

Cryptocurrency Portfolio Optimization Using Value-At-Risk Measure

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Abstract—Current research has led to a rejection of the hypothesis of a normal distribution of financial assets returns. Under these conditions, portfolio variance cannot serve as a good risk measure. In this paper analyzed the daily returns of the most common cryptocurrencies: Bitcoin, Bitcoin Cash, Litecoin, XRP, Ethereum, NEM. It is shown that the asset returns are not normally distributed, but with good precision follow the Cauchy distribution. The analytical expressions for risk measure were obtained using the Cauchy distribution function and the VaR technique. The efficient frontiers of cryptocurrencies portfolios were constructed using modified Markowitz model. The purpose of the article is to assess the risks of major cryptocurrencies and to diversify the risk of cryptocurrency investing by applying a portfolio model

Keywords—cryptocurrency, portfolio of assets, expected return, risk measure, variance, Value-at-Risk

I. INTRODUCTION

The first complete cryptographic currency appeared in 2008 thanks to the efforts of Satoshi Nakamoto. It was named Bitcoin. New varieties of digital currency appear each year due to the information technology active development and the globalization processes spread. The main advantages of cryptography are that the user controls them without any regulatory rules in the transaction. Third party costs on a transaction can be greatly reduced. This has been the main reason for the rapid development of the market for virtual currencies (crypto-currency) over the past 10 years. More than 2000 varieties of digital money have appeared on the market since the birth of Bitcoin for 5 years. Bitcoin remains the most widespread cryptocurrency: there is the largest market capitalization among other digital currencies (about \$220 billion) [1]. The first positions of the market capitalization rating as of July 2019 are the following cryptocurrencies: Ethereum (about \$33 billion), XRP (Ripple) - about \$17 billion, Litecoin and Bitcoin Cash (about \$7.5 billion each).

But investments in cryptocurrency can be quite risky as their price is very volatile. Thus, during the period from July 2018 to July 2019 there were significant changes in the exchange rate. Initially, the cost of one Bitcoin was \$6,600 (July 2018). There was a significant dropping in mid-December 2018 in the price - to \$3,200. Then there was a sharp increasing at the end of June 2019 - to \$13,000. The price of Bitcoin Cash fluctuated from \$869 per unit (July 2018) to \$77 per unit (mid-December 2018) to \$400 per unit in June 2019. The price of the unit XRP demonstrated a sharp jump from \$0.26 to \$0.58 during three weeks in

September 2018. Then it began to fall with slight fluctuations. The course of the ordinary currency (dollars, euros, etc.) strongly depends on inflation, politic factors and other economic conditions. Therefore, its calculations can be performed fairly accurately, taking into account the influence factors changing. Instead, fluctuations in the price of cryptocurrency are very difficult to forecast. Therefore, making the correct decisions in investing and trading cryptocurrency in order to get the most return is a rather difficult task. The interaction between supply and demand, the attractiveness for investors, macroeconomic conditions and financial events are important factors in the formation of the cryptocurrency price [2]. In addition, investors rely vastly on speculation and rumors that also affect the cryptocurrency price change.

II. LITERATURE REVIEW

Diversification is an important risk reduction tool. Creating a portfolio of financial assets is one of its instruments. In this paper, the formation of cryptocurrency investment portfolio based on the Markowitz model is investigated [3]. By changing the proportion of certain assets in a portfolio, it can be managed to maximize return or to minimize risk. The Markowitz model relies on the hypothesis of a normal distribution of returns. This hypothesis significantly simplifies the problem of choosing a portfolio for investing, since it allows you to compare alternative portfolios by just two criteria: standard deviation and mathematical expectation. However, numerous theoretical researches in the field of finance [4]–[10] and the events in the financial market at the end of 2008 - early 2009 are doubted the hypothesis of a normal distribution of return.

It has been shown that the distribution of financial assets contains so-called "heavy tails". It indicates a high likelihood of realization of very large and very small return values. The task of this work is investigating the distribution of the return of virtual currencies and using it to minimize the risk of working with portfolios of cryptocurrencies. The results of the study [11] are shown that the inclusion in the investment portfolio of several cryptocurrencies brings to investors the advantages of diversification for short term investments.

