

Formal Models of the Evolution of Scientific Theory

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Abstract—The article discusses various formal models of the evolution of scientific theory. The cumulative model based on the standard interpretation of a scientific theory presented the evolution of science as a continuous linear process. The abstract character of this model has become the object of fierce criticism from some representatives of the post-positivistic philosophy of science. This led to the statement about the incommensurability of alternating theories, of the unrealistic nature of the statements of scientific theories and the impossibility of applying to the theories of the natural sciences, and formally strict concepts of truth/falsity. The set-theoretic model of the development of science, based on the generalization of the concept of a model of theory in the sense of A. Tarski, allows avoiding the extremes of cumulative representations and at the same time allows demonstrating partial continuity between the models of alternating theories.

Keywords—theory; model; formalization; axiomatization; truth; verisimilitude

I. INTRODUCTION

One of the distinguishing features of the program of foundation of scientific knowledge put forward by logical positivism was a deliberate and persistent reliance on formal research methods in solving a number of pivotal problems in the philosophy of science [1]. The possibility of the successful solution of some of these problems (such as the problem of a strict distinction between analytical and synthetic statements of the theory, the formulation of criteria of the meaningfulness of statements etc.), was supposed to be conditioned by their preliminary explication in a certain artificial language [2]. In modern philosophical and logical literature the language of classical first-order predicate logic with equality (hereinafter referred to as F.O.P.L. — 1 =) is generally accepted as such a formalized language. At the same time, the scientific theory subjected to analysis was to be formulated as a partially interpreted axiomatic system, the axioms of which were the fundamental laws of the corresponding theory, expressed in some theoretical language LT. The observable consequences of these laws ought to be formulated in a separate observation language LO, and the relationship between concepts, belonging to LT and LO, was to be expressed via so-called correspondence rules C (or rules of reduction, as they were dubbed in later versions of this program). In this case, only concepts from the LO language had literal semantic meanings. The scientific theory was identified with a set of conjunctions of theoretical postulates of the theory and the correspondence

rules $T \wedge C$. The set $T \wedge C$ proved to be partially ordered by the relation of formal derivability.

The formalization of the theory in F.O.P.L. — 1 = taken to be called standard.

The interpretation of the theory as a set of statements of some formalized language (axioms and theorems) ordered by the deductive derivability relation is usually called the standard or "statement" view of the theory.

On the basis of the standard interpretation of scientific theory received in logical positivism, a very specific (cumulative) model of the development of scientific knowledge was proposed, which was subjected to fierce criticism in the works of a number of representatives of the so-called historical school in the philosophy of science. As a result, the very possibility of a logical reconstruction of the dynamics of scientific knowledge was called into question [3].

This article reviews the logical-positivist and model-theoretic strategies for analyzing of the development of scientific knowledge. As a result, the conclusion about the fruitfulness of using formal methods in the reconstruction of the processes of development and transformation of scientific theories [4] is drawn.

II. LOGICAL POSITIVISTIC MODEL OF DYNAMICS OF SCIENTIFIC KNOWLEDGE

According to logical-positivist ideas, theories that have successfully passed a series of empirical tests have a high degree of reliability. At the same time, further transformations of "pre-confirmed" theories can be implemented according to one of the following scenarios: 1) as the measuring and experimental equipment improves, at the theory's predictions emerge inconsistencies and errors that lower the degree of its confirmation by experience; this narrows the field of applicability of the theory and in some exceptional cases can lead to its abandonment; 2) the high degree of reliability of the original theory allows us to expand the scope of its application; 3) several disparate theories, each of which has a high degree of reliability, are included in a single, "generalizing".

Options 2) and 3) are considered as the most common.

In the second of the possible options for the development of the theory, the original set of theoretical postulates and

correspondence rules of the TC is replaced by the set of TC', in which the rules of correspondence C' are designed to "adapt" the conceptual apparatus of the original theory to a wider area of observable phenomena. The theory TC' should successfully pass an empirical test and, in addition, the terminological apparatus of the original TC theory should not differ significantly from the terminological apparatus of the TC' theory. The latter requirement determines the homogeneity of the dictionaries of both theories, which, in turn, makes it possible to derive deductively the statements of the TC theory from the set of statements of the TC' theory. Therefore, this model of the evolution of a theory is, in essence, a scheme for reducing the statements of one theory to statements of another.

