

Class Instructional Design of Advanced Mathematics Based on BOPPPS Model

—Taking "Definite Integral Concept" as an Example

Lei Yan*

Nanyang Institute of Technology
School of Mechanical & Automotive engineering
Nanyang Henan China

Meng Wang

Nanyang Institute of Technology
School of Mathematics and Statistics
Nanyang Henan China

Abstract—Higher Mathematics is a public compulsory course for college students. Traditional teaching methods only focus on teachers' teaching and neglect the problems of students' learning and use. In order to solve such problems and effectively improve class teaching efficiency, this paper proposes to integrate BOPPPS model into Higher Mathematics classroom teaching, and use this method to motivate students to autonomously participate in class and apply the knowledge they have learned to solve practical problems. In the practice of classroom teaching taking the concept of "Definite Integral" as an example, it shows that this new teaching method can effectively solve the problems existing in traditional teaching mode, and has good practical significance.

Keywords—BOPPPS Model; Higher Mathematics; Class Instructional Design; Definite Integral Concept

I. INTRODUCTION

In the new era national education work conference, the Minister of Education Chen Baosheng, pointed out that "the quality and effect of personnel training should be taken as the fundamental criterion to test all work". This speech embodies the state's requirements for higher education in China. In the actual teaching process, teachers should adhere to the principle of "Focusing on the Origin" and promote the "Four Regressions". In view of the reform for Higher Mathematics classroom teaching, this paper proposes to apply BOPPPS model to improve the classroom teaching effect.

The problems that are easy to appear in the traditional classroom of Higher Mathematics are as follows.

A. Problem 1

Teachers tend to adopt such forms as "preaching the whole class" and "one-speech show".

B. Problem 2

It is difficult for students to apply what they have learned to practice.

As a public compulsory course for university students of science and engineering, the advantages and disadvantages of teachers' teaching effects will directly affect the quality of talent training in colleges and universities. In actual teaching, BOPPPS model enables students to participate in classroom learning, gives full play to the initiative of students' learning,

and transforms students from passive knowledge recipients into real learning subjects^[1]. The integration of BOPPPS model in Higher Mathematics classroom teaching can enhance communication and sharing between students and teachers, and achieve good teaching results in practical classroom teaching.

II. OVERVIEW OF BOPPPS MODEL

BOPPPS model is a classroom teaching design model that emphasizes students' participatory learning. It is proposed by Canadian University Teachers' Skill Training Institutes and has been widely used in some universities in Europe and America in recent years. On the basis of cognitive theory that people's attention can only maintain for 15 minutes according to the law, BOPPPS model divides the classroom teaching process into several small units with 15 minutes as a teaching unit. Each teaching unit is centered on achieving teaching goals. Classroom teaching composed of all these small units follows the principle of "initiating and turning"^[2,3]. Specifically, BOPPPS model divides the classroom teaching process into six stages in order (see TABLE I). Linking the basic elements of these six stages constitutes the BOPPPS model, which then constitutes a complete classroom teaching.

TABLE I. TEACHING LINKS OF BOPPPS MODEL

No.	Full name	Abbr.
1	Bridge-in	B
2	Objective	O
3	Pre-assessment	P
4	Participatory Learning	P
5	Post-assessment	P
6	Summary	S

The process of incorporating BOPPPS model into classroom teaching design of Higher Mathematics is as follows. First, the Bridge-in process enables students to link new and old knowledge, and transfer relevant mathematical knowledge learned in junior and senior high schools to the study of Higher Mathematics. Next, the Objective part is not only the baton of teachers' lectures but can also increase students' enthusiasm for reaching the goals. Then, setting the Pre-assessment in Higher Mathematics class allows teachers to teach students in accordance with their aptitude and lay a foundation for students to participate in classroom discussions. Next, students' participation in classroom communication learning effectively

avoids the phenomenon of teachers' "one-speech show". Then, the Post-assessment part can apply the content of classroom learning to practice, thus solving the situation that students reflect that the mathematics courses are "learned and useless"^[4]. Finally, the Summary part is not only a restatement of important content of classroom teaching, but also lays a foundation for further learning, so that the whole teaching process is developing in a positive direction.

