Optimal Decisions for a Dual Channel Supply Chain

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Abstract. This paper considers a two-stage dual-channel supply chain consisting of a manufacturer and a retailer. The manufacturer produce both standard and customized products. The standard products sell through the retailer, and the customized products sell through the Internet. Due to the different ways of sales, the efforts of the two channels are shown as service level and lead time respectively. Both demands for the two types of products are linearly related to sales price, service level and lead time. The objective is to make the optimal decisions, including markup on the products, service level of the retailer, lead time for the customized products, production lots of the manufacturer, and order times of the retailer, so as to maximize the profit of the supply chain system. For the problem, the nonlinear constrained programming model is established, the solution is discussed, Matlab programs for solving the problem are developed. Through numerical experiments, useful conclusions are drawn for both supply chain managers and researchers.

Introduction

In recent years, with the rapid development of e-commerce and information technology, consumer behavior has changed and manufacturers' sales strategies have also changed. In addition to traditional sales channels, manufacturers have begun to sell their products through online channels or the coexistence of online channels and traditional channels. Many companies such as Dell, IBM, Nike, HP, Apple, Estee Lauder and Pioneer Electronics have adopted a dual-channel sales strategy [1-6]. With a single channel compared with dual-channel, the management of dual-channel supply chain is much more complicated, which needs to redesign the supply chain structure and determine its production inventory strategy.

The hot issues in the field of supply chain management include the impact of pricing, effort level and production inventory on supply chain decision-making. Chiang et al. [2] and Tsay, Agrawal[3] studied the dual-channel supply chain problem with pricing and channel structure as decision variables and found that the use of online channels improved the channel efficiency. Hua et al. [4] studied the pricing problem of dual-channel supply chain considering lead time under centralized and decentralized strategies, and found that lead time affects pricing strategy and supply chain profit. Xia and Rajagopalan [7] studied the standardization and customization of products of the two competing companies by integrating product categories, lead time and price, and found that increasing product categories or reducing delivery time can increase market share and profit. Dan et al. [8] studied the strategic role of retail services in the dual-channel supply chain, indicating that retail services directly affect the overall profits of the system and the pricing strategies of manufacturers and sellers. Xiao et al. [9] used the Stackelberg pricing model to study the type and structure of dual-channel products and found that the introduction of online channels for customized products makes the wholesale price and retail price of standard products increase when the booking price of retail channels is low enough. Li et al. [10] studied customers' sensitivity to channel acceptance and product differences, and found that manufacturers improved the profits of manufacturers and sellers by selling customized products online. Batarfi, Jaber et al. [11] studied the two-level dual-channel supply chain problem consisting of one manufacturer and one seller. The manufacturer sells standard products through traditional channels, namely the seller, and sells customized products through direct online channels. They built a maximum model of the problem under linear demand conditions and compared it with two single-channel cases through numerical experiments.

This paper is based on the research of Batarfi, Jaber et al., but considers the impact of sales efforts on the supply chain, including the order lead time of customized products and the service level of the seller. Some redundancy was removed, such as the degree of differentiation of the custom product, which was considered but treated as constant. In this way, there is no benefit other than increasing the complexity of the problem. In this paper, a more feasible solution method is given and more practical results are obtained.

Model Framework

Assumptions

(1) In the two-level dual-channel supply chain composed of a manufacturer and a seller, the manufacturer produces both standard products and customized products. Standard products are sold by sellers through traditional sales channels. Customized products are sold through (Internet) direct sales channels.

(2) Standard products are formed by adding basic features to core products, while customized products are produced by adding corresponding features to core products according to customer preferences. Different features added to core products produce different customized products that can be expressed as sets $G_M = \{G_i, i = 1, 2, \dots, M\}$.