Building a portfolio solely on the basis of cryptocurrencies [12] shows that a cryptocurrencies set increases investment opportunities with a low level risk. In contrast to our research, this work does not take into account the possible deviation of the distribution of the cryptocurrency return from the normal one. In the work [13]

researchers apply a portfolio diversification strategy that is based on several models of portfolio formation. So, on the basis of the modern portfolio theory, an optimal risk portfolio has been established and the effect of cryptocurrency on the usual investment portfolio of assets has been investigated. The results, obtained in the paper [14], show that the expected return on the cryptocurrency portfolio is greater than the return of separate cryptocurrency. The risk assessment was carried out according to the quantile method, but unlike our research, the distribution of assets return does not determined.

The authors of the work [15] emphasize the importance of modeling nonlinearity and taking into account the behavior of tail distribution in analyzing the causal relationships between Bitcoin revenues and trading volume. For analysis the Bitcoin behavior in the study [16] taking into account heavy tails of return distribution, quantile regression is used. This made it possible to determine that Bitcoin does act as a hedge against market uncertainty. Yet, the quantile method is applied only to Bitcoin analysis without specifying the asset return distribution ([15], [16]). In the paper [17] another approach is offered. It considers the decision-making process related to technological innovation is considered in the conditions of uncertainty and risk arising from incomplete information about the explored system. The proposed economic-mathematical model allows describing the dynamics of multi-stage control of the technological innovation process, depending on investment resources receipt.

III. DATA AND METHODOLOGY

Thus, as shown by the analysis of literary sources, in present-day conditions, not only currencies and valuable metals are used for investment, but also cryptocurrency assets are added to the portfolio. Our analysis was done on the basis of historical data on prices of 6 cryptocurrency (Bitcoin, Bitcoin Cash, Litecoin, XRP, Ethereum, NEM) for the period from July 5, 2018 to July 4, 2019. This data are freely available from the site Analytical Service CoinMarketCap www.coinmarketcap.com. For further processing, the calculation of the corresponding normalized cryptocurrency return is performed according to

$$x_{ni} = C_{ni+1} / C_{ni} - 1, \tag{1}$$

where

- x_n is the daily return of the n-th asset,
- C_n is the daily closing price of the n-th asset,
- i is the observation number.

The dynamics of cryptocurrency Bitcoin return is presented in Fig. 1. The main characteristics of the investigated cryptocurrency return for the observed period are given in Table 1.

From the correlation matrix (Table 2) it can be seen that the return of the cryptocurrency is sufficiently correlated with each other.

Let's introduce the concept of the risk zone boundary. In this capacity we will use the 5% quantile of return. To determine the risk zone boundary it is necessary to identify the distribution of returns. Under the investor risk we understand the difference between the most expected value

of cryptocurrency return (\bar{x}) and 5% quantile of return (risk zone boundary L), which is determined using the corresponding return distribution. If the distribution is normal, the most expected return value \bar{p} is the average value of sample. If the distribution is different from the normal one and is asymmetric, we will use the median return Me as an expected return. A significant asymmetry in the return distribution (Table 1) prompts as the most expected return value to choose the median sample, rather than the average value of sample.

Consequently, the value of the asset risk, in accordance with the above definition, can be estimated by the ratio

$$V_j = Me_j - L_j. \tag{2}$$

As a result of research of the cryptocurrencies Bitcoin, Bitcoin Cash, Litecoin, XRP, Ethereum, NEM, using the Pearson, Kolmogorov-Smirnov, and Shapiro-Wilk tests, in all cases the hypothesis of normal returns was rejected. Computer experiments showed that the return of the investigated cryptocurrency with good accuracy is described by the Cauchy distribution (Fig. 3). The Cauchy distribution function has the form

$$F(x) = \frac{1}{\pi} \arctg\left(\frac{x-\mu}{\gamma}\right) + \frac{1}{2}. \tag{3}$$

Here

- μ is the mathematical expectation (median) of return,
- γ is the coefficient of distribution function chosen by us for each case in accordance with the least squares method.