III. CLASSROOM TEACHING DESIGN WITH DEFINITE INTEGRAL CONCEPT AS AN EXAMPLE

This paper mainly analyses BOPPPS classroom teaching model. In the specific teaching process, the knowledge point of "Definite Integral Concept" in Higher Mathematic course is selected as an example for actual classroom teaching design^[5]. According to the six stages of BOPPPS model, general rules and applicable methods of BOPPPS model are explored.

A. Bridge-in

In the introduction of the concept of Definite Integral, a simple question is used to arouse students' enthusiasm for learning. The question used here is how to solve the area of plane graphics that students have learned in middle school. Most of the students can accurately state the area calculation formula of regular figure (such as triangle, circle, rectangle, trapezium, etc.). In this way, students can enhance their self-confidence in learning mathematics, and at the same time they can also mobilize classroom learning atmosphere. Finally, by introducing how to solve the problem of irregular graphics -- the curved trapezoid (see Fig. 1.), the learning of this new content of lesson can be smoothly carried out^[6,7].

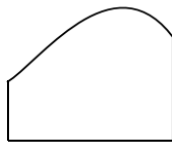


Fig.1. Curved trapezoid.

B. Objective

The target design for the concept of Definite Integral is as follows.

1) *Knowledge objective*: Let students understand the definition and geometric meaning of Definite Integral.

2) *Ability goal*: Let students learn to apply the definition of Definite Integral to represent the area of graph surrounded by the curve and calculate some simple function integrals using geometric meaning.

3) *Emotional attitudes and values*: By learning the definition and geometric meaning of Definite Integral, the goal of this lesson is to develop students' ability to abstract specific problems in the context of practical problems. Through learning, students will gradually develop the ability of dialectical thinking and knowledge transfer. In the meantime, students will be inspired to innovate and explore the spirit of knowledge and establish self-confidence to solve problems.

C. Pre-assessment

The Pre-assessment of the concept of Definite Integral is carried out by asking students to write on the blackboard to find graphic area.

1) Exercise 1.

Looking back at the early eighteenth century, we can find the way to solve the problem of circular regions by combining the exhaustive method (see Fig. 2.). The main purpose of this exercise is to deepen students' understanding of the ideas of "replace curves with straight lines" and "local linearization".

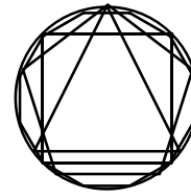


Fig.2. Finding Circle Area by Exhaustion Method.

2) Exercise 2.

According to the idea of applying exhaustion method in the calculation of circle area, students are guided to think about how to solve the area of curved trapezoid. This exercise is mainly to guide students to generalize the law of solving the curved trapezoidal area, which is also the main content of this section.

D. Participatory Learning

In participatory learning, students are mainly asked to discuss the following three aspects.

1) Area calculation of curved trapezoid.

In class, the content of this lesson is explained through the combination of PPT. First, the bottom edge of the curved trapezoid is divided into n cells. Then, at each of the points, a line perpendicular to x -axis is formed, thereby dividing the original large curved trapezoid into n small curved trapezoids. When the number of the above-mentioned points is enough and any two adjacent points are sufficiently small, the small-curved trapezoidal area may be approximately replaced by a small rectangular area for calculation. Then, sum the n small rectangular areas as an approximation of the original large curved side trapezoidal area. Finally, take the limit of the maximum inter-cell length λ of the segmentation. When limit process is $\lambda \rightarrow 0$, the exact value of trapezoidal area of the large curved edge can be finally obtained.

2) Calculation of Variable Force Work for Linear Moving Objects.

Prescribed here, the direction of variable force $f(x)$ is the same as the positive direction of x -axis, and a method similar to solving the problem of trapezoidal area is still adopted. In the process of class explanation, students are divided into groups. In the process of students' discussing and solving problems, teachers participate with students and provide timely guidance on students' discussions. Then use the big screen to show the discussion results for each group.

3) Definition and Geometric Meaning of Definite Integral.

The results discussions of each group are sorted out and common ideas of how to solve the curved trapezoidal area and the work of changing force are summarized. On the basis of this, the method of solving these two problems is the same as the process of "segmentation, approximation, summation, and taking limit". Finally, the contents summarized above are summed up, and then use this to generalize the definition of integral and the geometric meaning.

a) Definition of Definite Integral.

