

# Analysis on the Cooperation Between Independent Colleges and Enterprises Based on Game Theory

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**Abstract**—Independent college is a special product of the higher education development in China. Due to its own characteristics of independent college, there are many differences in cooperation with companies from the ordinary universities. In this paper, analyze the conditions of the college-enterprise cooperation, net income of two participants and other issues by using game theory. The research results show that the independent college and the enterprise can realize fully cooperation under certain conditions, obtain the "win-win" state. The strength of independent colleges is the key factor in deciding the different forms of cooperation with enterprises.

**Keywords**—*independent college; college-enterprise cooperation; Stackelberg model; complete cooperative game*

## I. INTRODUCTION

The independent college came into being in the late 90s of last century. At that time, in order to expand the resources of higher education, the state encouraged the public universities to organize independent colleges in combination with social funds. Since then, the independent college has developed rapidly. Independent college is a higher education institution organized by ordinary undergraduate universities (applicant) and social forces (the collaborators include enterprises, institutions, social groups or individuals and other organizations with the ability to cooperate.). It is a higher education institution carrying out undergraduate level education. It is a second level undergraduate college based on the new mechanism and mode. The difference between the independent college and the second level College of the university lies in its private nature. Its source of funding is not from national appropriations but from the sponsor of the Institute in various ways. The management of funds, tuition and other related management is also based on the way of private universities. The rapid development of independent colleges has played a very important role in improving the gross enrollment rate of higher education in China.

Due to the lack of independent college running in the process of government funding, many independent colleges put more funds to pay the wages of teachers and other aspects under the pressure of survival. Less investment in laboratory construction, scientific research and other aspects will inevitably lead to the lack of R&D ability and the decline of personnel training ability, resulting in insufficient

student resources. But the school is inadequate, the school is lack of effective sources of funds, and then the R&D investment has fallen into a vicious circle of further decline. It can be seen that if independent colleges can find appropriate enterprises to cooperate, it is essential for the development of the college. And for enterprises, they can cooperate with the independent college by a small amount of money. Independent colleges are responsible for personnel training, product development and so on. Enterprises will have more energy to invest in their core businesses so as to get more profits. Therefore, the cooperation of independent colleges and enterprises will bring about a win-win result.

This article will try to study the problem of independent college and enterprise cooperation. Since the scale of independent colleges is often relatively small, there are a large number of colleges and universities that can choose to cooperate. In many cases, enterprises are often in the dominant position. So, after a brief analysis of the non-cooperative Nash equilibrium between the two sides, this paper will focus on the Stackelberg model of the cooperation game analysis. Of course, with the strengthening of the independent college's own strength or a certain amount of binding agreements, the two sides may also cooperate fully. Therefore, after Stackelberg analysis, this paper will further explore the possibility of cooperation.

## II. LITERATURE REVIEWS

The theory of innovation is the origin of the theory of school enterprise cooperation. Bear Peter (1912) first proposed the concept of innovation. Since the 70s and 80s of last century, foreign researchers began to explore the issue of school enterprise cooperation from the perspective of industry university research or technological innovation, and formed rich results. Mansfield (1991) pointed out that if there is lack of academic research cooperation, 1/10 of the new products cannot be developed in the motor, chemical and pharmaceutical industries, and the rate of return on investment in academic research is about 28%. Geisler and Rubenstein (1995), based on the theory of inter organizational relationship, puts forward the basic theoretical framework of technological R&D cooperation, explores the causes and influencing factors of school enterprise cooperation, and how to improve the efficiency of cooperation. According to the investigation and Research on

1000 staff in the research university in the United States, Lee et al. (1996) found that American scholars in the 1990s were more inclined to strengthen cooperation between universities and industry than whom in 80s. And he found that advocating academic freedom does not affect the pursuit of cooperation with enterprises. The real obstacle to school enterprise cooperation lies in the lack of communication between the two cultures. Campbell (1997) puts forward a system to evaluate the cooperation between school and enterprise, and constructs an index system to evaluate the performance of school enterprise cooperation. Valent (2000) evaluated the benefits of school enterprise interaction and obstacles, and puts forward some countermeasures and suggestions to overcome various obstacles.