(3) Standard products are consigned, and the inventory is managed by the seller, who only pays the manufacturer for the products sold at the wholesale price. The quantity produced by the manufacturer per cycle is q_t , the sales cycle is the same as the production cycle, and the manufacturer delivers the same quantity to the seller each time, the number of deliveries per production cycle by the manufacturer is n_t , the production cycle of standard products is $q_t/D_t = n_t q/D_t$.

(4) The inventory of the core products used in the customized products is managed by the manufacturer. The quantity of core products produced by the manufacturer for each customized product is q_e , and the cycle of the customized products is q_e/D_e .

(5) No shortage is allowed in the sales process, that is, the productivity P_t and P_e of standard products and customized products are higher than the demand rate D_t and D_e .

(6) During the sales process, both the manufacturer and the seller adopt the same mark-up rate k , $0 \leq k \leq k_0$. Where k_0 is the price increase upper limit stipulated by the price management regulations or regulations.

(7) Traditional sales channels and network direct sales channels need to make corresponding efforts in order to increase the demand rate, but due to the different sales methods of the two channels, the efforts made are also different. The efforts of the seller are to optimize the service level s , while the efforts of the manufacturer are to optimize the order lead time l . The service cost of the seller per unit time is $C_s(s) = as^2/2$, $0 \leq s \leq 1$; The processing and delivery cost of customized products per unit is $C_l(l) = c_L + b(L-l)^2/2$, $0 < L_1 \leq l \leq L_2 < L$. Where $s = 0$ represents no service, $s = 1$ represents the highest level of service, and L_1 and L_2 are the minimum and maximum lead times (delivery delays) respectively.

Demand Rate Function

Assuming that the demand for standard and customized products is linearly related to the sales price, service level and order lead time, the demand rate function of standard and customized products is as follows:

$$D_i = \alpha_i - \beta_i c_i (1+k)^2 + \sum_{i=1}^M \mu_i \gamma_{it} c_{ei} (1+k) + \delta_i l + \tau_i s, \quad (1)$$

$$D_e = \alpha_e - \sum_{i=1}^M \mu_i \beta_{ei} c_{ei} (1+k) + \gamma_e c_t (1+k)^2 - \delta_e l - \tau_e s. \quad (2)$$

Among them, parameters α_t and α_e are the market shares of standard products and customized products in the potential market demand respectively. β_t and β_{ei} are the sensitivity coefficients of the demand for standard product and customized product i to the sales price. γ_{ti} and γ_e are the cross-sensitivity coefficients of the demand for standard product and customized product i to the sales price respectively, they reflect the degree of substitutability of products sold through the two channels. δ_t and δ_e respectively represent the cross sensitivity coefficient and sensitivity coefficient of the demand for standard product and customized product i to the lead time of ordering. τ_t and τ_e respectively represent the sensitivity coefficient and cross sensitivity coefficient of the demand for standard product and customized product i to the service level of the seller. c_t and c_{ei} are the production costs of unit standard product and unit customized product i . μ_i is the share of customized product i in all customized products.

Make $\beta_t = \beta_t c_t$, $\beta_e = \sum_{i=1}^M \mu_i \beta_{ei} c_{ei}$, $\gamma_t = \sum_{i=1}^M \mu_i \gamma_{ti} c_{ei}$, $\gamma_e = \gamma_e c_t$. So (1), (2) turns into

$$D_t = \alpha_t - \beta_t (1+k)^2 + \gamma_t (1+k) + \delta_t l + \tau_t s, D_e = \alpha_e - \beta_e (1+k) + \gamma_e (1+k)^2 - \delta_e l - \tau_e s.$$

In order to make sense of the problem, suppose $\gamma_{ti} < \beta_{ei}, \gamma_e < \beta_t, \delta_t < \delta_e, \tau_t > \tau_e$, this means that the influence of sales price, order lead time and service level on the product's own demand is greater than the cross-influence [6]. At the same time assume that $\gamma_{ti} < \beta_t$ and $\gamma_e < \beta_{ei}$, this means that the demand for a product is more affected by its own price than by cross-price. Easy to know $\gamma_i < \beta_j, i, j \in \{t, e\}$.

