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An Extended TODIM Method Under Uncertain and Risky Situations

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Abstract. The purpose of this paper is to solve the MADM problem under uncertain and risky situations by proposing an extended TODIM method. After considering about the attitude of decision makers on "risk" sufficiently, we introduce a new measure for IT2FS to describe the uncertain information. Taking into account the complexity of computation using T2FSs, the interval type-2 fuzzy sets can be used to dispose of the vagueness and uncertainty. Moreover, we illustrate the implementation of proposed extended TODIM method based on TrIT2FS by two case studies. The availability and the feasibility of the presented method are validated through a comparative analysis with other methods.

Introduction

Multi-Attribute Decision Making (MADM)[1] is a part of MCDM problem which is mainly focused on the problem of discrete decision space. The typical MADM problem is concerned with the evaluation of a limited number of predetermined alternatives according as a group of decision criteria. The relevant literature [2] about MADM is very extensive, also its theories and methods are widely applied to many fields such as engineering, technology, economics, management, military, and so on [3-5].

Nevertheless, most existing methods are based on expected utility model of rationality, but some researches about behavioural experiments [6, 7] have illustrated that the decision maker is bounded rational in the decision-making process. Therefore, how to solve the MADM problem when considering decision-maker's psychological behaviour is always an issue deserving of study.

TODIM, a valuable means to resolve the MADM problem, was early introduced by Gomes and Lima [8-13]. It can be effectively applied under the circumstance of risk. This method has been applied in many fields and is empirically validated to be efficient and feasible.

However, another difficulty in setting up mathematical models is how to express the DM's ideas and beliefs in a mathematical form. In reality, as the complexity of society continues to increase, the problems that people met are getting blurred. It is no longer a good solution to solve the decision problem with the exact number. To address this issue, Zadeh proposed T2FSs (the abbreviation for type-2 fuzzy sets) in which the membership function are fuzzy themselves. It uses both primary and secondary membership to provide us with additional degrees of freedom and greater flexibility particularly.

This paper will present an TODIM method based on T2FS and introduce a new distance measure based on the signed distance between two interval numbers. At the end of this paper, we will apply this proposed method to a multi-criteria investment selection problem to verify the practical application effect of it.

The Measure of IT2FS Considering Risk Preferences

The Fuzzy theory provides theoretical basis and technical support for uncertain information representation and organization.

[Definition 1] A type-2 fuzzy set, denoted $\widetilde{A}$, is characterized by a type-2 membership function $\mu_{\widetilde{A}}\left(x,\mu_A\right):\left(x,\mu_A\right)\to I\subseteq\left[0,1\right]$, where $x\in U$, $\mu_A\in J_x\subseteq\left[0,1\right]$,i.e.,

$$\widetilde{A}=\left\{\left(\left(x,\mu_A\right),\mu_{\widetilde{A}}\left(x,\mu_A\right)\right)\middle| x\in U,\mu_A\in J_x\subseteq[0,1]\right\}, \tag{1}$$

$$\widetilde{A}=\int_{x\in U}\int_{\mu_A\in J_x\subseteq[0,1]}\frac{\mu_{\widetilde{A}}\left(x,\mu_A\right)}{\left(x,\mu_A\right)}, \tag{2}$$

For discrete universes of discourse, $\int$ is replaced by $\sum$. A T2FS is three dimensional (3D).

[Definition 2] *An interval type-2 fuzzy set* (IT2FS) is defined to be a T2FS where all the secondary grades are at unity, i.e., $\forall f_x\left(\mu_A\right)=1, x\in U, \mu_A\in J_x\subseteq[0,1]$.

[Definition 3] (Perfectly Normal IT2FS). A IT2FS $\widetilde{A}$, is said to be perfectly normal if $\sup\left(\underline{\mu_{\widetilde{A}}(x)}\right)=\sup\left(\overline{\mu_{\widetilde{A}}(x)}\right)=1$. So the $\alpha-$cuts of IT2FS may be presented by corresponding intervals with interval valued bounds as:

$$\widetilde{A}_\alpha=\left[\left[\overline{x}_\alpha{}^{l},\underline{x}_\alpha{}^{l}\right],\left[\underline{x}_\alpha{}^{u},\overline{x}_\alpha{}^{u}\right]\right], \tag{3}$$

Then the perfectly normal interval type-2 fuzzy value (IT2FV) may be presented as follows:

$$\widetilde{A}=\bigcup_\alpha \alpha\widetilde{A}_\alpha=\bigcup_\alpha \alpha\left[\left[\overline{x}_\alpha{}^{l},\underline{x}_\alpha{}^{l}\right],\left[\underline{x}_\alpha{}^{u},\overline{x}_\alpha{}^{u}\right]\right], \tag{4}$$

To compare the IT2FS values in the extended TODIM method based on Type-2 Fuzzy Sets, one of the most critical issues is the defuzzification techniques. This paper adopts the signed distance-based (also known as oriented distances or directed distances) approach to eliminate the fuzziness.