In twenty-first Century, scholars began to work on the study of local school enterprise cooperation, aiming to improve the status of local school enterprise cooperation. D'este and Perkmann (2011) takes the survey data of British physics and engineering science as a sample, and studies show that most scholars tend to deepen their research rather than commercialize their knowledge. Therefore, policies should not pay too much attention to monetary incentives for industry participants, but should consider a wider range of incentives to promote interaction between academia and industry. According to the Fourth UK Community Innovation Survey (CIS4), Iammarino (2012) found that the level of cooperation between enterprises and universities has a significant correlation with the technical ability of the enterprise. However, if regard the UK as the whole of analysis, it will mask the significant regional differences between local and non-local cooperative relations, as well as the technological capabilities and the willfulness of enterprises. Based on the British business innovation survey during 2002-2005 year, Hewitt-Dundas (2013) investigated 16500 enterprises implanted in the innovative environment of University Research. It is found that there are obvious differences between the school enterprise cooperation with

local universities and school enterprise cooperation with foreign universities. These differences relate to business scale, sales status, geographical location, ability to absorb and innovate, and innovation activities. The study also found that cooperation is often local if it is close to a good research university. But distance is not the decisive factor for the enterprise to cooperate with the local universities. If the universities in the business area have a high density of research and development, they are more likely to cooperate with non-local universities.

To sum up, the domestic scholars' research on school enterprise cooperation mainly focuses on two aspects: the first is the alliance between universities and enterprises, and the second is the school enterprise alliance of higher vocational colleges. There are relatively few studies on the cooperation between school and enterprise in the independent colleges in China. Therefore, aiming at the characteristics of independent colleges, this paper analyzes the cooperation between independent colleges and enterprises, and concludes the conditions for cooperation between independent colleges and enterprises. The author hopes that these will serve as a theoretical guide for the development of independent colleges.

### III. THE MEANING OF THE PARAMETERS AND THE IDEA OF BUILDING THE MODEL

#### A. Parameter Meaning

Before introducing the model, the meaning of the use of symbols is explained first. The subscript  $c$  and  $f$  represent independent colleges and enterprises respectively. The superscript  $*$ ,  $**$  and  $***$  represent the three states of Nash equilibrium, Stackelberg and complete cooperation game, respectively. Other symbolic meanings are shown in "Table I".

TABLE I. RELATED PARAMETERS AND THE MEANING OF DECISION VARIABLES

Decision variables	
$x$	The amount of investment in product research and talent training in Independent Colleges
$\theta$	Investment proportion of independent college in enterprise decision sharing
Parameter	
$I$	The amount of funds needed to be invested in product development and talent training
$R_i$	Total benefit of participants $i$ , $i = c, f$
$\Pi_i$	Net income of participants $i$ , $i = c, f$
$\Pi$	Total net income of both parties
$S$	The strength of the Independent College
$p$	The probability of successful cooperation between the two sides

#### B. The Idea of Model Construction

Through game analysis, independent colleges and enterprises carry out the mode of school enterprise cooperation. Independent colleges, as the game side 1, are responsible for product research and development, and

"order type" talents training for enterprises. As a game player 2, the enterprise provides funds for the research and development of independent colleges and the training of talents. Considering the relatively small scale and relatively low prestige of independent colleges, enterprises are in the leading position in the process of cooperation. But in order to

better compare which is more suitable for the cooperation between the independent college and enterprise cooperation, this paper will compare the optimal way of cooperation through the construction of Cournot model and Stackelberg model and cooperation game model. In the process of school enterprise cooperation, the independent college has invested  $x$  in product development and talent training, and  $x > 0$ . In addition to  $x$ , it does not have any other input. Enterprises subsidize the industry research and development and the training of talents of independent colleges. The proportion of subsidies is  $\theta$ ,  $0 \leq \theta < 1$ .  $R_c$  and  $R_f$  respectively indicate the gains obtained by independent colleges and enterprises after the successful cooperation.  $\Pi_c$  and  $\Pi_f$  represent the expected net benefits of cooperation between the two sides after the success.  $\Pi$  is the system expected net income,  $\Pi = \Pi_c + \Pi_f$ .

Whether the cooperation can succeed depends on the input level of product development and personnel training. The amount of investment needed for product R&D and personnel training, and is subject to exponential distribution. Assuming  $P = \{I < i\} = 1 - e^{-si}$ ,  $s > 0$ .  $s$  indicates the strength of independent colleges, the larger the  $s$ , the stronger the strength of independent colleges, the stronger the ability of R&D and talent training, the greater probability of game cooperation to achieve final success.