Profit Function

(1) Manufacturer's profit function for standard products:

$$\Phi_t = c_t (1+k) D_t - [S_t D_t / q_t + c_t D_t + q_t h_1 D_t / 2 n_t P_t]. \quad (3)$$

It consists of two parts: the profit and cost of standard products per unit time. The cost part includes the preparation cost, production cost and inventory cost of standard products produced by manufacturers. Among them, S_t is the preparation cost of each production, and h_1 is the inventory cost per unit time of the product of the manufacturer.

(2) Manufacturer's profit function of customized products:

$$\Phi_e = c_e (1+k) D_e - [S_e D_e / q_e - (h_1 q_e / 2 - h_1 q_e D_e / 2 P_e) + c_e D_e + C_l(l) D_e]. \quad (4)$$

It is composed of two parts, namely, the profit and cost per unit time of the customized products. The cost part includes the preparation cost, inventory cost, production cost and processing and delivery cost of the core products of the customized products produced by the manufacturer. Among them, S_e is the preparation cost of each production, h_1 is the inventory cost per unit time of product of the manufacturer, and $c_e = \sum_{i=1}^M \mu_i c_{ei}$ is the average production cost of customized products.

(3) Seller's profit function:

$$\Phi_r = c_t (1+k)^2 D_t - [A n_t D_t / q_t + c_t (1+k) D_t + (h_2 q_t / 2 - h_2 (n_t - 1) q_t D_t / 2 n_t P_t) + C_s(s)]. \quad (5)$$

By the seller unit time sales revenue and cost two parts, the cost part includes the seller's order cost, inventory cost and service cost. Among them, A is the cost of each order, and h_2 is the inventory cost per unit time of the products sold by the seller.

(4) The total profit function of the supply chain:

$$\Phi = \Phi_t + \Phi_e + \Phi_r = K_t D_t + K_e D_e - (as^2 + h_2 q_t + h_1 q_e) / 2. \quad (6)$$

Where $K_t = c_t k(2+k) - (S_t + An_t) / q_t + q_t(h_2 - (h_1 + h_2) / n_t) / 2P_t$, $K_e = c_e k - S_e / q_e - c_L + q_e h_1 / 2P_e - b(L-l)^2 / 2$.

The purpose of this paper is to make the optimal decision, which is to determine the markup rate, the service level of the seller, the advance order period of the customized product, the production batch of the manufacturer's standard product and the core product of the customized product, and the order times of the seller in each production cycle, so as to maximize the profit of the supply chain system. Namely

$$\max \Phi(n_t, q_t, q_e, l, s, k) \quad (7)$$

$$s.t. \quad q_t > 0; q_e > 0; n_t > 0; L_1 \leq l \leq L_2; 0 \leq s \leq 1; 0 \leq k \leq k_0.$$

Optimal Decision

Because $\partial^2 \Phi / \partial n_t^2 = -q_t(h_1 + h_2)D_t / P_t n_t^3 < 0$, so for any given q_t, q_e, l, s, k there's a unique n_t that maximizes Φ to satisfy $\partial \Phi / \partial n_t = D_t[q_t(h_1 + h_2) / 2P_t n_t^2 - A / q_t] = 0$, namely

$$n_t = q_t \sqrt{(h_1 + h_2) / 2AP_t}. \quad (8)$$

From this we can see that for the standard product manufacturer the delivery quantity $q = q_t / n_t = \sqrt{2AP_t / (h_1 + h_2)}$ is a constant. Substitute equation (8) into equation (6) to get $K_t = c_t k(2+k) - S_t / q_t + h_2 q_t / 2P_t - 2A / q$, the profit function Φ becomes a function of q_t, q_e, l, s and k . Because Φ is negative about the Hessian matrix H_1 of q_t, q_e (see Appendix 1), so for any given l, s, k has a unique q_t, q_e to maximize Φ and satisfy $\partial \Phi / \partial q_t = K_1 D_t - h_2 / 2 = 0$, $\partial \Phi / \partial q_e = K_2 D_e - h_1 / 2 = 0$. Namely

$$q_t = \sqrt{2S_t D_t P_t / h_2 (P_t - D_t)}, q_e = \sqrt{2S_e D_e P_e / h_1 (P_e - D_e)}. \quad (9)$$