Generally, the distance from a point x to a set U in a matric space Ω is defined by:

$$\underset{x\in\Omega,U\subset\Omega}{dist}\left(x,U\right)=\inf_{y\in U}\left\|x-y\right\|, \tag{5}$$

The signed distance function $sd(\cdot)$ of U can be indicated globally on Ω by:

$$sd\left(x\right)=\begin{cases}dist\left(x,U\right), x\in U^c\\ -dist\left(x,U\right), x\in U\end{cases}, \tag{6}$$

[Definition 4] [16] Let R be a domain of discourse, if $\forall a_1,a_2\in R, 0\le a_1\le a_2, \left[a_1,a_2\right]$ is called interval number.

[Definition 5] [16,17] Let $a=\left[a_1,a_2\right], b=\left[b_1,b_2\right], 0=[0,0]\in Ns$, $p\ge 1, a\cap b=\varnothing$, $x,y\in[0,1]$, $a_x=xa_1+\left(1-x\right)a_2\in a, b_x=yb_1+\left(1-y\right)b_2\in b$, considering the difference between each point in the interval numbers and $0=[0,0]$, we define

(1) according to Eq. (4), the signed distance measured from $0=[0,0]$ to $a=\left[a_1,a_2\right]$ is

$$d_0\left(a,0\right)=\int_0^1\left(a_x-0\right)dx=\int_0^1\left(xa_1+\left(1-x\right)a_2\right)dx=\frac{a_1+a_2}{2}, \tag{7}$$

(2) the signed distance measured from $0=[0,0]$ to $a\cup b$ is

$$d_0\left(a\cup b,0\right)=d_0\left(a,0\right)+d_0\left(b,0\right)=\frac{a_1+a_2}{2}+\frac{b_1+b_2}{2}, \tag{8}$$

For $0\le\alpha\le h$, then, from Eq. (3), we have the signed distance measured from $0=[0,0]$ to $\widetilde{A}_\alpha$:

$$d_0\left(\widetilde{A}_\alpha,0\right)=d_0\left(\left[\overline{x}_\alpha^{\,l},\underline{x}_\alpha^{\,l}\right],0\right)+d_0\left(\left[\underline{x}_\alpha^{\,u},\overline{x}_\alpha^{\,u}\right],0\right)=\frac{\overline{x}_\alpha^{\,l}+\underline{x}_\alpha^{\,l}}{2}+\frac{\underline{x}_\alpha^{\,u}+\overline{x}_\alpha^{\,u}}{2}, \tag{9}$$

Similarly, for $h \le \alpha \le 1$, we can get the signed distance from [Definition 3] as

$$d_0\left(\widetilde{A}_\alpha,0\right)=d_0\left(\left[\overline{x}_\alpha^{\,l},\overline{x}_\alpha^{\,u}\right],0\right)=\frac{\overline{x}_\alpha^{\,l}+\overline{x}_\alpha^{\,u}}{2}, \tag{10}$$

Because the above-mentioned function is continuous over the closed interval $[0,h]\cup[h,1]$, we can obtain the weighted mean of the signed distances of $\widetilde{A}_\alpha$ measured to $0=[0,0]$ under full consideration of the decision-maker's attitude towards "risk" by introducing integral role.

$$\begin{aligned}d_0\left(\widetilde{A}_\alpha,\tilde{0}\right)=&\int_0^h\left[d_0\left(\left[\overline{x}_\alpha^{\,l},\underline{x}_\alpha^{\,l}\right],0\right)+d_0\left(\left[\underline{x}_\alpha^{\,u},\overline{x}_\alpha^{\,u}\right],0\right)\right]f(\alpha)d\alpha\Big/\int_0^h f(\alpha)d\alpha\\&+\int_h^1\left[d_0\left(\left[\overline{x}_\alpha^{\,l},\overline{x}_\alpha^{\,u}\right],0\right)\right]f(\alpha)d\alpha\Big/\int_h^1 f(\alpha)d\alpha\end{aligned}, \tag{11}$$

Here $f(\cdot)$, which regards as a weighted function, is a continuous positive function on $[0,1]$. Taking the decision-maker's attitude towards "risk" into consideration, the weighted function $f(\cdot)$ allows the decision-maker to be involved in a flexible approach.