Because the independent institute itself is relatively weak in R&D capability compared to ordinary colleges and Universities. Therefore, the cooperation between independent colleges and enterprises does not only existing in product R&D, but also in many aspects, such as order training, marketing strategy and so on. The cooperative benefit of this paper is not related to the output of the product, but is expressed by the quantity  $R$ . In the later analysis, a successful probability analysis is added to the uncertainty of the profit.

Because both independent colleges and enterprises have other sources of income, this paper uses net profit instead of profit to show profits between two sides in the game of school enterprise cooperation.

According to the general experience, the successful development of the number  $\xi$  of per unit time is a random variable, and it obeys the Poisson (Poisson) distribution with

the parameter of  $\lambda$ . That is  $P\{\xi = n\} = \frac{\lambda^n}{n!} e^{-\lambda}$ ,  $\lambda > 0$ ,  $n = 0, 1, 2, \dots$

Among them,  $\lambda$  is the average value of the number of successful R&D success per unit time, which can be regarded as an indicator of R&D capability. The greater the  $\lambda$  shows, the stronger the R&D ability. During a period of  $t \in [0, T]$ , the number of times  $N(t)$  obeys Poisson distribution with

the parameter of  $\lambda t$ . That is  $P\{N(t) = n\} = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$ . This

proved that the interval of time  $\eta$  between the last time of success and the next success follows the exponential distribution, and the  $P\{\eta \leq t\} = 1 - e^{-\lambda t}$ . If in the two consecutive successful development interval of time, as the capital flow, the success of R & D investment  $I$  is

proportional to the time interval  $\eta$ , that is  $I$  assumed that  $\eta = \phi I, I > 0$ . It can also be proved that  $I$  obeys exponential distribution. That is  $P\{I \leq i\} = 1 - e^{-si}$ , and  $s = \phi \lambda$ .  $s$  can be seen as an indicator of R&D capability.

#### IV. CONSTRUCTION AND COMPARISON OF INDEPENDENT COLLEGE AND ENTERPRISE COOPERATION MODEL

In this part, the best way of cooperation between the independent colleges and the enterprises through a comparative analysis of Cournot model and Stackelberg model and cooperation game model. No matter what kind of cooperation, the law is that when  $x$  is larger than  $I$ , the cooperation is successful. At the time of  $x < I$ , the cooperation between school and enterprise is failure, so the probability of the success of the cooperation between school and enterprise is:

$$p = 1 * P\{I \leq x\} + 0 * P\{I > x\} = 1 - e^{-sx} \quad (1)$$

The expected net income function of the two sides is, respectively,

$$\Pi_c = pR_c - (1 - \theta)x = (1 - e^{-sx})R_c - (1 - \theta)x \quad (2)$$

$$\Pi_f = pR_f - \theta x = (1 - e^{-sx})R_f - \theta x \quad (3)$$

The expected net income function of the whole system is as follows:

$$\Pi = \Pi_c + \Pi_f = (R_c + R_f)(1 - e^{-sx}) - x \quad (4)$$

##### A. The Analysis of the Cournot Model

The Cournot model is generally used that when one side of the game has no complete control over the other party, the two sides participating in the game are equal. When the two sides have not reached any binding agreement, the game is not cooperative. The goal of both sides is to choose a strategy that maximizes net income to satisfy their own constraints under the given strategy of the other side, and the equilibrium achieved by this strategy becomes Nash equilibrium. If the independent college and the enterprise are equal, can solve the optimal decision of both sides according to the Cournot model, and the theorem 1 can be obtained at this time.

##### • Theorem 1:

In the Cournot game, there is Nash equilibrium, that is, when the independent college and the enterprise are equal,

(I) the optimal input of the independent college is,

$$x^* = \frac{1}{s} \ln(sR_c)$$

(II) The optimal share ratio of the enterprise is,

$$\theta^* = 0$$

Prove:

Because of the independent decision of both parties, the first derivative of (2) and (3) can be obtained, respectively.

$$\frac{d\Pi_c}{dx} = (1 - e^{-sx})R_c - (1 - \theta) \quad (5)$$

$$\frac{d\Pi_f}{d\theta} = -x \quad (6)$$

Therefore, can get that  $\Pi_f$  is a minus function of  $\theta$ , that is, the greater  $\theta$ , the smaller  $\Pi_f$ . In order to maximize their net profit, enterprises will minimize the proportion  $\theta$  of enterprise cooperation, so when an enterprise makes independent decisions, the optimal proportion of enterprises are  $\theta^* = 0$ .