Other variations of this theory evolving suggest the possibility of expanding the list of theoretical postulates of the original theory while maintaining the reduction rules (transition from TC to T'C) or expansion of both lists of statements of the original theory (transition from TC to T'C').

The third of the possible options for the development of the theory suggests a slightly different type of reduction, in which (at least some) formulations of the laws of the original theory/theories contain descriptive terms that are absent in the formulations of theoretical postulates and/or correspondence rules of the "generalizing" theory. For the correct implementation of the reduction of this type, the following conditions must be satisfied: 1) the meanings of the theoretical terms of both theories must be unambiguously determined; 2) for each theoretical term α of the initial theory, which does not occur in the dictionary of the generalizing theory, a hypothetical correspondence between the meaning of the term α and the theoretical term β in the dictionary of the generalizing theory, (presumably) reflecting the properties of the value of α is postulated; 3) on the basis of hypothesis 2) all the laws of the original theory should be logically derived from theoretical postulates and the correspondence rules of the generalizing theory; 4) all additional hypotheses used in the conclusion should be empirically confirmed [5].

As a rule, a reduction of this type is an explanation of a certain phenomenological theory or a set of experimental laws relating to one subject area, using a theory formulated for another (albeit related) domain of exploration (for example, explanation of the laws of the motion of the planets according to Kepler by means of Newton's mechanics).

As a result, science appears to be a linear cumulative process of continuous accumulation of knowledge. The previous theories are not discarded, but are reduced to the new ones by the aforementioned ways [6].

This technique, rather cumbersome and excessively schematized, has become a favorite object of criticism of opponents of the logical-positivist program of foundation of scientific knowledge.

Thus, T. Kuhn's idea about the "incommensurability of paradigms", alternating fundamental scientific theories that determine the very way of positing and solving of scientific

problems in a certain field of knowledge — became a paradigm itself [7].

Kuhn's idea, in its own turn, aggravated the controversy between scientific realism and anti-realism in interpretation of the nature and functions of scientific theory.

Crucial ideas concerning those topics were presented in the classical work of Larry Laudan "A Confutation of Convergent Realism" [8].

The essence of scientific realism (or, in the terminology of L. Laudan, convergent epistemic realism) can be characterized as a sequence of the following theses:

- Mature scientific theories are (at least) approximately true in the sense of correspondence; at the same time, the subsequent theories are "closer to the truth" than the preceding theories which relate to the same subject area.
- Both observable and theoretical terms that are part of "mature" theories have real referents — there really are objects in the world that correspond to the postulated elements of the ontology of scientific theory.
- In mature science, the preceding theories turn out to be "limiting cases" of subsequent ones (subsequent theories in mature science retain the referents of previous theories and the interconnections that bind them).
- Each subsequent theory is capable of explaining why the previous theory was "successful" in predictive and explanatory terms (if, of course, the previous theory was indeed successful in the indicated sense).

The term "approximate truth" or "verisimilitude" is considered as some kind of "weakening" of the formally logical concept of the truth of the theory proposition [9].

As a result, from the point of view of epistemic realism, the development of science is a "convergent" process, which inevitably brings us closer to the "increasingly credible" picture of reality.

In particular, from the point of view of scientific realism, the following statements should be correct:

- If the theory is "plausible" / "approximately true," then it successfully fulfills its explanatory and predictive functions.
- If the theory successfully fulfills its explanatory and predictive functions, then it is approximately true.

With regard to the statement B), L. Laudan notes: one can easily name the number of theories that had successfully implemented their explanatory and predictive functions for quite a long period of time and, nevertheless, turned out to be virtually false eventually (Ptolemy and Aristotle's astronomy is a classic example of such a theory).

Regarding the statement A) L. Laudan drew readers' attention to the following fact: not a single attempt to define

strictly the notion of "approximate truth" of the theory has proved successful so far.

One of the most well-known variants of such a definition is the following formulation.