By substituting (8) and (9) into equation (6), the profit function Φ becomes a function of l, s and k , and the Hessian matrix of function Φ is H_2 (see Appendix 1). If it can be proved that H_2 is negative definite, we can conclude that its optimal solution exists and is unique.

But unfortunately, through a large number of numerical experiments, we find that even for some seemingly reasonable parameters, H_2 can not be guaranteed to be negative definite.

The problem must meet the following conditions:

- (1) $0 \leq s \leq 1, 0 \leq k \leq k_0, L_1 \leq l \leq L_2$;
- (2) $0 \leq D_t \leq P_t, 0 \leq D_e \leq P_e$;
- (3) $k(k+2)c_t - 2A / q - \sqrt{2S_t h_2 (1 / D_t - 1 / P_t)} - as^2 / 2D_t \geq 0$,
 $kc_e - \sqrt{2S_e h_1 (1 / D_e - 1 / P_e)} - c_L - b(L-l)^2 / 2 \geq 0$.

Whether traditional products or customized products, if there is no profit, they will not be produced. Therefore, the above conditions (3) must be guaranteed.

Since Φ is a continuous function of l, s and k in a bounded closed region, its maximum value exists and reaches at the stationary point, that is, the point satisfying

$$\partial \Phi / \partial l = K_t \delta_t - K_e \delta_e + b(L-l)D_e = 0, \quad (10)$$

$$\partial\Phi / \partial s = K_i\tau_i - K_e\tau_e - as = 0, \quad (11)$$

$$\partial\Phi / \partial k = 2c_i(1+k)D_i + K_iB_1 + c_eD_e + K_eB_2 = 0. \quad (12)$$

or the boundary point in the feasible region.

In summary, the optimal decision of the supply chain exists, which satisfies equations (8), (9), (10),(11) and (12), or at the boundary of l,s and k feasible regions.

Calculation and Analysis

The program pa_v.m for judging whether the parameters are appropriate, the program co_v.m for judging whether the profit function is concave, and the program phmax.m for finding the optimal solution (see Appendix 2) are compiled by MATLAB. These programs are used for numerical calculation and analysis.

Numerical Example

Case 1. $M=1$, $\alpha=[400,600]$, $P=[3000,3000]$, $\beta=[1.2,1]$, $\gamma=[0.4,0.4]$, $a=4$, $b=2$, $h=[4,5]$, $\delta=[0.4,0.5]$, $\tau=[0.7,0.3]$, $L=[1,10,14]$, $S=[400,200]$, $k_0=0.45$, $A=20$, $c=[30,50]$, $c_L=1.5$.

By judging and solving, it is concluded that the parameters are appropriate, the profit function Φ is non-concave, the optimal decision is $l^*=10$, $s^*=1$, $k^*=0.45$, $q_i=255$, $q_e=259$, $n_i=2.21$, and the maximum profit is $\Phi^*=1.2483 \times 10^4$.

Case 2. $M=3$, $\mu=[0.2,0.3,0.5]$, $\alpha=[15000,9000]$, $P=[18000,18000]$, $k_0=0.45$, $S=[1000,800]$, $A=300$, $\beta=[20,2,2,2]$, $b=0.4$, $c_L=20$, $L=[2,23,25]$, $c=[350,400,450,500]$, $\gamma=[0.12,0.12,0.12,0.36]$, $\delta=[8,68]$, $a=12000$, $\tau=[50,20]$, $h=[30,45]$.