Let the function $f(x)$ be meaningful on $[a, b]$. If the limit of integral sum shown in formula (1) is presented, the limit is called the definite integral of function $f(x)$ on $[a, b]$. The definite integral can be written as formula (2).

$$\lim_{\lambda \rightarrow 0} \sigma_n = \lim_{\lambda \rightarrow 0} \sum_{k=1}^n f(\xi_k) \Delta x_k \quad (1)$$

$$\int_a^b f(x) dx = \lim_{\lambda \rightarrow 0} \sum_{k=1}^n f(\xi_k) \Delta x_k \quad (2)$$

In the above two formulas, The integral number is \int . $[a, b]$ is called the interval. The integral lower limit and integral upper limit are a and b respectively. $f(x)$ is called the integral function and x is the integral variable, simultaneously, dx is called the integral micro-element. The form of the expressed product is $f(x) dx$.

b) Geometric meaning of Definite Integral.

$$f(x) > 0, \quad \int_a^b f(x) dx = A; \quad (3)$$

$$f(x) < 0, \quad \int_a^b f(x) dx = -A. \quad (4)$$

Where, A represents the area of curved trapezoid and is a positive value. $-A$ represents the negative value of curved trapezoidal area.

E. Post-assessment

The specific ways of Post-assessment include classroom observation, classroom exercise (including quizzes, random questions, open-ended discussions, student summaries and feedback information forms, etc.), card feedback, and self-examination, and so on. In this lesson, use classroom exercise to check students' learning.

1) Example 1.

Use the expression of Definite Integral to represent the area of graph surrounded by the following curves.

$$y = x^2 + 1, \quad x = 0, \quad x = 3, \quad y = 0.$$

2) Example 2.

Use geometric meaning of Definite Integral to prove that the following formula is established.

$$\int_a^b x dx = \frac{1}{2}(b^2 - a^2).$$

F. Summary

The Summary part is designed to help learners to summarize and reflect classroom knowledge points according to specific content learned in class. Effective summaries cannot only help students to have a general grasp of knowledge and extend the application of what they have learned, but also achieve the effect of emphasizing ideas and expanding methods. This lesson uses the method of PPT demonstration to help students summarize the key points of Definite Integral, emphasize the dialectical thinking method of "turning whole into zero, turning zero into whole" in the definition of Definite Integral. Using this approach to help students build a clear knowledge framework for this lesson.

The specific teaching process using BOPPPS model is shown in TABLE II.

TABLE II. SPECIFIC TEACHING PROCESS UNDER BOPPPS MODEL

Abbr.	Communication Content	Communication Strategy
B	[T] Do you still remember the problem of flat graphic area that you have learned in middle school? [S] Students will actively answer the area calculations of regular graphics such as triangles, circles, rectangles, and trapezoids. [T] If it is the area of an irregular graph, how should we calculate it? [S] After segmentation and splicing, it can be solved as a rule graph. [T] If it is an irregular curved edge graphic? [S] We can use the exhaustive method of previous study to calculate.	In Bridge-in process, take the history of mathematics as entry point to arouse students' interest in learning. Use courseware to promote teacher-student communication.
O	[T] After reviewing the area of flat graphic area, we will discuss the Concept of Definite Integral by solving trapezoidal area of irregular curved edge.	According to the review of problem of flat graphic area, the classroom teaching objectives are reasonably determined.
P	[T] Before learning the concept of Definite Integral, let's first recall the idea of using exhaustive method to find the area of round circle. [S] The slashing of the smear, the loss of the smear, the cutting and cutting, so that it cannot be cut and the circle is combined without loss. [T] So how do we use this idea to solve the curved trapezoid? [S] The curved trapezoid is infinitely subdivided into a plurality of small curved trapezoids until the division is impossible, then the area of small curved trapezoid can be approximated as the area of the small rectangle.	Through students' writing on the blackboard, the teacher can understand students' ability to use knowledge. This can also lay the foundation for further communication between teachers and students.

