

# Modeling of Enterprise Development on the Basis of Approximation of Methods of Innovation Dynamics

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**Abstract.** The article discusses modeling algorithms based on the approximation of innovative dynamics methods. It is implemented using a system of differential equations and functionality to determine the optimal period for the transition of an enterprise system to dynamic development. For this, the equilibrium states and the necessary values of the indicators are determined. On the basis of the developed model, the equilibrium conditions of PJSC “Bashtansky Cheese Factory” were calculated and a phase portrait of its states was formed. The authors proposed to highlight seven possible equilibrium states that can be determined using financial and economic indicators that summarize a certain level of managerial and technological maturity of an enterprise.

**Keywords** — *innovation dynamics, modeling, equilibrium states, system behavior, enterprise development.*

## I. INTRODUCTION

To ensure the dynamic development of the enterprise, it is important to define the management of a new type that implements modern formats of possible variants of enterprise development as a socio-economic, which is constantly changing under the influence of factors of the business environment.

Given that the company always operates under the influence of endogenous and exogenous factors of the external business environment, the authors believe that there will be no absolute stability and balance for the enterprise. The system loses equilibrium as a result of influence on it, in inconsistency process, uncertainty, deviation from the planned development targets, there are contradictions in the mechanism of action of the system, fluctuations are created and the system fluctuates shown in the papers [1, 3, 9].

If we consider the process of dynamic enterprise development as a transition from one level of stability to another, then we define the criteria for the loss of stability. The matrix of space of equilibrium states of the enterprise allows us to characterize enterprise equilibrium state. We have identified seven basic levels of enterprise system stability based on work [2, 6, 7, 9].

## II. MATERIALS AND METHODS

### A. Description of the model, subject of the model, and research methods

We are invited to apply the mathematical model of optimal rapid transition of the enterprise system to an effective state, which will allow us to formulate scenarios for how to achieve the planned result in a short time by effective methods.

Consider the behavior model of an enterprise system that is under the influence of management activity and is described by the system of differential equations on the basis of work [1, 6, 8].

$$\frac{dY_i}{dt} = f_i(t, y_1, y_2, \dots, y_k, u_1, u_2, \dots, u_r), i = \overline{1, n} \quad (1)$$

or in a vector form

$$\frac{dY}{dt} = f(t, y, u) \quad (2)$$

where:  $Y (y_1, y_2, y_k, \dots, y_n)$ ,  $f(f_1, f_2, \dots, f_n)$  –  $n$ -dimensional vectors of functions that describe the state of the enterprise system;

$U(t)$  –  $r$  dimensional vector whose function is described as:  
 $u(t) = (u_1, u_2, \dots, u_r(t))$  – enterprise system management through parameters  $(u_1, u_2, \dots, u_r)$ ;

$y(t)$  – the decision in line the management

$u(t)$  – system trajectory;

numbers  $(y_1, y_2, \dots, y_n)$  – phase coordinates;

$y (y_1, y_2, \dots, y_n)$  – phase point - financial and economic indicators of the system state.

$t$  - independent variable of the differential equation.

The efficiency of the enterprise system can be described as functional (3):

$$Y = \int_{t_0}^T f_0(t, y_1(t), \dots, y_n(t), u_1(t), \dots, u_r(t)) dt \quad (3)$$

We introduce Y as a parameter of system efficiency. The smaller Y, the higher the efficiency of the economic system of the enterprise. The function  $f_0$  is determined by the desired state to which the system must reach some state.

Let's consider the problem of optimal performance in linear form, which translates the state of the enterprise for min time into an effective state. To determine the task, modeling the state of the enterprise, we propose to apply a system of differential equations (linearly effective control).

$$\frac{dY}{dt} = A(t)Y + C(t)u, \quad (4)$$

where:  $A(t) = \|a_{ij}\|$  – square matrix, the elements of which characterized the impact

on the economic state of the enterprise system.

where:  $i, j = 1, n$ ;  $Y(t)$ ,  $C(t)$  – matrix n-dimensional vectors, the elements of which reflect the economic state of the enterprise system. Example,

$y_1$  – profit of the enterprise system;

$y_2$  – the number of resources needed to produce a unit of product, etc.

