuang (2005) pointed out that China's family planning policy began in the 1970s, but since the founding of the People's Republic of China, the fertility rate has declined. He believed that it was not the family planning policy but the economic factor that caused the fertility rate to decline. Li Xinyun et al. (2014) pointed out that the policy of " two-child fertility policy for couples where either the husband or the wife is from a single-child family " can alleviate the problems of low fertility, aging and labor shortage in China. Yuanxin (2016) believes that the " the universal two-child policy " policy is the continuation and improvement of family planning policy, and has positive significance for the coordinated development of population, economy, society, resources and environment. However, Zhou Changhong and Chen Youhua (2013), Zhang Liping and Wang Guangzhou (2015) and Song Jian (2016) respectively studied from the perspective of fertility model prediction, fertility willingness survey, fertility policy environment and objectives, and found that the current low fertility problem could not be solved immediately by two-child fertility policy for couples where either the husband or the wife is from a single-child family or the universal two-child policy.

Generally speaking, the use of endogenous population growth theory to study fertility and population growth mechanism has been unanimously agreed by scholars. However, there are great differences on the effectiveness of fertility policy. Incorporating fertility policy into endogenous population growth model is a feasible way to evaluate the effectiveness of fertility policy. However, few studies have been done in this field. The following research arrangements are as follows: Firstly, the endogenous population growth model including family, enterprise and government departments is constructed, and the behavior decision-making process of each department is analyzed by general equilibrium method; secondly, the endogenous population growth mechanism is deduced by numerical simulation technology, and then the fertility policy is evaluated. Finally, the main conclusions and policy implications are summarized.

3. Theoretical Analysis Framework of Endogenous Population Growth Incorporating Fertility Policy

In this part, a dynamic general equilibrium (DGE) model including family, enterprise and government departments is established to describe the economic system including consumption, investment, fertility and production, and the behavior decision equation of each department is obtained through general equilibrium analysis. Under the balanced and unbalanced conditions of the economic system, this paper analyses the endogenous population growth mechanism incorporated into the fertility policy.

3.1 Model Construction and Solution

In the DGE model constructed in this paper, the behavioral decision-making process of three departments is as follows: Families determine fertility, consumption and investment under the constraints of fertility policies, and pay for the economic and time costs of childbearing. Enterprises employ family members and lease family capital to organize production. The government's policy of

giving birth to children through external supply. It should be noted that the fertility policy refers to the norms formulated by the state or under the guidance of the state to regulate the reproductive behavior of couples of childbearing age (including the quantity and quality of fertility). However, the fertility policy discussed in this paper has a broader meaning. It refers to various laws, regulations and policies that can influence the decision-making of family fertility behavior, such as the number of children born, the economic and time cost of raising children.

3.1.1 Family

Assuming that there are many families living indefinitely in the economic system, there is no gender difference among family members. The behavioral decisions of typical families in the t period are as follows: At the beginning of the period, families provide labor L_t and capital K_{t-1} to enterprises with wage rate $w_t$and rent rate r_t respectively. At the end of the period, wages $w_t L_t$ and capital rent $r_t K_{t-1}$ are obtained, and family consumption C_t, capital stock K_t and fertility n_t under the restriction of fertility policy are determined at the same time. And the economic and time cost of raising the new population is paid. Family behavior decision-making aims at maximizing the present value of utility. After eliminating population factors, the per capita intensive form of this optimization problem can be expressed as follows:

$$U = \max_{\{c_t, n_t\}} \sum_{j=0}^{\infty} \beta^j \left[\ln c_{t+j} + \chi \ln\left(1 + n_{t+j}\right) \right]$$

$$\text{s.t.} \begin{cases} \dfrac{w_t l_t + \left[r_t + (1-\delta) - \eta n_t - \phi\left(n_t - n\right) \right] k_{t-1}}{1 + n_t} = c_t + k_t \\ l_t = 1 - \theta n_t \end{cases}$$