At this time, on the basis of (5), the derivative of the two orders is obtained by  $\Pi_c$  to the  $x$ , which can be obtained.

$$\frac{d^2\Pi_c}{dx^2} = -R_c s^2 e^{-sx} < 0$$

So  $\Pi_c$  is a concave function of  $x$ , that is, the unique  $x$  can make  $\Pi_c$  reach the maximum value. The order (5) is equal to zero, and the response function of the independent institute's R & D input to the enterprise subsidy can be obtained.

$$x = \frac{1}{s} \ln \frac{sR_c}{1-\theta} \quad (7)$$

The optimum R & D input of the independent college is obtained by replacing  $\theta^* = 0$  into the upper form.

$$x^* = \frac{1}{s} \ln(sR_c)$$

Theorem 1 has been proved.

At this time, the profits of independent colleges, enterprises and the whole system are:

$$\Pi_c^* = \frac{1}{s}(sR_c - 1 - \ln(sR_c)) \quad (8)$$

$$\Pi_f^* = R_f(1 - \frac{1}{sR_c}) \quad (9)$$

$$\Pi^* = \Pi_c^* + \Pi_f^* = R_c + R_f - \frac{1}{s}(1 + \frac{R_f}{R_c} + \ln(sR_c)) \quad (10)$$

The first derivative of (2) - (4) formula for  $S$  is obtained, which can be obtained.

$$\frac{dp}{ds} = x e^{-sx} > 0, \quad \frac{d\Pi_c}{ds} = R_c x e^{-sx} > 0,$$

$$\frac{d\Pi_f}{ds} = R_f x e^{-sx} > 0, \quad \frac{d\Pi}{ds} = (R_c + R_f) x e^{-sx} > 0,$$

Therefore, get the corollary 1.

- Corollary 1:

$p$ ,  $\Pi_c$ ,  $\Pi_f$  and  $\Pi$  were increasing function of  $S$ .

Inference 1 indicates that the strength of independent college is positively related to the expectation of cooperative success. The expected net income of the independent college, the expected net income of the enterprise and the expected net income of the whole system are all increasing functions

of the independent college's own strength. Thus, the independent institute strength will help improve the development and cultivation of talents so as to increase the likelihood of success, the ultimate success of the possibility of cooperation, and then improve the net income of its expected net income, the expected net income of the enterprise and the expected net income of the whole system. Therefore, the strength of the independent college is of great significance to itself, the enterprise and the whole system.

The first derivative of the (2) - (4) formula to  $\theta$  is obtained, get

$$\frac{d\Pi_c}{d\theta} = x > 0, \quad \frac{d\Pi_f}{d\theta} = -x < 0, \quad \frac{d\Pi}{d\theta} = 0$$

Therefore, get corollary 2.

Corollary 2.

$\Pi_c$  is an increasing function of  $\theta$ , and  $\Pi_f$  is a subtraction function of  $\theta$ , and  $\Pi$  is not related to  $\theta$ .

From inferential 2, can see that the expected net income of independent colleges is an increasing function of the proportion of corporate subsidies, and the expected net income of enterprises is a decreasing function of the subsidy ratio, but the net income of the whole system is not affected by the proportion of subsidies. Thus, it can be seen that in the case of mutual strength between the two sides, the independent college hopes that the more subsidies the enterprises give, the more the better, while what enterprise hope is that the less the proportion of the subsidy is, the better it is. Because of the existence of cooperative risk, enterprises do not want to cooperate with independent colleges, R&D cooperation does not exist, that is,  $\theta = 0$ .

## B. Analysis of Stackelberg Model Enterprises

Stackelberg model refers to that one side of the game is in the dominant position, while the other is in the position of the followers in the process of game. According to the previous correlation analysis, because the strength of the independent college is relatively weak, it is in a weak position in the whole system, and the enterprise is in the leading position. Therefore, the Stackelberg model was adopted to analyze. Because the enterprise is in the dominant position, the enterprise first chooses the subsidy ratio that maximizes the net income of the enterprise. After observing  $\theta$ , the independent institute, chooses the investment  $x$  that maximizes the income of the college to develop product development and talent training. The inverse recursive method is used to solve this game equilibrium, and theorem 2 can be obtained.