$u(t)$  – vector function, which expresses the ratio of the amount of investments in the innovation development of the enterprise to the total cost of production. This is management limitation  $u(t) \leq l$ .

The initial and final position of the enterprise system can be specified by the equation:

$$Y(0) = a, \dots Y(T) = 0 \quad (5)$$

It should be noted that a is an integral indicator of the state of the enterprise's economic system at the time of the beginning of the implementation of management actions. We are tasked to find the optimal set of management actions  $u(t)$ , which translates the state of the enterprise described by the changes in the equation (4), from the initial position in the most efficient operation for the shortest period of time T. The statement of the problem is considered in a scalar form. The differential equation describes the dynamics of processes in the enterprise system:

$$\frac{dY}{dt} = kY, \quad (6)$$

where:  $k$  – coefficient of proportionality, which gives a characteristic of the change in the enterprise system.

Let,  $k=m-n$ , where:  $m$  – coefficient of growth of market value of the enterprise,

$n$  – loss ratio of the enterprise.

If at a certain point in time  $t=0$ , the state of equilibrium of the enterprise system, which is characterized by the parameter  $y_0$ , then the equation leads to an exponential change in the characteristics of the enterprise system.

$$y(t) = y_0 e^{kt}, \quad (7)$$

Coefficient gains the book value of the enterprise ( $k=m-n$ )

$$m = b_1 - b_2 y \quad (8)$$

$$n = b_3 + b_4 y \quad (8 a)$$

where:  $b_1$  – resources for production;

$b_2$  – the required amount of resources for a unit of output;

$b_3$  – the volume of production which is not in demand or is not competitive;

$b_4$  – resources used per unit of products that are not successful in the market.

Then  $k$ - the coefficient that gives the characteristics of changes in the enterprise system can be filed as follows (9):

$$k = m - n = b_1 - b_3 - (b_2 + b_4)y = (b_2 + b_4)((b_1 - b_3) / (b_2 + b_4) - y) = \alpha(A - y), \quad (9)$$

$$\text{where: } \alpha = b_2 + b_4, \quad A = (b_1 - b_3) / (b_2 + b_4).$$

The equation of change in the state of the enterprise system in this model has the form (10):

$$\frac{dY}{dt} = \alpha(A - y)Y \quad (10)$$

If we divide the variables in this equation we get (11)

$$\frac{dY}{(A - y)y} = \alpha dt. \quad (11)$$

Under the conditions on which the enterprise system was analyzed  $y(t_0) = y_0$ , you can enter additional variables into an equation that describes changes in the enterprise system (12):

$$y(t) = \frac{A}{(1 + (\frac{A}{y_0 - 1})e^{-\alpha t})}, \quad (12)$$

$$\text{At } \alpha \geq 0 \text{ or } A \geq 0,$$

we will get  $y(t) \rightarrow A$  at  $t \rightarrow \infty$ .

The defined equation contains two parameters A and  $\alpha$ , for their definition it is necessary to have two additional values  $y(t)$  when given  $t_1$  and  $t_2$ .

In the equation A is called the potential opportunity systems of the company to transition to the desired state of equilibrium, in which it is possible to implement the most optimal managerial impacts on the enterprise system, that is the parameter A is the determining criterion for the effective operation of the enterprise. The criterion for such an activity is the functional, which is described by the expression (13):

$$Y = \int_0^{\tau} [x(t) - y(t)] dt \rightarrow \min. \tag{13}$$

Variable control  $y(t)$  limited  $y(t) \leq x(t)$  (the dynamic development of the enterprise system at the expense of its own resources, is defined as a functional)(14):

$$Y = \int_0^{\tau} [A - x(t)] dt \rightarrow \min. \tag{14}$$

This functional (14) expresses the criterion of optimality of the enterprise as an economic system, which is expressed in the approximation of the equilibrium state indicators to its limit value.