Among them, in the utility function U of family members, β and χ represent the subjective discount rate of the family and the degree of altruism of parents towards children respectively; in the budget constraint s.t., $c_t, k_t$and l_t represent the per capita consumption, capital stock and labor input in the t period, and θ represents the time cost ratio of raising a single child in the family, and the δ Indicate the depreciation rate of capital, $\eta n_t k_{t-1}$ represents the direct cost of raising children, $\phi(n_t - n)k_{t-1}$ represents the additional cost or benefit of family fertility n_t deviating from policy fertility n, which constitute the economic cost of raising children. The first-order conditions of c_t and $n_t$can be obtained by solving the dynamic optimization problems of families. See formulas (1), (2).

$$\frac{c_{t+1}}{c_t} = \frac{\beta}{1 + n_{t+1}} \left[r_t + 1 - \delta - \left(\eta + \phi\right) n_{t+1} + \phi n \right] \tag{1}$$

$$\left(\chi - 1\right) c_t = \theta w_t + \left(\eta + \phi\right) k_{t-1} + k_t \tag{2}$$

Formula (1) denotes the behavioral decision-making equation of per capita household consumption-investment. Its economic implication is that the loss of marginal utility of abandoning one unit of consumption in the t period is equal to the present value of marginal utility obtained by investing in it and consuming in the $t + 1$ period. Formula (2) denotes the decision-making equation of family reproductive behavior. Its economic implication is that the marginal benefit of childbearing is equal to the marginal cost of childbearing. Among them, the marginal cost includes the per capita consumption c_t and capital stock k_t of the decrease of the new born population, the marginal economic cost of raising children $(\eta + \phi)k_{t-1}$and the time cost θw_t.

3.1.2 Enterprise

At the beginning of the period, enterprises employ labor L_t and lease capital K_{t-1} to organize production. At the end of the period, enterprises pay wages $w_t L_t$ and capital rent $r_t K_{t-1}$ to families. Cobb-Douglas Production Function Form with no Technical Factor and constant Scale Return in Selection of Production Function:

$$Y_t = K_{t-1}^{\alpha} L_t^{1-\alpha}$$
$$\text{s.t.} \begin{cases} L_t = N_{t-1}\left(1-\theta n_t\right) \\ N_t = N_{t-1}\left(1+n_t\right) \end{cases}$$

Among them, Y_t represents the total output in the t period and alpha represents the marginal output elasticity of capital. In the budget constraint s.t., N_{t-1} denotes the total population at the beginning of the t period; $L_t$denotes the total working population at the beginning of the t period, that is, the total working population at the beginning of the period minus the population who raises children but does not participate in labor; N_t denotes the total population at the end of the t period, that is, the growth rate of the total population is equal to the family fertility rate. Under the above conditions, enterprises pursue profit maximization. Considering population growth, the per capita intensive form of this optimization problem can be expressed as:

$$\Pi = \max_{\{k_{t-1}, l_t\}} \left(\frac{k_{t-1}^{\alpha} l_t^{1-\alpha} - r_t k_{t-1} - w_t l_t}{1+n_t} \right)$$

The first-order conditions of k_{t-1} and $l_t$can be obtained by solving the optimization problem of enterprise production. See formulas (3), (4).

$$r_t = \alpha k_{t-1}^{\alpha-1} l_t^{1-\alpha} \tag{3}$$

$$w_t = \left(1-\alpha\right) k_{t-1}^{\alpha} l_t^{-\alpha} \tag{4}$$

Formulas (3) and (4) represent the demand function of capital and labor respectively. Their economic meaning is that the demand for capital (labor) is equal to the amount of marginal income and cost of capital (labor).