Let T_1 be a standardly formalized theory; Let $CtT(T_1)$ be the cardinality of the set of its true conclusions (true propositions logically derivable from the postulates (axioms) of T_1), $CtF(T_1)$ is the cardinality of the set of false conclusions of T_1 .

Then the T_1 theory is verisimilar (approximately true) if and only if the cardinal number of the set of its true corollaries is greater than the cardinal number of the set of its false corollaries:

$$Ct T(T_1) > Ct F(T_1)$$

As L. Laudan notes, this definition is unsatisfactory: we can consider some randomly selected class of observable consequences of the theory following from its postulates. It may well be that the theory is approximately true in the above-stated sense, and, nevertheless, all its observable consequences belonging to the class in question will actually be false.

The absence of a correct definition of the notion of approximate truth automatically makes intractable the task of comparing changing theories by their "degree of verity" and casts doubt on the central thesis of epistemic realism about the continuity in the development of scientific knowledge and the steady increase of its "true content" [10].

It is easy to see that the above-stated definition of the concept of verisimilitude is based on the standard interpretation of scientific theory as a set of statements, ordered by the derivability relation. In other words, this definition totally neglects the notion of model that mediates the relationship between the postulates of a theory and its subject area in the set-theoretic program of the formalization and foundation of scientific knowledge, which is the main alternative to the program of logical positivism.

As a result, the concept of truth/verisimilitude of a theory is interpreted as a literal (full or partial) correspondence of the fundamental postulates of the theory to the elements of its subject area, provided by specific "translation rules" — reduction statements.

III. SET-THEORETICAL MODEL OF DYNAMICS OF SCIENTIFIC KNOWLEDGE

Within the framework of the set-theoretic program of foundation scientific knowledge, two fundamentally different concepts should be distinguished: a purely formal concept of truth of a theory's statement in a model and a concept of structural correspondence between elements of models of a theory of a various levels and of elements of a certain subject area (moreover, it is the second concept that is considered as a correct clarification of the concept of truth in natural science theory in the sense of "correspondence").

As a result, in our view, the aforementioned definition of the verisimilitude of a theory implicitly mixes these two

concepts of truth — namely, it implicitly uses the first concept instead of the second one.

This fact is due to the insufficiently developed semantic problematics in the logical-positivist program of formalization of scientific knowledge.

The concept of partial truth/verisimilitude of the theory has been successfully clarified in the set-theoretic approach to the foundation of scientific knowledge. The crucial notion underlying the set-theoretic approach is the generalization of the semantic concept of the model in the sense of A. Tarski.

Generally speaking, in formal semantics a model of a theory is usually understood as some possible realization of a theory that satisfies its axioms. A possible realization of the theory, in its own turn, is a set-theoretic object of the corresponding logical type — for example, an ordered sequence of elements $\langle D, R, F \rangle$, where D is some arbitrary non-empty set of objects, R is a non-empty set of relations defined on D , F is a set (possibly empty) of functions (operations) defined on D . This construction is a model of the theory, if only all the statements (axioms) of the theory are valid when interpreted in terms of $\langle D, R, F \rangle$.

These definitions imply the fulfillment of the following conditions.

First, all terms included in expressions of the corresponding formal language must take definite meanings from domain D .

Second, for each predicate symbol defined on D (n -placed relation) R_i^n ($n \geq 1, i \geq 1$), the sets of sequences of objects from D , satisfying/ not satisfying the predicate must be uniquely determined. That is, for each such relationship, the sets of its "truth" $T(R_i^n)$ and "falsity" $F(R_i^n)$ must be uniquely defined in the following way:

- $T(R_i^n) \cap F(R_i^n) = \{\emptyset\}$
- $T(R_i^n) \cup F(R_i^n) = D$

Opponents of the use of formal methods in the philosophy of science usually point to the excessively "rigorous", normative nature of these requirements in comparison to the peculiarities of the construction and modification of the theories of the natural sciences.

In particular, the first requirement does not adequately reflect conceptual changes that accompany the evolution of natural science theories: some hypothetical concepts postulated by theory may later turn out to be "empty" (consider, for example, the fate of such hypothetical concepts as "calorie" and "phlogiston").