Cont.to TABLE II

P	<p>[T] First, let's analyze the area solving the problem of curved trapezoid (using PPT display). Then, we will discuss how to solve the problem of variable force work for linear moving objects.</p> <p>[S] Sub-group discussion.</p> <p>[T] Inspect the situation of students' discussing. Correct and guide students in time.</p> <p>[T] Through discussion, can we sum up the commonalities of these two problems when they are resolved?</p> <p>[S] They all get the final result by means of "segmentation, approximation, summation, and taking limit".</p> <p>[T] From this we can summarize the definition of Definite Integral (using PPT display).</p> <p>[T] Please think about the geometric meaning of Definite Integral.</p> <p>[S] It should be the area of curved trapezoid.</p> <p>[T] Guide students to classify the trapezoidal area of curved edge in the plane rectangular coordinate system.</p>	<p>First, through teacher and student question and answer, cleverly set questions. Then, through the process of knowledge explanation, teacher submits questions for all students, guides students to explore problems, and encourages them to actively participate in class, so that they can communicate more closely.</p>
P	<p>[T] Through PPT display, let students solve problems in the group discussion.</p> <p>[S] Answer questions.</p> <p>[T] Inspect the situation of students' discussing. In response to students' problem, timely comment it on the blackboard.</p>	<p>By answering questions, teacher can detect whether students master the knowledge. That is the effect of teacher and student interaction in this section.</p>
S	<p>[T] In this lesson, we learned Definite Integral and its geometric meaning. Please use the small card in your hand to summarize.</p> <p>[S] Hands-on summary.</p> <p>[T] Next, based on Definite Integral, what kind of properties can we derive from Definite Integral? And this is what we are going to learn in the next lesson.</p>	<p>Do a good job in class summary, use the flow chart to construct student's mind map, and arrange after-school homework to pave the way for teacher-student communication.</p>

*T is short for Teacher; S is short for Students.

IV. CONCLUSION

Soviet Mathematician Stolyar once said that "Mathematics teaching is the teaching of mathematical thinking activities and not just the result of mathematical activities". The practical application of BOPPPS model in Higher Mathematics classroom can enrich the content of classroom teaching^[8]. The introduction of this new model cannot only guide students to actively participate in classroom. Moreover, it can effectively solve the phenomenon of traditional classroom "the teacher is on the stage, and students are dead and down". Through the process of students' participating in classroom, they can gain a deeper understanding of what they have learned. In group discussions and classroom tests, students learn to apply what they have learned to solve practical problems. Later, we will continue to explore the deeper integration of BOPPPS model with Higher Mathematics classroom teaching. By continuously improving the quality of classroom teaching, an active role in promoting the development of new talents with innovative consciousness and innovative ability will come true in China.

REFERENCES

- [1] Joanna Allan. Learning outcomes in higher education[J]. Studies in Higher Education, 1996, 21(1), pp.93-108.
- [2] Chu Yawei, Ye Weiwei, Wang Haikun. Teaching Design of Higher Mathematics Based on BOPPPS Model—Taking "Solution of First Order Nonhomogeneous Linear Differential Equation" as an Example[J]. Journal of Shandong Agricultural Engineering University, 2016, 33(9), pp.153-156.(In Chinese)
- [3] Liu Bin, Li Shuqin, Han Hong. Teaching Design of C Language Course Based on BOPPPS Model[J]. Education and Teaching Forum, 2017(44), pp.248-250. (In Chinese)
- [4] Aithal S, Kumar S. Student performance and Learning Outcomes in Higher Education Institutions[J]. Social Science Electronic Publishing, 2016, pp.2455-5430.
- [5] Zhang Wei, Li Hongxia. The Teaching Design of Higher Mathematics Based on BOPPPS Mode—Taking "Number Limit" as an Example[J]. West Quality Education, 2017, 3(2), pp.163-164. (In Chinese)
- [6] Guo Xuejun. Advanced Mathematics. Volume [M]. Science Press, 2012. (In Chinese)
- [7] Tian Shiqin, Wang Yuwen. A Comparative Study of the Contents of Chinese and American Advanced Mathematics Textbooks—A Case Study of the Pennsylvania Nine Editions and the Chinese Tongji Seven Editions[J]. Journal of Mathematics Education, 2017, 26(2). (In Chinese)
- [8] Dahl, Bettina. What is the problem in problem-based learning in higher education mathematics[J]. European Journal of Engineering Education, 2018, 43(1), pp.1-14.