### III. RESULTS AND DISCUSSION

#### A. Formation of the phase portrait of the state of the enterprise for PJSC "Bashtany cheese factory"

Parameter A – provides a characteristic of the most effective development in the given parameters of the enterprise system (this is a dynamic indicator). To optimization the current state to  $A_{opt}$  the current state of the economic system of the enterprise should approach the species A at the expense of the parameters that can be managed. These parameters, which are objects of administrative actions, include parameters, which determine the innovative activity of the enterprise and the main indicators determining the enterprise as a social and economic system. Indicators for constructing a model of the field of bifurcation and attractors are given in the table 1.

TABLE I. TABLE I. MAIN MANAGEMENT INDICATORS

Conformity	Indicator
<i>Indicators of the financial and economic group</i>	
$b_1 \rightarrow E_{21}$	Indicator total liquidity
$b_2 \rightarrow E_{14}$	Indicator of maneuverability of equity
$b_3 \rightarrow E_{11}$	Indicator of financial stability
$b_4 \rightarrow E_{42}$	Indicator Asset profitability ratio (ROA)
<i>Indicators of the production group</i>	
$b_1 \rightarrow P_{11}$	The coefficient of efficiency of use of the basic production assets of the enterprise
$b_2 \rightarrow P_{14}$	Indicator the share of production working capital in working assets
$b_3 \rightarrow P_{16}$	Indicator fixed asset renewal
$b_4 \rightarrow P_{22}$	Indicator loading of production facilities

The methodological composition of the economic-mathematical modeling is based on the data of each PJSC "Bashtany cheese factory" [4].

For each enterprise indicator of optimal transition are defined. The choice of metrics was determined by criterion is small errors and great one authenticity.

$A$  – static coefficients;  $\alpha$  – coefficient system movement:

$$1) A = \frac{b_1 - b_3}{b_2 + b_4}, \tag{15}$$

$$\alpha = b_2 + b_4. \tag{16}$$

1. The first stage of the calculation of the model, the calculation of the indicator A.
2. The second stage of model calculation, calculation  $\alpha$ .
3. Calculation  $Y_0$  – carried out by the formula previously derived.
4. To calculate the corridor of states of equilibrium, define a portrait of the states of the cheese factory. For this we will select indicators of equilibrium states of the enterprise. A.

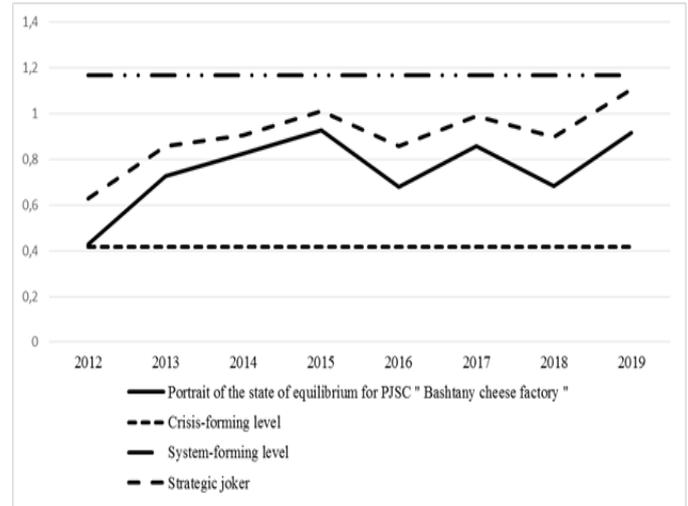


Fig. 1. Portrait of the state of equilibrium for PJSC "Bashtany cheese factory"

Based on the above formulas and the use of statistical information from the open source, we calculated the phase portrait of the enterprise. Such a phase portrait suggests the allocation of bifurcation points, i.e., points at which a change in the equilibrium of the enterprise occurs. The portrait consists of: graphics of the actual state of the enterprise. The level after which the crisis at the enterprise occurs. The level at which there is a qualitative change of the enterprise system; the level that is called the joker (this is the level of the possible alternative development of the enterprise, which will help it to achieve dynamic development in the shortest possible time).