For risk-neutral decision-makers, $f(\alpha)=\alpha$ seems to be more reasonable. While for risk-averse decision-makers, they may put more weight on information at a higher α level. Therefore, we can use $f(\cdot)$ such as $f(\alpha)=\alpha^2$ or a higher power of α. For risk-prone decision-makers, a constant ($f(\alpha)=1$), or even a decreasing function $f(\cdot)$ can be utilized.

Because the signed distances $d_0\left(\widetilde{A},\tilde{0}\right)$ and $d_0\left(\widetilde{B},\tilde{0}\right)$ are real numbers for two IT2FS $\widetilde{A}$, $\widetilde{B}$, they satisfy linear ordering. That is to say, one of the following three conditions must be met:

$$\widetilde{A}\prec\widetilde{B}\ \ iff\ \ d_0\left(\widetilde{A},\tilde{0}\right)<d_0\left(\widetilde{B},\tilde{0}\right),\ \widetilde{A}\approx\widetilde{B}\ \ iff\ \ d_0\left(\widetilde{A},\tilde{0}\right)=d_0\left(\widetilde{B},\tilde{0}\right),\ \widetilde{A}\succ\widetilde{B}\ \ iff\ \ d_0\left(\widetilde{A},\tilde{0}\right)>d_0\left(\widetilde{B},\tilde{0}\right),$$

We can demonstrate that the signed distance mentioned above satisfies the metric space axioms. A comparison of the IT2FS ratings can be drawn via the signed distance from IT2FS to $\tilde{0}$.

TODIM Method Based on Type-2 Fuzzy Sets

The Classical TODIM Method

TODIM is a discrete multi-criteria method. The essence of TODIM is to selecting the most preferred alternatives by adopting the paired-comparison method on the basis of gains and losses. Then calculate the partial dominance based on the Prospect Theory and aggregate the partial dominance to obtain the final dominance. We can achieve the overall dominance degree of TODIM by aggregating all measures of gains and losses regarding to each criterion. [8-13].

The classical TODIM method is often used in the format of accurate value. For further analysis and expansion, we conclude an algorithm for the TODIM method as follows.

Step 1: Define decision matrix. In order to eliminate the influence of the dimension, we normalize decision matrix $DM=\left[x_{ij}\right]_{m\times n}$ into $DM_{Normalize}=\left[y_{ij}\right]_{m\times n}$ by normalization method.

Step 2: Calculate the relative weight ω_{jr} of attribute C_j to the reference attribute C_r, i.e., $\omega_{jr}=\omega_j/\omega_r$, $j,r\in N$ where $\omega_r=\max\left\{\omega_j \mid j\in N\right\}$

Step 3: Calculate the dominance degree of alternative S_i over alternative S_k, i.e.,

$$\Phi_j(S_i,S_k)=\begin{cases}\sqrt{\left(y_{ij}-y_{kj}\right)\omega_{jr}\Big/\sum_{j=1}^{n}\omega_{jr}}, & \left(y_{ij}-y_{kj}\right)>0,\\ 0, & \left(y_{ij}-y_{kj}\right)=0,\\ -\frac{1}{\theta}\sqrt{\left(y_{kj}-y_{ij}\right)\left(\sum_{j=1}^{n}\omega_{jr}\right)\Big/\omega_{jr}}, & \left(y_{ij}-y_{kj}\right)<0\end{cases}, \tag{12}$$

$\Phi_j(S_i,S_k)$ means dominance degree concerning attribute C_j. θ is the attenuation factor of the losses, $\theta>0$. The greater θ is, the lower the degree of loss aversion is. If $\left(y_{ij}-y_{kj}\right)>0$, DM will gain more. If $\left(y_{ij}-y_{kj}\right)=0$, the DM will get balance between gain and loss. If $\left(y_{ij}-y_{kj}\right)<0$, DM will bear more losses.