By judging and solving, it is concluded that the parameters are appropriate, the profit function Φ is concave, the optimal decision is $l^*=21.19$, $s^*=0.8556$, $k^*=0.3185$, $q_i=420.41$, $q_e=540.6256$, $n_i=1.1079$, and the maximum profit is $\Phi^*=1330236$.

Comparison with a Single-channel Supply Chain

Consider the case of two single channels. One is that the demand for customized products will be lost and the demand for standard products will be increased in the traditional situation of selling only through vendors. The other is to sell the standard products in the same way as the customized products only in the case of online sales (see Appendix 3).For the above examples, we obtain the optimal solution and maximum profit for two single channel cases, as shown in Table 1.

Table 1. Optimal decision and maximum profit of single channel supply chain

Optimal solution and maximum profit		k	l	s	n_i	q_i	q_e	Φ
Example 1	Traditional channels	0.45	-	1	2.21	255	-	10 529
	Network channels	0.45	10	-	-	295	257	5 396
Example 2	Traditional channels	0.268 4	-	0.875 3	1.3	493.54	-	866 899
	Network channels	0.45	18.143 8	-	-	526.58	701.05	1 417 449

It can be seen that the dual-channel supply chain is superior to the traditional channel supply chain, but not necessarily superior to the network channel supply chain. In some environments, the network channel is superior to the dual-channel supply chain, and in other environments, it is inferior to the network channel. Among the three sales modes, the online sales rate is the highest, while the traditional sales rate is the lowest. Sales volume is also the largest online sales, the smallest traditional sales. The lead time of customized products in network sales is less than that in dual-channel sales, while the service level of sellers in dual-channel sales is lower than that in traditional channels.

Conclusion

(1) The optimal solution can be reached at the boundary or within the feasible region. The increase of mark-up rate will increase the profit of a single product while reducing the demand for the product, while the increase of service level and the shortening of lead time will increase the demand while increasing the cost. Therefore, it is very important to find a balance among several aspects in practice. The method provided in this paper can help find them.

(2) In the current environment, supply chain should be chosen between two kinds of supply chains: dual-channel supply chain and online sales supply chain. Which one to choose depends on the specific environmental conditions.

(3) Online sales make more demand satisfied. It reduces the selling price of standard products. Although it also increases the selling price of customized products, the sensitivity of customized products to price is generally lower than that of standard products.

(4) Compared with online sales, dual-channel sales extend the lead time of customized products. Compared with traditional sales, dual-channel sales reduces the service level of retailers.

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Appendix 1 Expressions of H_1, H_2 and Φ

$$H_1 = (-S_t D_t / q_t^3 \quad 0 \quad ; \quad 0 \quad -S_e D_e / q_e^3),$$

$$H_2 = -\Delta^T H_1^{-1} \Delta + H_3, \text{ Among}$$

$$\Delta = (K_1 \delta_t \quad K_1 \tau_t \quad K_1 B_1 \quad ; \quad -K_2 \delta_e \quad -K_2 \tau_e \quad K_2 B_2),$$

$$K_1 = \partial K_t / \partial q_t = S_t / q_t^2 + h_2 / 2P_t, K_2 = \partial K_e / \partial q_e = S_e / q_e^2 + h_1 / 2P_e.$$

$$H_3 = \begin{pmatrix} -2b(L-l)\delta_e - bD_e & -b(L-l)\tau_e & 2c_t(k+1)\delta_t - c_e\delta_e + b(L-l)B_2 \\ -b(L-l)\tau_e & -a & 2c_t(k+1)\tau_t - c_e\tau_e \\ 2c_t(k+1)\delta_t - c_e\delta_e + b(L-l)B_2 & 2c_t(k+1)\tau_t - c_e\tau_e & z \end{pmatrix},$$