The practical implementation of the model was carried out in 6 stages. The calculation of indicators was made for 12 years from 2008-2019. In the third stage we calculated  $Y_{2008-2019}$ . It had certain meanings:  $Y_{2008} - 2,4386$ ;  $Y_{2009} - 1,7151$ ;  $Y_{2010} - 0,8317$ ;  $Y_{2011} - 0,463$ ;  $Y_{2012} - 0,7501$ ;  $Y_{2013} - 0,8795$ ;  $Y_{2014} - 1,0242$ ;  $Y_{2015} - 1,0529$ ;  $Y_{2016} - 0,6347$ ;  $Y_{2017} - 0,8795$ ;  $Y_{2018} - 1,0242$ ;  $Y_{2019} - 1,0529$ .

On the base of the calculations, a movement schedule of enterprise change was built and bifurcation points are pointed out. On the graphs they are presented as monobifurcation points, that is, those key moments at which the enterprise lost its balance and could fall to the crisis-forming level, and moments when the enterprise could reach a higher system-forming level of managerial and technological maturity to go into the flow of dynamic development of all enterprise systems.

5. Calculation of mathematical expectation  $M(A)$  the state of equilibrium of the enterprise and the phase of the cycle in which the enterprise is located.

$$M(A) = \frac{\sum A}{9} = 1,166 \quad (17)$$

6. Structural-functional modeling of minimization of the determined functional of the system's behavior and possible equilibrium states.

I. Determining the optimal transition to the most effective state of the enterprise development by the given parameters for a certain period of time. This state is determined with the help of  $u(t) = ky^*(t)$ ;  $y^*(t) - I_{HAK}$  - innovative activity of the system;  $k$  - coefficient of proportionality

II. The justification of the choice of a tunnel transition in the most effective state of the enterprise, as an economic system, is carried out with the help of a functional by the choice of the period of implementation of the transition. To do this, use the value of the functional.

$$\int_{t_0}^T \frac{dt}{1 + \beta e^{-\nu t}} = \left[ t + \frac{1}{\nu} \ln(1 + \beta e^{-\nu t}) \right]_{t_0}^T \quad (18)$$

The calculation is made using a table of integrals and other mathematical formulas [14].

III. We will calculate the term of the possible transition of the system by the defined parameters of functioning in the most optimal state. The term is determined depending on the main parameter of innovative capabilities - innovation activity.

7. Structural-functional modeling of the possible transition period of the economic and social system of the enterprise to the optimal set state of equilibrium, that is, the definition and formation of the pool attractor PJSC "Bashtany cheese factory".

IV. Consider the function  $y^*(t)$  where  $t_0$  accept as  $f$  2016.

The measurement effect from the introduction of innovative management technologies will be achieved in the period from 1 to 15 years. That's why the dimension of the temporal parameter is given in  $T = 1,5$  years. To understand how effective was the innovative technology that we implemented, it takes time. Usually, companies see it only after 7-9 months. We took a period of 1,5 years to understand how long the enterprise would need to go into a dynamic development mode if it remains in the same mode of innovation movement.

Let us note  $T$ - the period for which the enterprise may make active changes.