- Theorem 2:

When  $\frac{1-\theta^{**}}{R_c} < s < \frac{1}{R_c} e^{\frac{R_f}{R_c}}$ , it exists Stackelberg Equilibrium  $(x^{**}, \theta^{**})$ , that is the optimal solution of the two sides of the game meet the following conditions:



$$x^{**} = \frac{1}{s} \ln \frac{sR_c}{1-\theta^{**}}$$

$$\frac{R_f}{R_c} - \frac{\theta^{**}}{1-\theta^{**}} - \ln \frac{sR_c}{1-\theta^{**}} = 0$$

Prove:

First, after observing the decision variables  $\theta$  of the enterprise, the independent college chooses  $x$  to maximize its net income. Like the Cournot model, the formula (2) is applied to the first order partial derivative of  $x$  and make it as 0, the response function of the independent college to the enterprise subsidy is (7) formula.

$$x = \frac{1}{s} \ln \frac{sR_c}{1-\theta}$$

Because  $x > 0$ ,  $\frac{sR_c}{1-\theta} > 1$ , then  $s > \frac{1-\theta}{R_c}$ .

After predicting the response function of the independent college, enterprises choose  $\theta$  to maximize their expected net income. The formula (7) is brought into the formula (3), and the expected net income function of the enterprise is obtained.

$$\Pi_f = \left(1 - \frac{1-\theta}{sR_c}\right) R_f - \frac{\theta}{s} \ln \frac{sR_c}{1-\theta} \quad (11)$$

The formula (11) is given to  $\theta$  for the first derivative and the two order derivative, respectively. Get

$$\frac{d\Pi_f}{d\theta} = \frac{1}{s} \left[ \frac{R_f}{R_c} + 1 + \ln(1-\theta) - \ln(sR_c) - \frac{1}{1-\theta} \right] = \frac{1}{s} \left[ \frac{R_f}{R_c} - \frac{\theta}{1-\theta} - \ln \frac{sR_c}{1-\theta} \right]$$

$$\frac{d^2\Pi_f}{d^2\theta} = -\frac{1}{(1-\theta)^2} < 0$$

So,  $\Pi_f$  is a concave function of  $\theta$ , there is only  $\theta$  that can make  $\Pi_f$  reach the maximum value,  $\theta$  satisfies  $\frac{d\Pi_f}{d\theta} = 0$ .

$\ln(1-\theta) - \frac{\theta}{1-\theta} < 0$ , in order to ensure that  $\frac{d\Pi_f}{d\theta}$  can be equal to 0,  $\frac{R_f}{R_c} - \ln(sR_c) > 0$  is required, then  $s < \frac{1}{R_c} e^{\frac{R_f}{R_c}}$  is obtained.

$\ln \frac{sR_c}{1-\theta} > 0$  has been obtained according to the preceding derivation. So if  $\frac{d\Pi_f}{d\theta} = 0$ , then  $\frac{R_f}{R_c} - \frac{\theta}{1-\theta} > 0$  is required, therefore,  $\theta < \frac{R_f}{R_c + R_f}$ . The further hypothesis is that

$$f(\theta) = \frac{1}{s} \left[ \frac{R_f}{R_c} + \ln(1-\theta) - \ln(sR_c) - \frac{\theta}{1-\theta} \right]$$

Then

$$f(0) = \frac{1}{s} \left[ \frac{R_f}{R_c} - \ln(sR_c) \right] > 0$$

$$f\left(\frac{R_f}{R_c + R_f}\right) = \ln \frac{1}{s(R_c + R_f)} < 0$$

Therefore,  $\frac{1-\theta^{**}}{R_c} < s < \frac{1}{R_c} e^{\frac{R_f}{R_c}}$ . Equation  $f(\theta) = 0$  has a unique solution at  $\theta \in (0, \frac{R_f}{R_c + R_f})$ .

Theorem 2 has been proved.

According to the above analysis, the expected net income of the independent college, enterprise and the overall system under the Stackelberg Equilibrium is as follows:

$$\Pi_c^{**} = R_c \left(1 - \frac{1-\theta^{**}}{sR_c}\right) - (1-\theta^{**}) \frac{1}{s} \ln \frac{sR_c}{1-\theta^{**}} \quad (12)$$

$$\Pi_f^{**} = R_f \left(1 - \frac{1-\theta^{**}}{sR_c}\right) - \frac{\theta^{**}}{s} \ln \frac{sR_c}{1-\theta^{**}} \quad (13)$$

$$\Pi^{**} = (R_c + R_f) \left(1 - \frac{1-\theta^{**}}{sR_c}\right) - \frac{1}{s} \ln \frac{sR_c}{1-\theta^{**}} \quad (14)$$

### C. Complete Cooperative Game Analysis

If the independent colleges and enterprises can reach a binding cooperation agreement, the game participants will maximize the net profit of the whole system. When the game between enterprises is cooperative, Theorem 3 can be obtained according to the complete cooperative game model.