$$B_1 = \partial D_t / \partial k = -2\beta_t(k+1) + \gamma_t, B_2 = \partial D_e / \partial k = -\beta_e + 2\gamma_e(k+1),$$

$$z = 2c_t D_t + 4c_t(k+1)B_1 - 2\beta_t K_t + 2c_e B_2 + 2\gamma_e K_e.$$

$$\Phi(l, s, k)$$

$$= [k(k+2)c_t - 2A/q - \sqrt{2S_t h_2(1/D_t - 1/P_t)}]D_t + [kc_e - \sqrt{2S_e h_1(1/D_e - 1/P_e)} - c_L - b(L-l)^2/2]D_e - as^2/2.$$

Appendix 2 Solving optimal solution and several related Matlab programs

```
1. Program pa_v.m for judging whether the parameters are suitable
dm1=a(1)+min(-b(1)*(1+k0)^2+r(1)*(1+k0),-b(1)+r(1))+d(1)*L(1);
[~,b1]=fminbnd(@(k)-b(2)*(1+k)+r(2)*(1+k)^2,0,k0);
dm2=a(2)+b1-d(2)*L(2)-t(2);
dM2=a(2)-b(2)+r(2)+max(k0*(2*r(2)-b(2)+k0*r(2)),0)-d(2)*L(1),
[~,p1]=fminbnd(@(k)b(1)*(1+k)^2-r(1)*(1+k),0,k0);
dM1=a(1)-p1+d(1)*L(2)+t(1);
```



```

T=(dm1>=0)*(dm2>=0)*(p(1)>=dM1)*(p(2)>=dM2)
if T>0
T1=c(1)*(k0*(k0+2))-(2*o*(h(1)+h(2))/p(1))^0.5-(2*S(1)*h(2)*(1/dM1-1/p(1)))^0.5;
T2=c(2)*k0-(2*S(2)*h(1)*(1/dM2-1/p(2)))^0.5-cL-0.5*B*(L(3)-L(2))^2;
T=(T1>=0)*(T2>=0)
if T<1
M=rand(3,10000);M(1,:)=L(1)+(L(2)-L(1))*M(1,:);M(3,:)=k0*M(3,:);l=M(1,:);s=M(2,:);k=M(3,:);
d1=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;
d2=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;q=(2*o*p(1)/(h(1)+h(2)))^0.5;
sp1=(2*S(1)*h(2)*(1./d1-1/p(1))).^0.5;sp2=(2*S(2)*h(1)*(1./d2-1/p(2))).^0.5;
T3=(k.*(k+2)*c(1)-2*o/q-sp1).*d1-0.5*A*s.^2;T4=k*c(2)-sp2-cL-0.5*B*(L(3)-l).^2;
T=(sum((T3>=0).*(T4>=0))>0);
end end

```

2. Conditional program mycon.m for objective function

```

function [cc,ceq]=mycon(x)
global a b r t d L c cL h S o A B p
l=x(1);s=x(2);k=x(3);
d1=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;
d2=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;q=(2*o*p(1)/(h(1)+h(2)))^0.5;
sp1=(2*S(1)*h(2)*(1./d1-1/p(1))).^0.5;sp2=(2*S(2)*h(1)*(1./d2-1/p(2))).^0.5;
cc(1)=(-k.*(k+2)*c(1)+2*o/q+sp1).*d1+0.5*A*s.^2;
cc(2)=(-k*c(2)+sp2+cL+0.5*B*(L(3)-l).^2).*d2;
cc(3)=-d1;cc(4)=-d2;cc(5)=d1-p(1);cc(6)=d2-p(2);ceq=[];