Step 4: Calculate the overall dominance degree, the final dominance matrix is derived by summing the partial dominance matrices of each attribute.

$$\delta(S_i,S_k)=\sum_{j=1}^{n}\Phi_j(S_i,S_k),\ \mathrm{i},k\in M, \tag{13}$$

Step 5: Obtain the overall dominance of S_i, i.e.,

$$\xi(S_i)=\frac{\sum_{k=1}^{m}\delta(S_i,S_k)-\min_{i\in M}\left\{\sum_{k=1}^{m}\delta(S_i,S_k)\right\}}{\max_{i\in M}\left\{\sum_{k=1}^{m}\delta(S_i,S_k)\right\}-\min_{i\in M}\left\{\sum_{k=1}^{m}\delta(S_i,S_k)\right\}},\ \mathrm{i}\in M, \tag{14}$$

Step 6: Rank all the alternatives according to $\xi(S_i)$. The greater $\xi(S_i)$ is, the better the alternative S_i will be.

Extended TODIM Method Based on IT2FS

Because of the subjective and motivating behavior in human thinking, we always encounter the uncertain, imprecise, and ambiguous circumstances when we evaluate criteria and alternatives. We use type-2 fuzzy sets for multi-attribute decision making problem with uncertainty. In particular, this method works better when we have some subjective criteria. Considering the computational complexity of this theory, this paper uses the interval type-2 fuzzy sets (IT2FSs) to deal with the practical multiple criteria decision-making problem. Based on IT2FSs, the extended TODIM method can be summarized in the following steps:

Steps 1 & 2: Problem formulation and input stage

Step 1: The same as discussed in Section 3.1.

Step 2: Select the appropriate linguistic variables or other data collection tools to establish the IT2FS rating x_{ij} in accordance with [Definition 2] for the alternative S_i with respect to the criterion C_j and the importance weight ω_j.

Steps 3-7: TODIM computation stage

Step 3: The same as discussed in Section 3.1.

Step 4: Calculate the dominance degree.

$$\Phi_j(S_i,S_k)=\begin{cases}\sqrt{\left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)\omega_{jr}/\sum_{j=1}^{n}\omega_{jr}}, & \left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)>0,\\ 0, & \left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)=0,\\ -\frac{1}{\theta}\sqrt{\left(d_0\left(y_{kj},\tilde{0}\right)-d_0\left(y_{ij},\tilde{0}\right)\right)\left(\sum_{j=1}^{n}\omega_{jr}\right)/\omega_{jr}}, & \left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)<0\end{cases}, \tag{15}$$

If $\left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)>0$, it denotes the gain , while $\left(d_0\left(y_{ij},\tilde{0}\right)-d_0\left(y_{kj},\tilde{0}\right)\right)<0$, it denotes the loss.

Steps 5-7: The same as discussed in Section 3.1.

The Signed Distance of TrIT2FS

Up to now, there have been many distance calculation methods for interval fuzzy numbers. Here, we follow the results of Chen [14, 15, 18], who adopted trapezoidal interval type-2 fuzzy set (TrIT2FS) for expressing the uncertainty, imprecision, and subjectiveness in multi-attribute decision making problem.

A TrIT2FS $\tilde{A}$ in the universe of discourse U is expressed in the form:
$\tilde{A}=\int_{x\in U}\left[\underline{\mu_{\tilde{A}}(x)},\overline{\mu_{\tilde{A}}(x)}\right]=\left[\left(a_1^D,a_2^D,a_3^D,a_4^D;h_{\tilde{A}}^D\right),\left(a_1^U,a_2^U,a_3^U,a_4^U;h_{\tilde{A}}^U\right)\right]$, where $\forall f_x(\mu_A)=1$, $\mu_A\in J_x\subseteq[0,1]$, $a_1^D,a_2^D,a_3^D,a_4^D,h_{\tilde{A}}^D,a_1^U,a_2^U,a_3^U,a_4^U,h_{\tilde{A}}^U$ are all real numbers and which satisfy the inequality $a_1^D\le a_2^D\le a_3^D\le a_4^D$, $a_1^U\le a_2^U\le a_3^U\le a_4^U$, $0\le h_{\tilde{A}}^D\le h_{\tilde{A}}^U\le 1$. The upper membership function (UMF) $\overline{\mu_{\tilde{A}}(x)}$ and lower membership function (LMF) $\underline{\mu_{\tilde{A}}(x)}$ are defined in the following way:

$$\overline{\mu_{\tilde{A}}(x)}=\begin{cases}\dfrac{\left(x-a_1^U\right)h_{\tilde{A}}^U}{a_2^U-a_1^U} & a_1^U\le x\le a_2^U\\ h_{\tilde{A}}^U & a_2^U\le x\le a_3^U\\ \dfrac{\left(a_4^U-x\right)h_{\tilde{A}}^U}{a_4^U-a_3^U} & a_3^U\le x\le a_4^U\\ 0 & otherwise\end{cases}\quad\text{and}\quad \underline{\mu_{\tilde{A}}(x)}=\begin{cases}\dfrac{\left(x-a_1^D\right)h_{\tilde{A}}^D}{a_2^D-a_1^D} & a_1^D\le x\le a_2^D\\ h_{\tilde{A}}^D & a_2^D\le x\le a_3^D\\ \dfrac{\left(a_4^D-x\right)h_{\tilde{A}}^D}{a_4^D-a_3^D} & a_3^D\le x\le a_4^D\\ 0 & otherwise\end{cases},\tag{16}$$

According to (9), the signed distance from $\tilde{A}$ to $\tilde{0}$ with the weighting functions $f(\alpha)=1$ is as follows:

If $h_{\tilde{A}}^D\ne h_{\tilde{A}}^U$

$$d_0\left(\tilde{A},\tilde{0}\right)=\frac{\left(h_{\tilde{A}}^U+2h_{\tilde{A}}^D\right)}{4h_{\tilde{A}}^U}\left(a_2^U-a_1^U-a_4^U+a_3^U\right)+\frac{1}{4}\left(a_2^D+a_1^D+a_4^D+a_3^D+4a_1^U+4a_4^U\right),\tag{17}$$

If $h_{\tilde{A}}^D=h_{\tilde{A}}^U$

$$d_0\left(\tilde{A},\tilde{0}\right)=\frac{1}{4}\left(a_1^D+a_2^D+a_3^D+a_4^D+a_1^U+a_2^U+a_3^U+a_4^U\right),\tag{18}$$

If the weighting functions $f(\alpha)=\alpha$, $d_0\left(\tilde{A},\tilde{0}\right)$ is as follows:

If $h_{\tilde{A}}^D\ne h_{\tilde{A}}^U$

$$\begin{aligned}d_0\left(\tilde{A},\tilde{0}\right)=&\frac{\left(h_{\tilde{A}}^U\right)^2+2h_{\tilde{A}}^Uh_{\tilde{A}}^D+2\left(h_{\tilde{A}}^D\right)^2}{3h_{\tilde{A}}^U\left(h_{\tilde{A}}^U+h_{\tilde{A}}^D\right)}\left(a_2^U-a_1^U-a_4^U+a_3^U\right)+\frac{1}{3}\left(a_2^D-a_1^D-a_4^D+a_3^D\right)\\&+\frac{1}{2}\left(2a_1^U+a_1^D+2a_4^U+a_4^D\right)\end{aligned},\tag{19}$$

If $h_{\tilde{A}}^D=h_{\tilde{A}}^U$

$$d_0\left(\tilde{A},\tilde{0}\right)=\frac{1}{3}\left(a_2^U-a_1^U-a_4^U+a_3^U+a_2^D-a_1^D-a_4^D+a_3^D\right)+\frac{1}{2}\left(a_1^U+a_1^D+a_4^U+a_4^D\right), \tag{20}$$

If the weighting functions $f\left(\alpha\right)=\alpha^2$, $d_0\left(\tilde{A},\tilde{0}\right)$ is as follows:

$$d_0\left(\tilde{A},\tilde{0}\right)=\frac{3h_{\tilde{A}}^D\left(h_{\tilde{A}}^U\right)^3-6\left(h_{\tilde{A}}^D\right)^4+3\left(h_{\tilde{A}}^U\right)^4}{8h_{\tilde{A}}^U\left(\left(h_{\tilde{A}}^U\right)^3-\left(h_{\tilde{A}}^D\right)^3\right)}\left(a_2^U-a_1^U-a_4^U+a_3^U\right)+\frac{3}{8}\left(a_2^D-a_1^D-a_4^D+a_3^D\right) +\frac{1}{2}\left(2a_1^U+a_1^D+2a_4^U+a_4^D\right), \tag{21}$$

If $h_{\tilde{A}}^D=h_{\tilde{A}}^U$

$$d_0\left(\tilde{A},\tilde{0}\right)=\frac{3}{8}\left(a_2^U-a_1^U-a_4^U+a_3^U+a_2^D-a_1^D-a_4^D+a_3^D\right)+\frac{1}{2}\left(a_1^U+a_1^D+a_4^U+a_4^D\right), \tag{22}$$

Case Illustration

This section illustrates the implementation of proposed extended TODIM method based on TrIT2FS by the following case study. The decision maker has to seek the best strategy from a given set of alternatives according to the decision criterion (usually multiple conflicting criteria) under the risk condition. For the sake of simplicity, we take $f\left(\alpha\right)=\alpha\ \&\ \theta=3$, which means that the decision maker is risk-neutral; $f\left(\alpha\right)=\alpha^2$ (or a higher power of α) $\&\ \theta=1$ for a risk-averse decision maker; $f\left(\alpha\right)=1\ \&\ \theta=5$ for a risk-prone decision maker.