3.1.3 Government

The government is the exogenous supply department of fertility policy, which decides three kinds of policy parameters: (1) policy fertility rate n, that is to say, the number of family children is directly stipulated in the regulations of fertility policy is called quantitative fertility policy; (2) the penalty cost (incentive benefit) coefficient ϕ of fertility policy, in which the penalty cost is expressed as family violation. The economic punishment when the fertility policy (over-birth) is imposed, and the reward income is expressed as the economic reward income when the family is in compliance with the fertility policy (less births), which is called the economic fertility policy; (3) the time-cost ratio of single child's upbringing θ; the state's fertility policy for female workers, basic education and domestic service The development policy will affect the time cost of raising children, which is called the time-based childbearing policy. The government adjusts different types of Fertility Policies by changing the values of three kinds of parameters.

3.2 Equilibrium of Economic System

The DGE model of endogenous population growth incorporating fertility policy includes equation (1) - (4) and four behavioral equations of economic agents, involving exogenous parameters $\{\alpha, \beta, \delta, \chi, \eta, n, \phi, \theta\}$ and endogenous variables $\{c_t, k_t, n_t\}$. When the economic system reaches equilibrium, the products, labor and capital markets are cleared at the same time, and the analytical equations of the main endogenous variables are obtained when the model is stable, as follow:

$$z_{ss} = \frac{1-\beta\left(1-\delta+\phi n\right)}{\alpha\beta} + \frac{\left[1+\beta\left(\eta+\phi\right)\right] n_{ss}}{\alpha\beta} \tag{5}$$

$$n_{ss} = \frac{\alpha\beta\left[\left(\chi-1\right)\varphi_{ss} - 1 - \eta - \phi\right] - \theta\left(1-\alpha\right)\left[1-\beta\left(1-\delta+\phi n\right)\right]}{\theta\alpha\beta\left[\left(\chi-1\right)\varphi_{ss} - 1 - \eta - \phi\right] + \theta\left(1-\alpha\right)\left[1+\beta\left(\eta+\phi\right)\right]} \tag{6}$$

$$\frac{z_{ss}+1-\delta-(\eta+\phi)n_{ss}+\phi n}{1+n_{ss}}-\varphi_{ss}=1 \tag{7}$$

Among them, x_{ss} denotes the stable value of endogenous variable x, and $z_{ss}=\frac{y_{ss}}{k_{ss}}$ denotes the input-output ratio of capital, that is, the degree of economic development. Formula (5) is the household consumption-investment behavior decision equation when the economic system is balanced, which indicates the impact of population growth on economic development. $\frac{dz_{ss}}{dn_{ss}}>0$ indicates that population growth has a restrictive effect on economic development. Its endogenous mechanism is that population growth not only reduces the per capita capital stock k_{ss}, but also increases the economic cost of raising new population $[\eta n_{ss}+\phi(n_{ss}-n)]k_{ss}$ and the time of raising θn_{ss}, and reduces the speed of capital accumulation and labor time input, which ultimately restricts the economic development. $\varphi_{ss}=\frac{c_{ss}}{k_{ss}}$ indicates the ratio of income to cost of family birth. Formula (6) is the decision-making equation of family birth behavior when the economic system is balanced. Formula (6) indicates that family chooses the fertility rate according to the income and cost of child birth. There should be a positive correlation betweenφ_{ss} and n_{ss}. However, as the birth parameters in Formula (6) are to be determined, The sign of $\frac{dn_{ss}}{d\varphi_{ss}}$ cannot be determined, so the correlation between them can not be confirmed. Form (7) is the family budget constraint when the economic system is balanced, which indicates the impact of economic development on the income and cost of family child-bearing. There are two endogenous mechanisms: on the one hand, the savings effect of economic development accelerates capital accumulation, increases unit cost k_{ss} of newborn population upbringing and reduces φ_{ss}; on the other hand, the savings effect of economic development accelerates capital accumulation; On the one hand, the income effect of economic development improves the unit income c_{ss} of the new-born population and improves theφ_{ss}, but it is not yet clear which mechanism is stronger, and because the relationship between_φ_{ss} and n_{ss} is uncertain, the impact of economic development on population growth is not clear.