```

3. Program co_v.m for judging whether the objective function is concave

```

N=input('Please enter the number of detection points: ');
M=rand(3,N);M(1,:)=L(1)+(L(2)-L(1))*M(1,:);M(3,:)=k0*M(3,:);l=M(1,:);s=M(2,:);k=M(3,:);
d1=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;d2=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;
q1=(2*S(1)*d1*p(1)/(h(1)*(p(1)-d1))).^0.5;q2=(2*S(2)*d2*p(2)/(h(1)*(p(2)-d2))).^0.5;
q=(2*o*p(1)/(h(1)+h(2)))^0.5;
Ke=c(2)*k-S(2)./q2+0.5*(h(1)/p(2))*q2-cL-0.5*B*(L(3)-l).^2;
Kt=k.*(k+2)*c(1)-2*o/q-S(1)./q1+0.5*q1*h(2)/p(1);
K1=S(1)./q1.^2+0.5*h(2)/p(1);K2=S(2)./q2.^2+0.5*h(1)/p(2);
B1=-2*b(1)*(1+k)+r(1);B2=-b(2)+2*(1+k)*r(2);
H=zeros(3,3,N); E=zeros(3,N);
h1=K1.^2.*q1.^3./(S(1)*d1);h2=K2.^2.*q2.^3./(S(2)*d2);
H(1,1,:)=h1*d(1)^2+h2*d(2)^2-2*B*d(2)*(L(3)-l)-B*d2;
H(1,2,:)=h1*d(1)*t(1)+h2*d(2)*t(2)-B*t(2)*(L(3)-l);
H(1,3,:)=h1*d(1).*B1-h2*d(2).*B2+2*c(1)*d(1)*(k+1)-c(2)*d(2)+B*(L(3)-l).*B2;
H(2,1,:)=H(1,2,:);
H(2,2,:)=h1*t(1)^2+h2*t(2)^2-A;
H(2,3,:)=h1.*B1*t(1)-h2.*B2*t(2)+2*c(1)*t(1)*(k+1)-c(2)*t(2);
H(3,1,:)=H(1,3,:);H(3,2,:)=H(2,3,:);
H(3,3,:)=h1.*B1.^2+h2.*B2.^2+2*c(1)*d1+4*c(1)*(k+1).*B1-2*b(1)*Kt+2*c(2)*B2+2*r(2)*Ke;
for w=1:N E(:,w)=eig(H(:,w));end
F=E<+0.0001;G=sum(F)>2;V=sum(G)
v=N-V,i=find(G);if ~isempty(i),M(:,i)=[];H(:,i)=[];E(:,i)=[];end

```

4. Program ph.m for solving the value of object function

```

function f=ph(l,s,k)
global a b r t d L c cL h S o A B p
d1=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;
d2=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;q=(2*o*p(1)/(h(1)+h(2)))^0.5;

```

```

sp1=(2*S(1)*h(2)*(1./d1-1/p(1))).^0.5;sp2=(2*S(2)*h(1)*(1./d2-1/p(2))).^0.5;
f=(k.*(k+2)*c(1)-2*o/q-sp1).*d1+(k*c(2)-sp2-cL-0.5*B*(L(3)-l).^2).*d2-0.5*A*s.^2;
5. Find the optimal solution of the program phmax.m
M=rand(3,10000);M(1,:)=L(1)+(L(2)-L(1))*M(1,:);M(3,:)=k0*M(3,:);l=M(1,:);s=M(2,:);k=M(3,:);
d1=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;
d2=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;q=(2*o*p(1)/(h(1)+h(2))).^0.5;
sp1=(2*S(1)*h(2)*(1./d1-1/p(1))).^0.5;sp2=(2*S(2)*h(1)*(1./d2-1/p(2))).^0.5;
T3=(k.*(k+2)*c(1)-2*o/q-sp1).*d1-0.5*A*s.^2;T4=k*c(2)-sp2-cL-0.5*B*(L(3)-l).^2;
T=(T3>=0).*(T4>=0).*(d1>=0).*(d1<=p(1)).*(d2>=0).*(d2<=p(2));
if sum(T)>0
i=find(1-T);M(:,i)=[];l=M(1,:);s=M(2,:);k=M(3,:);
y=ph(l,s,k);[v,i]=max(y);
[v,w]=fmincon(@(x)-ph(x(1),x(2),x(3)),M(:,i),[],[],[],[],[L(1),0,0],[L(2),1,k0],@(x)mycon(x));
w=-w;
else v=nan*[1;1;1];w=nan; end
l=v(1);s=v(2);k=v(3);
dt=a(1)-b(1)*(1+k).^2+r(1)*(1+k)+d(1)*l+t(1)*s;
de=a(2)-b(2)*(1+k)+r(2)*(1+k).^2-d(2)*l-t(2)*s;q=(2*o*p(1)/(h(1)+h(2))).^0.5;
qt=(2*S(1)*dt*p(1)/(h(2)*(p(1)-dt))).^0.5;qe=(2*S(2)*de*p(2)/(h(1)*(p(2)-de))).^0.5;nt=qt/q;
l,s,k,qt,qe,nt,w