Case Study : This case is about a multi-criteria supplier selection problem presented by Yanbing [19]. In order to find one of the most important parts of a device, an manufacturing company is looking for the suitable global supplier. After screening all the candidate supplier, the company finally choose three potential suppliers S_1, S_2 and S_3. The main factors which the company taken into consideration are divided into four attributes.

C_1: service performance of supplier, C_2: profile of the supplier, C_3: quality of the product, C_4: risk factor. The weight vector is $\omega=\{0.20,0.35,0.30,0.15\}$. For comparison and analysis, this paper utilizes directly the collective normalized TrIT2FS decision matrix shown in Table 1[19]. Based on the formula shown in section 4.2 and 5.1, the overall value of alternative S_i can be calculated, shown in Table 2.

Table 1. The best global supplier's decision matrix

	C_1	C_2	C_3	C_4
S_1	[(0.76,0.93,0.93,1;1),(0.845,0.93,0.93,0.965;0.9)]	[(0.76,0.93,0.93,1;1),(0.845,0.93,0.93,0.965;0.9)]	[(0.59,0.79,0.79,0.945;1),(0.69,0.79,0.79,0.8675;0.9)]	[(0,0.055,0.055,0.21;1),(0.0275,0.055,0.055,0.1325;0.9)]
S_2	[(0.84,0.97,0.97,1;1),(0.905,0.97,0.97,0.985;0.9)]	[(0.79,0.945,0.945,1;1),(0.8675,0.945,0.945,0.9725;0.9)]	[(0.61,0.81,0.81,0.955;1),(0.71,0.81,0.81,0.8825;0.9)]	[(0.03,0.135,0.135,0.31;1),(0.0825,0.135,0.135,0.2225;0.9)]
S_3	[(0.44,0.64,0.64,0.84;1),(0.54,0.64,0.64,0.74;0.9)]	[(0.84,0.97,0.97,1;1),(0.905,0.97,0.97,0.985;0.9)]	[(0.71,0.88,0.88,0.975;1),(0.795,0.88,0.88,0.9275;0.9)]	[(0,0.025,0.025,0.15;1),(0.0125,0.025,0.025,0.0875;0.9)]

Table 2. The overall value of alternative S_i of $f(\alpha)\&\theta$

Risk Attitude	S_1	S_2	S_3	Ranking orders of alternatives
risk-prone $f(\alpha)=1\ \&\ \theta=5$	0.5726	0	0.7288	$S_2 \succ S_1 \succ S_3$
risk-neutral $f(\alpha)=\alpha\ \&\ \theta=3$	0.5712	0	0.7594	$S_2 \succ S_1 \succ S_3$
risk-averse $f(\alpha)=\alpha^2\ \&\ \theta=1$	0.5705	0	0.7297	$S_2 \succ S_1 \succ S_3$

That is to say, the best desirable global supplier among S_1, S_2 and S_3 is S_3. A comparative study can be shown that the ranking order is consistent with Yanbing[19], et al.'s method.

Conclusions

Indeed, it is not easy to determine the accurate attribute values in MADM, it is very appropriate to describe them in other formats. Through this paper, we extended the TODIM method with the Type-2 Fuzzy Sets format of attribute values in the situation of taking into account the risk attitude of decision making. And we introduce a new distance measure based on the signed distance between two interval numbers. At last, we illustrate the implementation of proposed extended TODIM method based on TrIT2FS by case. Moreover, the effectiveness of the proposed method is verified by comparing with other methods.

In conclusion, the decision result obtained by the proposed method under the circumstance of risk is more in line with people's actual decision-making behaviour. Furthermore, the signed distance from the origin 0 points is a very common concept and can be easily understood by the decision maker(s). The calculation procedure is straightforward and effective in the current method and easy to program implementation. This is a very good practical aspect which is not seen in several other methods due to its simplicity, ease of interpretation, and effectiveness.

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