The partial derivatives of n_{ss} for n,ϕ and θ can be used to evaluate the effects of three fertility policies. $\frac{\partial n_{ss}}{\partial n}>0$ shows that relaxing the quantitative policy fertility can improve family fertility. As increasing n reduces the economic cost of raising children, families are encouraged to increase n_{ss}. The sign of $\frac{\partial n_{ss}}{\partial \phi}$ is undetermined, which indicates that the effect of economic fertility policy is still uncertain. Because the relationship between n and n_{ss} is undetermined when the economic system is balanced, the punishment or incentive effect of economic fertility policy is not clear, so it is impossible to determine whether ϕ has restrictive or encouraging effect on n_{ss}. The sign of $\frac{\partial n_{ss}}{\partial \theta}$ is undetermined, indicating that the effect of the time-dependent fertility policy is uncertain. There are two endogenous mechanisms for family fertility decision-making in the time-dependent family-rearing policy. The one is that saving family-rearing time will increase labor input and family wage income in production, under the effect of income effect, family consumption will increase; the other is that under the savings effect, family investment will also increase, resulting in changes in φ_{ss} and n_{ss} are uncertain.

3.3 Disequilibrium of Economic System

Taking the adjustment of fertility policy as an exogenous shock factor, under the condition of unbalanced economic system, exploring the transition dynamic path of main endogenous variables $\{c_t,k_t,n_t\}$ can analyze the transmission mechanism of fertility policy and evaluate its effect. Referring to Liu Bin's (2008) research method on the unbalance of economic system, the first-order Taylor expansion of endogenous variables near the steady state is carried out, and the equation of its linear transfer dynamic path is obtained as follow:

$$\hat{c}_t = \hat{c}_{t-1} + \frac{(1-\alpha)\alpha\beta z_{ss}}{1+n_{ss}}\hat{k}_{t-1} - \left\{1+\beta\left[\theta\alpha(1-\alpha)\frac{z_{ss}}{1-\theta n_{ss}} + \eta + \phi\right]\right\}\frac{n_{ss}}{1+n_{ss}}\hat{n}_t \tag{8}$$

$$\hat{k}_t = -\varphi_{ss}\hat{c}_t + \frac{1}{\beta}\hat{k}_{t-1} - \left[1+\eta+\phi+\varphi_{ss}+\frac{\theta(1-\alpha)z_{ss}}{1-\theta n_{ss}}\right]\frac{n_{ss}}{1+n_{ss}}\hat{n}_t \tag{9}$$

$$\hat{z}_t = -(1-\alpha)\hat{k}_{t-1} - \frac{\theta(1-\alpha)n_{ss}}{1-\theta n_{ss}}\hat{n}_t \tag{10}$$

$$\hat{\varphi}_t = \hat{c}_t - \hat{k}_{t-1} \tag{11}$$

$$\hat{n}_t = \frac{(1-\theta n_{ss})^2}{\theta^2\alpha(1-\alpha)z_{ss}n_{ss}}\left\{(\chi-1)\varphi_{ss}\hat{c}_t - \hat{k}_t - \left[(\chi-1)\varphi_{ss}-1\right]\hat{k}_{t-1}\right\} \tag{12}$$

In summary, this part builds a DGE model to analyze the behavior decision-making process of family, enterprise and government. In the following section, numerical simulation method will be used to analyze it.

4. Model Parameter Estimation and Numerical Simulation

Firstly, the exogenous parameters in DGE model are estimated empirically, and then the endogenous mechanism of economic development and population growth under the equilibrium and non-equilibrium state of economic system is analyzed by numerical simulation method according to the results of parameter estimation, and then the effectiveness of different types of Fertility Policies is evaluated.

4.1 Estimation of Model Parameters

This paper uses the typical method of DSGE literature for reference, and uses direct calibration method to obtain the clearer parameters. Referring to Zhang's research (2009), the subjective discount rate beta of family is set to 0.945 [5]. According to the estimation results of He et al. (2007), the elasticity of capital output is set to 0.6 [6]. Drawing on Wang Wenfu's research in 2010, the value of annual depreciation rate delta is 0.1 [7]. According to the viewpoint of Duyang (2005), the 1980s was the period when the effect of China's family planning policy was outstanding. The average natural growth rate of population from 1980 to 1990 was calculated as the value of the policy fertility rate n.