```

Appendix 3 Single channel problem model

1. In the traditional case, it's only through the seller

At this time, the demand for customized products is lost, and the demand for standard products is

$D_t = \alpha_t - \beta_t(1+k)^2 + \gamma_t(1+k_0) + \delta_t L + \tau_t s$, The total channel profit function becomes

$$\Phi(n_t, q_t, s, k) = K_t D_t - (as^2 + h_2 q_t) / 2.$$

For any given s, k , Φ is a concave function of n_t and q_t . Therefore, there exists a unique solution

$n_t = q_t \sqrt{(h_1 + h_2) / 2AP_t}$, $q_t = \sqrt{2S_t D_t P_t / h_2(P_t - D_t)}$ that maximizes Φ . Here

$$\Phi = [k(k+2)c_t - 2A/q - \sqrt{2S_t h_2(1/D_t - 1/P_t)}]D_t - as^2 / 2.$$

Condition (1) becomes $0 \leq s \leq 1$, $0 \leq k \leq k_0$, condition (3) becomes

$$k(k+2)c_t - 2A/q - \sqrt{2S_t h_2(1/D_t - 1/P_t)} - as^2 / 2D_t \geq 0.$$

2. E-commerce, that is, sales only through the Internet

At this point, the demand for customized products is

$$D_e = \alpha_e - \beta_e(1+k) + \gamma_e(1+k) - \delta_e l.$$

The demand for the standard product is

$$D_l = \alpha_l - \beta_l(1+k) + \gamma_l(1+k) + \delta_l l.$$

The profit function of the manufacturer for standard products becomes

$$\Phi_t = [c_t(1+k) - S_t / q_t - c_t - c_L + q_t h_1 / 2P_t]D_t - h_1 q_t / 2.$$

The total channel profit function is

$$\Phi = [c_t k - S_t / q_t - c_L + q_t h_1 / 2P_t]D_t + [c_e k - S_e / q_e + h_1 q_e / 2P_e - C_l(l)]D_e - h_1(q_t + q_e) / 2.$$

For any given k, l , Φ is a concave function of q_t and q_e . Therefore, there exists a unique solution $q_t = \sqrt{2S_t D_t P_t / h_1 (P_t - D_t)}$, $q_e = \sqrt{2S_e D_e P_e / h_1 (P_e - D_e)}$ that maximizes Φ . Here

$$\Phi = [c_t k - c_L - \sqrt{2S_t h_1 (1/D_t - 1/P_t)}] D_t + [c_e k - \sqrt{2S_e h_1 (1/D_e - 1/P_e)} - C_l(l)] D_e.$$

Condition (1) becomes $L_1 \leq l \leq L_2$, $0 \leq k \leq k_0$, condition (3) becomes

$$kc_t - c_L - \sqrt{2S_t h_1 (1/D_t - 1/P_t)} \geq 0, kc_e - \sqrt{2S_e h_1 (1/D_e - 1/P_e)} - c_L - b(L-l)^2 / 2 \geq 0.$$

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