For this goal, an interval distribution table was constructed. The role of the minimized value was the sum of the squares of the differences between the theoretical and actual values of the frequency at different intervals. The parameters μ , γ for the various cryptocurrencies are shown in Table 3.

Using the form of the Cauchy distribution function (3), we can find an analytic expression for risk degree at a given confidence level α . From equality

$$\frac{1}{\pi} \arctg\left(\frac{x-\mu}{\gamma}\right) + \frac{1}{2} = \alpha \tag{4}$$

we define

$$L_\alpha = \mu + \gamma \cdot \text{tg}\left(\pi\left(\alpha - \frac{1}{2}\right)\right). \tag{5}$$

Using (2), we calculated the risk V at the level of 5% for each cryptocurrency. As the most expected value of return, we used the median of the appropriate distributions. The results of calculations are shown in the last line of Table 1. We optimize the portfolio by applying the "modified Markowitz model". The obtained risk assessments (risk

measures) are used instead of the standard deviation in the cryptocurrency portfolio optimization.

IV. PORTFOLIO OPTIMIZATION

For the building the cryptocurrencies portfolio, let's used the technique, described in the work [18]. Assuming that cryptocurrency returns $r_i(t)$ are poorly stationary random processes, each of which is characterized by mathematical expectations μ_i and a degree of risk V_i , then for portfolio

optimization, a modified Markowitz model can be used. In this case, the mathematical description of the problem at the maximum portfolio return will have the form:

$$\begin{cases} R_p = w_i \times \mu_i \rightarrow \max; \\ V_p = \sqrt{\sum_{i=1}^6 \sum_{j=1}^6 (w_i \times V_i \times w_j \times V_j \times \rho_{ij})} \leq V_{req}; \\ w_i \geq 0; \sum w_i = 1. \end{cases} \quad (6)$$