The other parameters in this paper are estimated by Bayes. The model involves three endogenous variables: per capita consumption, per capita capital stock and natural population growth rate. The annual data of resident consumption, total population, fixed investment and natural population growth rate from 1996 to 2015 are selected. The data are from China National Bureau of Statistics and Wind macroeconomic data database. Among them, consumption and total population are used to calculate per capita consumption; capital stock data are estimated by Shan Haojie (2008) in 1996 as the base period, and combined with the fixed assets investment data of the whole society from 1996 to 2015. It is estimated by the perpetual inventory method. In order to expand sample size, data frequency conversion method is used to convert annual data into quarterly data. ADF test shows that the data after frequency conversion belong to the first-order single-integer sequence, and the data obtained from the first-order difference of the original data and the removal of the mean value correspond to the intrinsic variables in the model. The posterior mean and confidence interval of each parameter are obtained by using Metropolis-Hastings algorithm. See Table 1:

Table 1. Bayes Estimated results of Model Parameters

Parameter	Definition	prior distribution	Posterior mean	95% confidence interval
η	Cost Ratio of Maintenance Funds	Beta(0.3,0.1)	0.201	[0.153,0.441]
ϕ	Economic penalty (incentive) ratio	Beta(0.04,0.02)	0.022	[0.013,0.052]
θ	Maintenance time cost ratio	Beta(0.25,0.1)	0.198	[0.166,0.287]
χ	Parental altruism	Normal(10,2)	8.411	[4.883,11.939]

Note: Bayes estimation needs to set the prior distribution of the parameters to be estimated. For the parameters whose values are between 0 and 1, the prior distribution is set to Beta distribution; for the parameters whose values are not necessarily between 0 and 1, the prior distribution is set to Normal distribution. The values in parentheses of prior distribution represent the prior mean and standard deviation, respectively.

From the Bayes estimation results, there are obvious differences between the posterior mean and the prior mean of all the parameters to be estimated, and 95% of the posterior confidence interval is different from the prior confidence interval, which shows that the estimated parameters are robust relative to the prior distribution, and that the collected data can better reflect the real information of the parameters to be estimated.

4.2 Numerical Simulation of Economic System Equilibrium

The equilibrium of economic system corresponds to the steady state of DGE model. The steady state values of main endogenous variables can be calculated by using the estimated values of each exogenous parameter of the model, and the long-term equilibrium relationship between economic development and population growth can be determined. The simulation results are shown in Table 2:

Table 2. State Values and Derivatives of Main Endogenous Variables in the Model

Endogenous variables	c_{ss}	k_{ss}	z_{ss}	φ_{ss}	n_{ss}	$\frac{dz_{ss}}{dn_{ss}}$	$\frac{dn_{ss}}{d\varphi_{ss}}$	$\frac{d\varphi_{ss}}{dz_{ss}}$	$\frac{dn_{ss}}{dz_{ss}}$
Steady state value	4.1886	24.9321	0.2761	0.1680	0.0060	2.1353	42.6083	0.0110	0.4684