This value shows that for the transition and preservation of the dynamic development of the enterprise it is necessary to form innovative activity at the level not less than 63.76% in the total volume, in order to transfer in 1,5 years to the system of the enterprise in the planned state of development.

$$\left. \begin{aligned} \nu &= +A\alpha = 1,166\alpha, \\ \beta &= \frac{A}{y_0} - 1 = \frac{1,166}{0,6347} - 1 = 0,8371, \\ \nu &= 1,166 * 2,25 = 2,6235. \end{aligned} \right\} \quad (19)$$

V. Calculate the coefficient of innovation activity  $y^*(t)$ :

$$\left. \begin{aligned} y^*(t) &= 1.5 + \frac{1}{2,6235} \ln(1 + 0.8371e^{-2,6235*1,5}) - \\ &\quad - \frac{1}{2,6235} \ln(1 + 0,8371) = 1,2752, \\ y^* &= 1,2752; \\ k &= 0.5 \rightarrow k, \\ u(t) &= k \bullet y^*(t), \\ u(t) &= 0.5 * 1,2752 = 0.6376. \end{aligned} \right\} \quad (20)$$

To monitor the state of the system and to prevent adverse effects in a timely manner, it is necessary to constantly analyze the state of the enterprise. Method of assessing the level of financial and economic and social stability of the enterprise on the basis of qualitative and quantitative analysis of economic, production, personnel, informatization and innovative processes, which allows to form analytical forecasts of the enterprise development in the current conditions of the business environment. [2,5, 6].

Parameters indicate the range of enterprise system fluctuations. But the company's location on the rise of the amplitude or on the downturn depends on the level of managerial and technological maturity, and the stage of development or decline of the enterprise. The state of normal equilibrium shows that the system is in recession and is in a state of oscillation, fluctuations are insignificant (Figure 2).

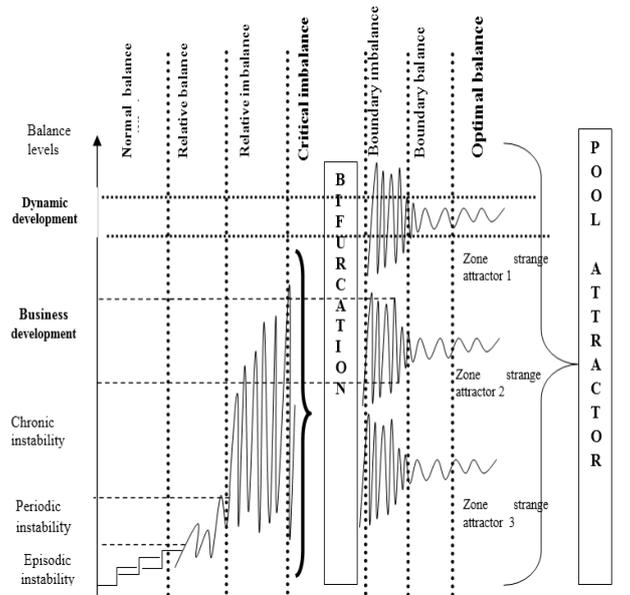


Fig. 2. The matrix of the zoning of equilibrium states, and the definition of the bifurcation point that arise during the operation of the enterprise (developed by the author)

The state of optimum equilibrium shows the rise of the system and the ability of the enterprise to dynamic development.

The state of relative equilibrium is quasi-development, and the marginal imbalance is the attenuation of fluctuations and the entry of the system into its development corridor.

#### IV. CONCLUSIONS

On the basis of the performed structural and functional modeling of the implementation, it is proposed to use the mathematical model of the optimal rapid transition of the enterprise system to an effective state, which will allow the formation of scenarios, how to achieve the planned result in a short time by effective methods.

Systems approach requires to analyze the system of the enterprise with an extrapolation-cyclic position, that is, based on the analysis of the real state of the enterprise to form a model of the space of states and to identify the opportunities and the tendency to dynamic development.

When forecasting changes in enterprise parameters, you can identify the prevailing trends in the vector of motion enterprises, possibilities of influence on separate elements of the system or in general the whole system, and possibilities of regulating fluctuations in the system and creating contour conditions for controlled attractors to ensure the dynamic development of the enterprise.

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