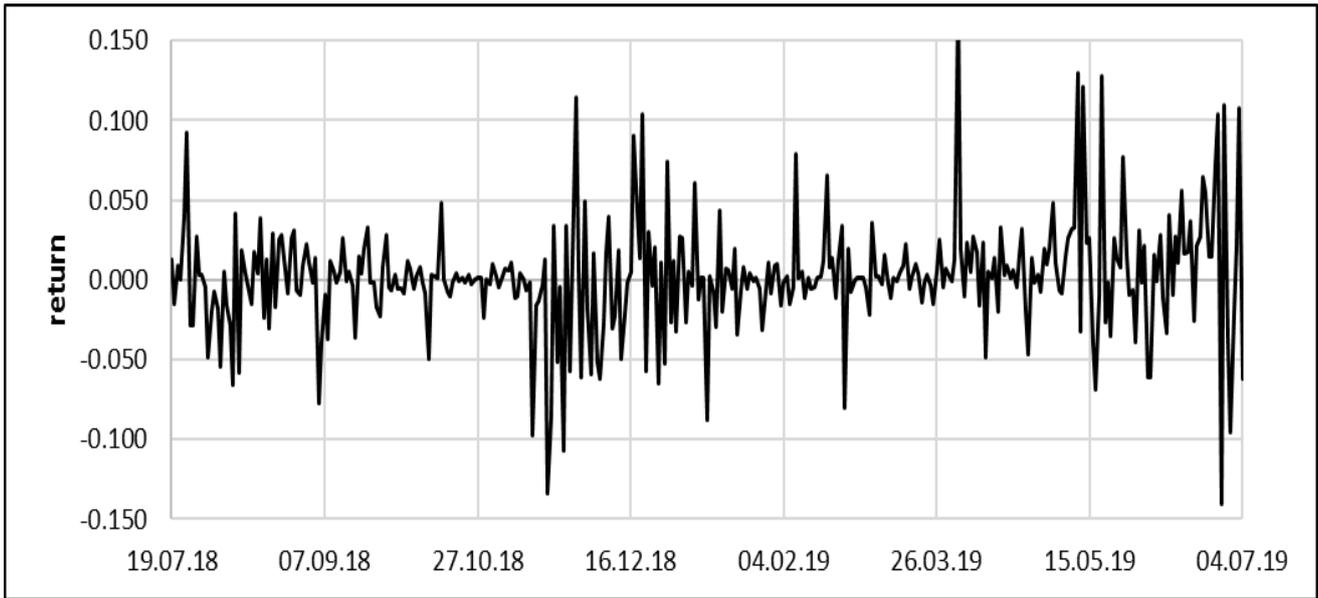


Fig. 1. Dynamics of return cryptocurrency Bitcoin (06.07.2018-04.07.2019)

TABLE I. STATISTICAL CHARACTERISTICS OF CRYPTOCURRENCY RETURN FOR THE PERIOD 07/05/2018 TO 07/04/2019

	Bitcoin	Bitcoin Cash	Litecoin	XRP	Ethereum	NEM
Distribution form Factor γ	0.0140	0.0234	0.0252	0.0205	0.0227	0.0257
Minimum	-0.141	-0.336	-0.137	-0.171	-0.187	-0.175
Maximum	0.174	0.512	0.308	0.380	0.181	0.253
Median, μ	0.0014	-0.0039	-0.0026	-0.0023	-0.0012	0.0001
Average	0.0021	0.0006	0.0023	0.0007	-0.0002	-0.0008
Standard deviation	0.036	0.069	0.051	0.053	0.049	0.051
Skewness	0.352	1.705	1.030	1.890	0.043	0.802
Return, per year	0.0021	0.0006	0.0023	0.0007	-0.0002	-0.0008
Return, per last 3 months	0.0101	0.0051	0.0050	0.0029	0.0075	0.0049
The risk zone boundary, $\alpha = 0.95$	-0.0869	-0.1514	-0.1615	-0.1317	-0.1444	-0.1624
Risk measure, %	9.696	15.653	16.642	13.446	15.177	16.724

TABLE II. CORRELATION MATRIX OF RETURN

	Bitcoin	Bitcoin Cash	Litecoin	XRP	Ethereum	NEM
Bitcoin	1	0.74	0.77	0.62	0.81	0.69
Bitcoin Cash	0.74	1	0.75	0.59	0.75	0.64
Litecoin	0.77	0.75	1	0.66	0.82	0.67
XRP	0.62	0.59	0.66	1	0.76	0.65
Ethereum	0.81	0.75	0.82	0.76	1	0.77
NEM	0.69	0.64	0.67	0.65	0.77	1

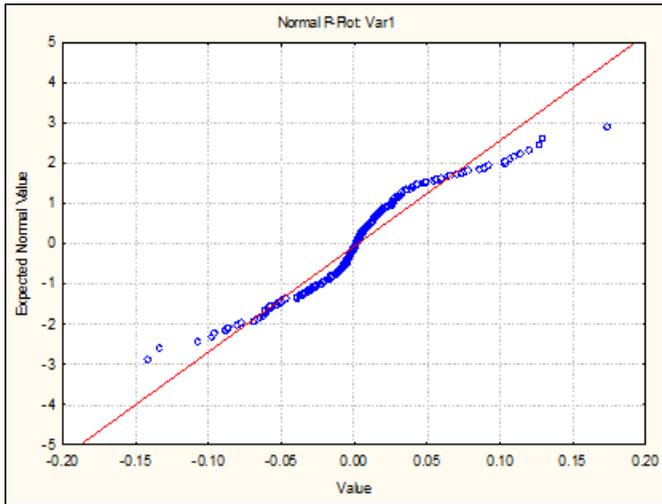


Fig. 2. Hypothesis testing on normal distribution of the Bitcoin return

We used an approach similar to the Markowitz approach to assess portfolio risk V_p , but instead of a standard deviation of stock return on the risk measure V_i we got. In contrast to the mean square deviation that describes the average deviation of stock return from its mathematical expectation, the risk measure V_i evaluates the deviation of VaR from the mathematical expectation of stock return. The correctness of such approach to optimizing the portfolio substantiated in the works [10], [19]. The mathematical description of the problem for a minimum portfolio risk will have the form:

$$\begin{cases} V_p = \sqrt{\sum_{i=1}^6 \sum_{j=1}^6 (w_i \times V_i \times w_j \times V_j \times \rho_{ij})} \rightarrow \min; \\ R_p = w_i \times \mu_i \geq R_{req}; \\ w_i \geq 0; \sum w_i = 1. \end{cases} \quad (7)$$