Table 2 shows that the ratio of income to capital stock is 0.2761 corresponding to the real interest rate of 6.57%, which is in line with the general experience of the rate of return on capital investment. The ratio of consumption to capital stock is 0.168, which is slightly lower than the result calculated by using the empirical data of household consumption and the estimated data of capital stock. However, considering that households hold financial assets such as deposits, bonds and stocks that do not enter the production sector, the calculation result of φ_{ss} is still reasonable. The family fertility rate is 6, which is close to the natural growth rate of 5.5 annual population in China from 2000 to 2015. Moreover, $n_{ss} < n$ reflects the reality of low fertility rate in China at present, and also shows that the economic fertility policy shows incentive effect. Generally speaking, the steady value of the main endogenous variables in the model is in good agreement with China's macroeconomic reality. In addition, $\frac{dz_{ss}}{dn_{ss}} > 0$ confirms the endogenous mechanism of population growth restricting economic development. $\frac{dn_{ss}}{d\varphi_{ss}} > 0$ confirms the previous conclusion that there is a positive correlation between φ_{ss} and n_{ss}. $\frac{d\varphi_{ss}}{dz_{ss}} > 0$ and $\frac{dn_{ss}}{dz_{ss}} > 0$ jointly demonstrate the endogenous mechanism of economic development for population growth: the savings effect of economic development is stronger than the income effect, which results in the reduction of the ratio of income to cost of child-bearing, thereby reducing the family fertility rate. Therefore, there are two-way endogenous constraints between population growth and economic development.

Taking the quantitative fertility policy parameter n fluctuating (+100%) and the economic fertility policy parameter ϕ and the time-dependent fertility policy θ as the upper and lower limits

of Bayes estimation posterior interval respectively as different fertility policy shocks scenarios, the effects of various fertility policies are simulated again. The simulation results are shown in Table 3:

Table 3. The effect of fertility policy under different impact scenarios

Endogenous variables	n		ϕ		θ		Initial steady state
	0.006	0.024	0.013	0.052	0.166	0.287	$n = 0.012; \phi = 0.022; \theta = 0.198$
c_{ss}	4.1906	4.1858	4.2984	3.8508	4.2316	4.0686	4.1886
k_{ss}	24.944	24.916	25.785	22.362	25.263	24.018	24.9321
z_{ss}	0.2760	0.2762	0.2725	0.2882	0.2747	0.2801	0.2761
φ_{ss}	0.1680	0.1680	0.1667	0.1722	0.1675	0.1694	0.1680
n_{ss}	0.0059	0.0063	0.0043	0.0117	0.0054	0.0079	0.0060

Note: Initial steady state represents the steady state value of each endogenous variable in Table 2, which is used to compare the effects of the same kind of fertility policy.

Table 3 shows that under different levels of quantitative fertility policy shocks, the optimal fertility rate of family choice is lower than the policy parameters, and with the increase of policy parameters, the family fertility rate, per capita consumption and capital accumulation decrease. The endogenous mechanism is that relaxing the quantitative fertility policy will reduce child rearing cost and increase family fertility. Population growth will reduce per capita consumption and capital accumulation. The numerical simulation results show that the effect of the former is greater than that of the latter, which leads to an increase in the ratio of income to cost of raising children, and encourages families to increase fertility again. Therefore, the relaxation of the quantitative fertility policy can have an encouraging effect on family fertility, which is consistent with the original intention of implementing the two-child policy since 2013. However, from the perspective of policy effect, the policy parameters are increased or decreased by 100%, and the family fertility rate is only increased by 5% or decreased by 1.67%. This shows that the effect of the current quantitative fertility policy is extremely limited, which is consistent with the fact that the actual fertility rate has not increased significantly after the implementation of the two-child policy.

To sum up, the endogenous population growth mechanism incorporating fertility policy is shown in the following figure. From the perspective of the effects of various fertility policies, in the predictable policy space, the quantitative fertility policy has been difficult to affect population growth; the economic fertility policy has the strongest effect; and the time-dependent fertility policy has the second effect.