- Theorem 3:

$$s > \frac{1}{R_c + R_f}$$

When  $s > \frac{1}{R_c + R_f}$ , the two sides can carry out complete cooperation. At this time, the optimal R & D input is as follows:

$$x^{***} = \frac{1}{s} \ln[s(R_c + R_f)]$$

Prove:

The game participants both aim to maximize the net profit of the whole system, so the objective function is formula (4). The formula (4) is obtained for the first derivative and the two derivative of the X, respectively.

$$\frac{d\Pi}{dx} = s(R_c + R_f)e^{-sx} - 1$$

$$\frac{d^2\Pi}{d^2x} = -s^2(R_c + R_f)e^{-sx} < 0$$

So  $\Pi$  is a concave function on the  $x$ , there is only  $x$  can make  $\Pi_c$  maximum. If  $\frac{d\Pi}{dx} = 0$ , can get

$$x = \frac{1}{s} \ln[s(R_c + R_f)] \quad (15)$$

Because  $x = \frac{1}{s} \ln[s(R_c + R_f)] > 0$ ,  $x > \frac{1}{R_c + R_f}$ .

Theorem 3 has been proved.

The formula (15) is brought into the formula (4), and the expected net income of the system is obtained when the system is fully cooperative.

$$\Pi^{***} = (R_c + R_f) \left( 1 - \frac{1}{s(R_c + R_f)} \right) - \frac{1}{s} \ln[s(R_c + R_f)] \quad (16)$$

Similarly,

$$\Pi_f^{**} - \Pi_f^* = \frac{\theta^{**}}{s} \left[ \frac{R_f}{R_c} - \ln \frac{sR_c}{1-\theta^{**}} \right] = \frac{(\theta^{**})^2}{s(1-\theta^{**})} > 0$$

In the above proof, apparently  $\Pi^{**} - \Pi^*$  has been set up.

The Corollary 3 has been proved.

Corollary 3 shows that in Stackelberg Equilibrium, whether it's an independent college, an enterprise, or the whole system, the expected net income they have achieved are higher than the Nash equilibrium. The Stackelberg cooperation is beneficial to both sides. It can not only achieve a "win-win" state, but also can improve the system profit.

2) *The optimal comparison of the three cooperative model systems:* According to the previous model derivation, the corollary 4 can be obtained,

- Corollary 4:

$$\Pi^{***} > \Pi^{**} > \Pi^*$$

Prove:

According to Theorem 3, the optimal profit of the system is,

$$x^{***} = \frac{1}{s} \ln[s(R_c + R_f)]$$

When  $x < x^{***}$ , then  $\Pi(x) < \Pi(x^{***})$ . Because

$$\theta^{**} < \frac{R_f}{(R_c + R_f)} \Rightarrow 1 - \theta^{**} > \frac{R_c}{(R_c + R_f)} \Rightarrow \frac{sR_c}{1 - \theta^{**}} < s(R_c + R_f)$$

Then

$$x^{**} = \frac{1}{s} \ln \frac{sR_c}{1 - \theta^{**}} < x^{***} = \frac{1}{s} \ln[s(R_c + R_f)] \Rightarrow \Pi(x) < \Pi(x^{***})$$

Therefore,  $\Pi^{***} > \Pi^{**}$ , in addition to the conclusion of the corollary 3,

The corollary 4 has been proved.

The corollary 4 shows that the expected net income of the system is the maximum when the two sides fully cooperate.

#### D. The Comparison of the Three Modes of Cooperation and the Improvement of Pareto

1) *Comparison of Cournot model and Stackelberg game model:* According to the previous model derivation, the Corollary 3 can be reached.