The second requirement does not allow us to describe properly the change in the structural relations between the elements of the subject area of the theory; in general case, for empirical theories (at least at the early stages of their development), it is impossible to provide strict implementation of conditions a) and b) (domains of truth/falsity of relations R_i^n , included in the formulation of empirical generalizations and the laws of the theory, can change).

N. Da Kosta, S. French, I. Maikenberg, R. Chuaqui proposed a generalization of the concept of a model that would successfully neutralize the described technical difficulties and naturally explicate the concept of approximate truth of the theory, which demonstrates the validity of the main theses of convergent realism [11] [12].

The starting points for this approach are the concepts of a partial relation (defined on some non-empty domain D), a partial structure, and a quasi-truth. The latter concept is a formally precise explication of the concept of approximate truth.

Let D be a non-empty domain, some objects from which may eventually prove to be non-existent.

The partial N -place relation on D is a triple $\langle R_1^n, R_2^n, R_3^n \rangle$, where R_1^n, R_2^n, R_3^n are mutually disjoint sets, the union of which coincides with D : $R_1^n \cup R_2^n \cup R_3^n = D$

R_1^n is such a set of n from D , which satisfies R , R_2^n is a set of n , which does not satisfy R , and R_3 is the set of n , relative to which it is not figured out yet, whether it satisfies R or not.

A partial structure is an ordered pair $\langle D, R_i^n \rangle$ ($n \geq 1, i \geq 1$), where D is a non-empty set, and R_i^n is a family of partial relations on D .

The next necessary notion in this approach is the concept of *A-normal structure*.

Let $A = \langle D, R_i^n \rangle$ be a partial structure. Then $B = \langle D', R_i^{n'} \rangle$ is an *A-normal structure*, if only:

- $D = D'$,
- in structures A and B the same constants designate the same objects,
- $R_i^{n'}$ is an extension of R_i^n

Since several *A-normal structures* prove possible for each partial structure, the concept of a *pragmatic structure* is introduced to specify the "admissible" extensions.

The pragmatic structure is the triple $\langle D, R_i^n, P \rangle$, where D, R are understood as before, and P is a set of laws or observation data, specific to a given subject area. P naturally imposes restrictions on the allowable extensions of the original partial structure. The adjective "pragmatic" in the name of this type of structure reflects the non-logical, factual character of the set of restrictions R .

On the basis of the concept of pragmatic structure, the conditions for the existence of an *A-normal structure* $\langle D', R_i^{n'} \rangle$ for some partial structure $\langle D, R_i^n \rangle$ can be defined as follows.

Let $\langle D, R_i^n, P \rangle$ be a pragmatic structure. For each partial relation R_i , one constructs the set M_i of atomic sentences and their negations, which satisfy/ does not satisfy each of the relations R_i^n . $M = \cup_{i \in I} M_i$ is the union of all sets M_i .

Then the pragmatic structure $\langle D, R_i^n, P \rangle$ admits the existence of an *A-normal structure*, if and only if (iff) the set $M \cup P$ is consistent.

The statement α is quasi-true in the pragmatic structure $\langle D, R_i^n, P \rangle$, i. e. there is an *A-normal structure* $\langle D', R_i^{n'} \rangle$, in which α is true (in the sense of Tarski). Otherwise, α is quasi-false in a pragmatic structure.

Finally, some statement α is quasi-true (approximately true), if there is some pragmatic structure A and the corresponding *A-normal structure* B such that α is true in B (in the sense of Tarski). Otherwise, α is quasi-false.

IV. CONCLUSION

The formal apparatus proposed in the framework of the set-theoretic strategy of analyzing of scientific knowledge allows us to construct adequate models of the dynamics of science, reflecting the possibility of conceptual and structural changes of theories and, at the same time, preserving (partial) continuity between them. Thus, it can be said that the thesis of the "confutation" of convergent realism and, in particular, the complete incommensurability of changing fundamental theories appears to be the result of an incorrect absolutization of the standard interpretation of scientific theory proposed by logical positivism.

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