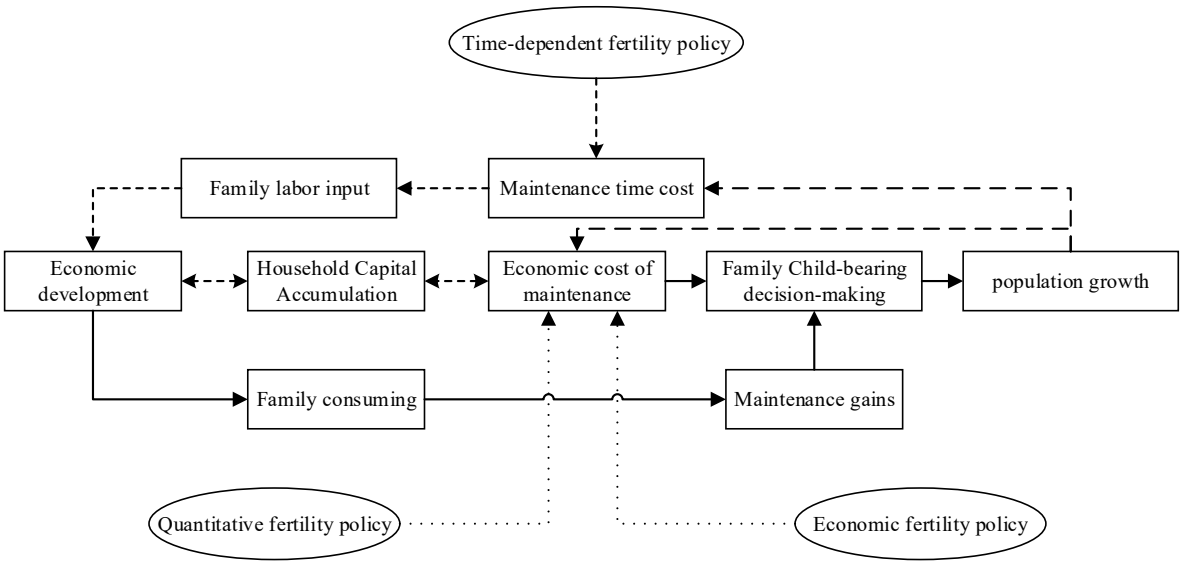

Figure 1. the endogenous population growth mechanism incorporating fertility policy

4.3 Numerical Simulation of Inequilibrium in Economic System

In order to deduce the short-term non-equilibrium correction mechanism between economic development and population growth, three kinds of upper bounds of operation space of fertility policy are used as shock scenarios of fertility incentive policy to simulate the dynamic path of endogenous variables from non-steady state to steady state. See Figure 2:

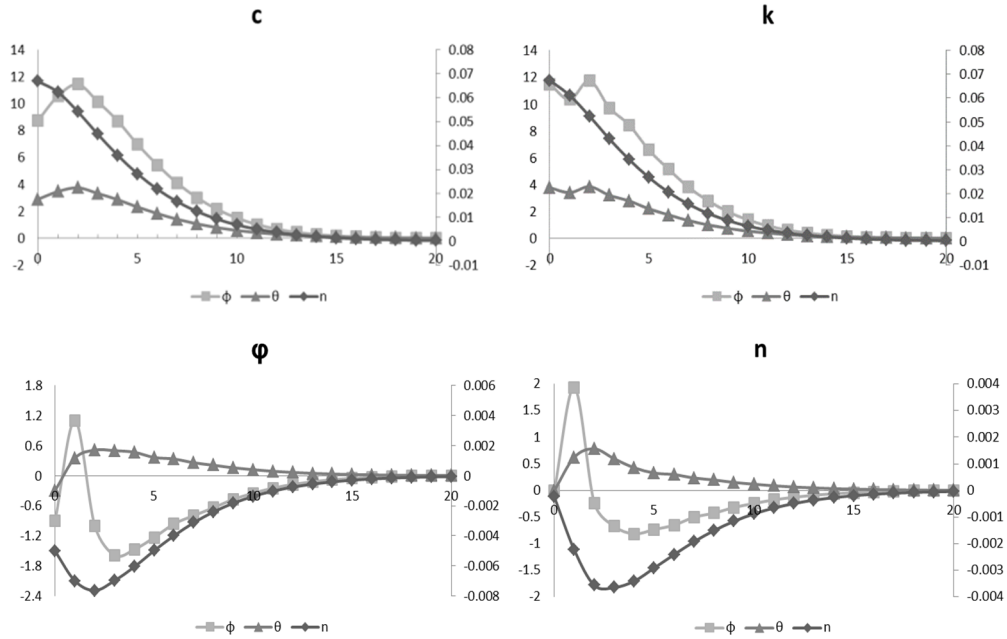

Figure 2. Dynamic Path of Endogenous Variable Transfer under Different Kinds of Fertility Policy Shocks