Here

- w_i is the weight of the i -th financial asset in portfolio,
- V_p is the general portfolio risk,
- V_{req} is the recommended portfolio risk,
- R_p is the overall portfolio return,
- R_{req} is the recommended portfolio return.

For portfolio optimization we will use the expected cryptocurrency stock returns $\mu_1, \mu_2, \mu_3, \mu_4, \mu_5, \mu_6$, previously found risk estimates $V_1, V_2, V_3, V_4, V_5, V_6$, and a pseudo-covariance $cov(r_i, r_j) = \rho_{ij} \cdot V_i \cdot V_j$, where $\rho_{ij}, i = 1, 6; j = 1, 6$ is a Pearson correlation coefficient between the two time series of cryptocurrency stock return.

Using (7) without regard to second condition we obtain the minimum possible risk level $V_p = 9.62\%$. The return of corresponding portfolio is $R_p = 0.00926$. Using (6) without regard to second condition we obtain the portfolio with

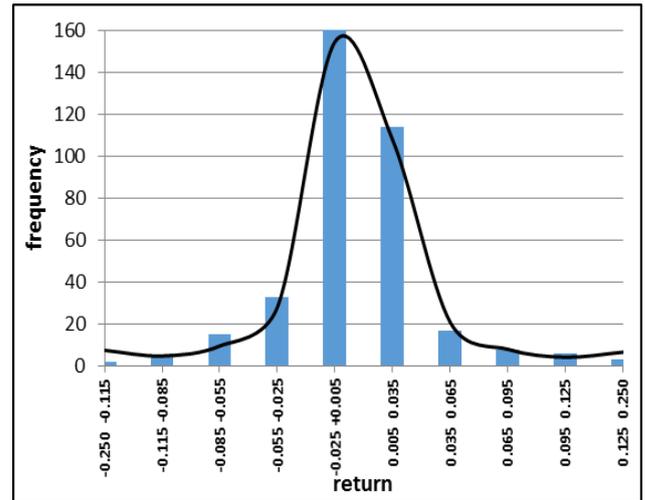


Fig. 3. Actual form of Bitcoin return distribution

maximum possible risk level $V_p = 9.70\%$. The return of corresponding portfolio is $R_p = 0.00996$. By changing the risk level V_{req} from minimum to maximum value in step 0.01% and using (6), we received the set of effective portfolios that meet the condition of maximum return R_p .

So, using the obtained above cryptocurrency risk estimates (Table 1), we constructed the set of optimal portfolios (the efficient frontier). Each such portfolio gives maximum return at the established risk level. For the first time, the concept of optimal portfolios set was introduced by Markowitz [3]. The table 3 presents the portfolio structure for each, obtained by us, optimal solution. The analysis of the table confirms the well-known statement that a higher return level always requires a higher risk degree.

The high return and low risk of Bitcoin predetermine its dominance in the cryptocurrency portfolio. At the same time, this limits the possibility of diversifying the portfolio.