- Corollary 3:

$$\Pi_c^{**} > \Pi_c^*, \Pi_f^{**} > \Pi_f^*, \Pi^{**} > \Pi^*$$

Prove:

According to the previous calculation, the formula (8) is subtracted from the formula (12), can get,

$$\Pi_c^{**} - \Pi_c^* = \frac{1}{s} \left[ \theta^{**} - (1 - \theta^{**}) \ln \frac{sR_c}{1 - \theta^{**}} + \ln(sR_c) \right] = \frac{1}{s} \left[ \theta^{**} + \theta^{**} \ln(sR_c) + (1 - \theta^{**}) \ln \frac{1}{1 - \theta^{**}} \right] > 0$$

With corollary 4 know that  $\Delta\Pi = \Pi^{***} - \Pi^{**} > 0$ . But  $\theta$  is changeable,  $\theta \in [\theta_{min}, \theta_{max}]$ . Based on the difference of  $\theta$ , there are many feasible Pareto improvement schemes. The independent college wants  $\theta$  closer to  $\theta_{max}$ , and the enterprise wants  $\theta$  close to  $\theta_{min}$ . Obviously,  $\theta_{max}$  and  $\theta_{min}$  can be calculated, but the core issue here is not  $\theta$ , but how to allocate  $\Delta\Pi$ . About how to allocate the  $\Delta\Pi$ , the references are Rubinstein bargaining method, Nash bargaining method and Shapley value correction method. In this paper, the Rubinstein bargaining model is used to allocate the net income of the increased cooperation. Rubinstein proves that in an indefinite round out bid game, the only result of the sub game refined Nash equilibrium is:

$$k = \frac{1 - \delta_c}{1 - \delta_c \delta_f}$$

Among them,  $\delta_c$  and  $\delta_f$  represent the discounting factor based on the degree of patience in Independent Colleges and enterprises, respectively. Patience is the degree of risk aversion of negotiators, negotiations cost, negotiation ability and competitive advantage. The greater the patience is, the greater the share of the negotiation has. The degree of patience is negatively related to risk aversion and negotiation cost, which is positively related to negotiation ability and competitive advantage.

The net income increased it is assumed from the Stackelberg Equilibrium to complete cooperative game. If it is from the Cournot equilibrium to the cooperative equilibrium, net income increase is  $\Delta\Pi = \Pi^{***} - \Pi^{**}$ .

Given  $\delta_c$  and  $\delta_f$ , through the Rubinstein bargaining model, the net income of the independent colleges and enterprises is as follows:

$$\Delta\Pi_c = (1 - k)\Delta\Pi = \frac{\delta_c(1 - \delta_f)}{1 - \delta_c \delta_f} \Delta\Pi \quad (17)$$

$$\Delta\Pi_c = k\Delta\Pi = \frac{(1 - \delta_c)}{1 - \delta_c \delta_f} \Delta\Pi \quad (18)$$

The formula (17) and the formula (18) show that when the independent college and the enterprise are negotiating, if

its discounting factor is large, the greater its income is. That is, if the risk aversion is small, the negotiation cost is small, the negotiation ability is strong, and the competitive advantage is obvious, the more profits are divided. On the contrary, the less the income is.

## V. CONCLUSION

Three Game Analysis of school enterprise cooperation between independent colleges and enterprises is carried out in this paper. Through the analysis, it proves existence of the Stackelberg Equilibrium and cooperative equilibrium. And it also confirms that fully cooperative decision-making can improve the net income of both independent colleges and businesses. Therefore, in the process of school enterprise cooperation with independent enterprises, independent colleges can not only improve their own situation, but also provide profits for enterprises, so the cooperation between colleges and enterprises in Independent Colleges and enterprises is of great significance. At the same time, there are some meaningful conclusions and understandings in the process of the analysis.

First, the strength of the independent college is an important factor affecting the form of cooperation with the enterprise. Independent colleges should first try to improve their basic strength. In addition, the improvement of Independent Colleges' own strength helps increase profits of independent colleges, enterprises and the whole system.

Second, cooperative R&D is better than non-cooperative R&D. This paper has proved that complete cooperation is better than that of Stackelberg cooperation, and is better than the non-cooperative. Therefore, for independent colleges, choosing appropriate enterprises to cooperate is more meaningful than the independent development.

Third, the independent college should have a clear plan for R&D and talent training. No matter whether the work can be carried out smoothly, independent colleges should clear the training plan and development plan of their talent. Only in this way can constantly improve their own strength and better cooperate with enterprises to provide negotiated capital. And the cooperation will also promote the further improvement of the independent college's strength, so as to form a virtuous cycle.

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