The results of Figure 2 show that the impact of economic fertility policy causes the most violent fluctuation of endogenous variables, followed by time-dependent fertility policy and quantitative fertility policy, which is consistent with the conclusion of steady-state analysis. However, under three kinds of policy shocks, the deviation degree of each endogenous variable from the steady-state value tends to converge over time, which indicates that there exists a short-term correction mechanism for the economic system to return from non-equilibrium to equilibrium.

Under the impact of economic fertility policy, there is excessive capital accumulation in the family's initial consumption-savings decision-making. Families will increase consumption and reduce investment, resulting in an increase in the ratio of income to cost of child-rearing and a sharp increase in fertility, thus forming a "fertility accumulation" phenomenon. In addition, the cost of maintenance saved by economic fertility policy increases household consumption and investment through income and savings effects, and the new-born population will have a compulsory saving effect on households, aggravate household capital accumulation and reduce fertility. Subsequently, households slowly adjust consumption-savings decision-making and reproductive decision-making, leading the endogenous variables to converge to a new steady state. Under the impact of quantitative fertility policy, the short-term revision mechanism of main endogenous variables is similar to economic incentive fertility policy. However, the impact of quantitative fertility policy is extremely weak, resulting in very low volatility of various endogenous variables.

Under the impact of the time-dependent fertility policy, households also make consumption-savings decision-making adjustments to increase consumption and reduce investment. In addition, the decrease of labor input time leads to the decrease of wage income, which reduces household consumption and investment through the effect of income and savings, resulting in an increase in the ratio of income to cost and fertility of child-rearing. Similarly, the compulsory savings effect of the new population will accelerate capital accumulation and reduce fertility. However, because the impact of the time-dependent fertility policy is far less than that of the economic incentive fertility policy, the fertility rate does not fluctuate dramatically, but converges slowly to a steady state.

5. Conclusion and Policy Enlightenment

The study finds that there are two-way endogenous constraints between population growth and economic development. The policies of increasing economic incentives, increasing the time of raising children and relaxing the number of births can change the economic and time cost of raising children, increase family fertility and promote population growth. Among them, the economic incentive fertility policy has the most obvious effect, but in the short term it will cause the phenomenon of "birth accumulation"; the effect of the time-based fertility policy Secondly, the effect of quantitative fertility policy is the weakest. The policy implications are as follows: Firstly, we should build and improve the policy environment to comprehensively improve the quality of population. On the basis of continuing to pursue and improve eugenic and eugenic policies, we should actively explore policy innovations in the areas of maternal and child health care, infant and child care, protection of women and children's rights and interests, basic education and vocational education, and create a policy environment conducive to the improvement of population quality through the combination and collocation of various policies. Secondly, we should explore a compound fertility policy with economic guidance as the main factor and quantity control as the supplement. This paper finds that it is difficult to achieve the ideal effect of fertility regulation by using a single fertility policy. We should consider the co-operation of the two policies. Specifically, the quantitative fertility policy represented by two-child and extra fertility arrangement mainly emphasizes the orientation of fertility regulation, with the collection of social support fees, fertility insurance and only child. Diversified economic fertility policies, such as child insurance and pension subsidies, achieve the desired fertility regulation objectives. Thirdly, we should design more diversified, flexible and humanized infant and child rearing policies to extend the necessary rearing time. While continuing to improve the policy of maternity leave and breastfeeding leave, we should strengthen the policy support to infant housekeeping service, social care and other industries, and introduce relevant policies such as infant rearing leave and early education leave as soon as possible, so as to prolong the time of child rearing through various ways.

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