To increase the weight of alternative cryptocurrencies in the portfolio, it is necessary to introduce an additional limitation on the Bitcoin weight $w_1 \leq w_{1req}$. Taking into account this condition the model (6) takes the form

$$\begin{cases} R_p = w_i \times \mu_i \rightarrow \max; \\ V_p = \sqrt{\sum_{i=1}^6 \sum_{j=1}^6 (w_i \times V_i \times w_j \times V_j \times \rho_{ij})} \leq V_{req}; \\ w_i \geq 0; w_1 \leq w_{1req}; \sum w_i = 1. \end{cases} \quad (8)$$

TABLE III. SET OF OPTIMAL PORTFOLIOS (THE RISK MEASURE VAR)

w_1	w_2	w_3	w_4	w_5	w_6	V_p	R_p
0.890	0.000	0.000	0.110	0.000	0.000	9.6167	0.00926
0.906	0.002	0.000	0.090	0.001	0.000	9.6300	0.00939
0.920	0.002	0.000	0.074	0.001	0.002	9.6400	0.00950
0.937	0.003	0.000	0.057	0.001	0.002	9.6500	0.00962
0.950	0.003	0.000	0.044	0.001	0.002	9.6600	0.00971
0.960	0.003	0.000	0.033	0.001	0.002	9.6700	0.00979
0.970	0.003	0.000	0.024	0.001	0.002	9.6800	0.00986
0.978	0.003	0.000	0.016	0.001	0.002	9.6900	0.00991
0.981	0.003	0.000	0.012	0.001	0.002	9.6957	0.00994
0.983	0.004	0.000	0.009	0.001	0.002	9.7000	0.00995

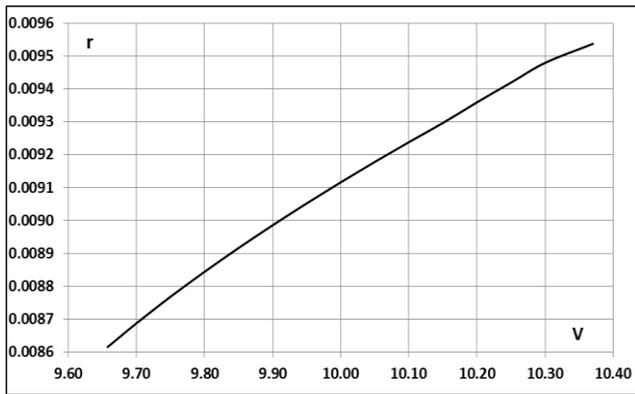


Fig. 4. Set of optimal cryptocurrency portfolios, taking into account the Bitcoin share limitation $w_1 \leq 0.8$

The set of optimal cryptocurrency portfolios with Bitcoin share limitation $w_1 \leq 0.8$ is presented in Fig.4. Using (8) we received a set of scenarios, each of which corresponds to the optimal portfolios set at different w_{1req} . The graphical representation of the corresponding sets for $w_{1req} = 0.6; 0.7; 0.8; 0.9; 1.0$ is presented in Fig.5.

V. CONCLUSION

We have shown that the cryptocurrency returns are not subject to normal distribution, but they can be described by the Cauchy distribution. Using the Cauchy distribution function, we obtained the analytical expressions for VaR risk measures and performed calculations of cryptocurrencies risk assessment using approach VaR. As a result of optimization, the sets of optimal cryptocurrency portfolios were built. The high return and low risk of Bitcoin predetermines its dominance in the cryptocurrency portfolio.

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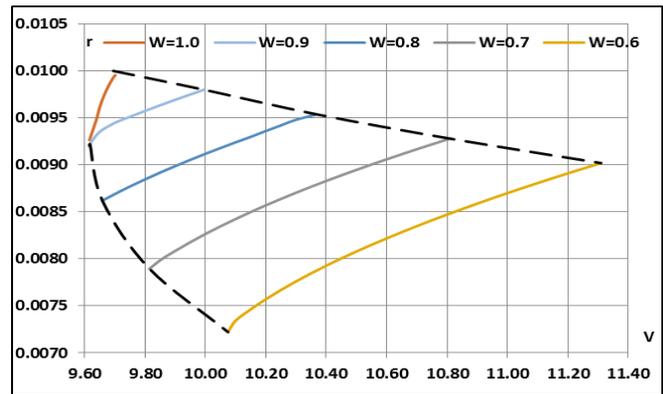


Fig. 5. Set of optimal cryptocurrency portfolios, taking into account the Bitcoin share limitation. The dashed line is the boundary of optimal portfolios area

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