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Conceptual Bases of the Structuralist Program of Scientific Knowledge Analysis

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***Abstract*—The so-called structuralist program of substantiation scientific knowledge is one of the most technically developed variants of the set-theoretic strategy of formalization and axiomatization of scientific knowledge. A distinctive feature of this version of the specified strategy is the use of a powerful mathematical apparatus, which often leads to an incorrect interpretation of its essence. This paper is devoted to a detailed description of the conceptual apparatus of the structuralist strategy for the substantiation of scientific knowledge, as well as the characteristics of the underlying substantive premises.**

Keywords—axiomatization; formalization; structuralism; theory; potential model; partial potential model; actual model

I. Introduction

This paper discusses some of the philosophical and methodological foundations of the so-called structuralist program of the axiomatization of scientific theories, and also describes the features of the technical tools used to analyze the structure and dynamics of scientific knowledge in the framework of this program.

The structuralist version of the set-theoretic strategy for the axiomatization of scientific theories in its "canonical" form was described in the works of J. Sneed [1] [2], W. Stegmüller [3] [4], W. Balzer [5] [6], C. U. Moulines. [7]

Of all the available versions of the set-theoretic strategy of axiomatization and analysis of scientific theories, the structuralist methodological program is the most elaborated and very convincingly supported by numerous reconstructions of concrete scientific theories.

This strategy was successfully applied to the description of structure of more than forty scientific theories from the most diverse branches of knowledge — physics, chemistry, biology, psychology, economics, sociology, and even theory of literature [8].

A distinctive feature of the structuralist strategy of the axiomatization of scientific theories is the use of a powerful mathematical apparatus. Probably, that's why, the core of this methodological program has been repeatedly misunderstood. In particular, F. Suppe, one of the supporters of the semantic interpretation of scientific theories, comepletely excludes the structuralist program from a list of versions of the set-theoretic strategy for analyzing scientific theories. According to F. Suppe, the structuralist strategy of the axiomatization of scientific theories is an insignificant revision of the traditional, so-called "statement" interpretation of scientific theories, as far as it heavily draws on the idea of distinguishing between empirical and theoretical terms, firstly proposed in the neo-positivistic methodology, and uses the technique of rules of correspondence [9].

Such an interpretation of the structuralist strategy of analyzing scientific knowledge seems to us to be utterly incorrect.

In order to prove this thesis in the maximally exhaustive manner, we will adhere to the following strategy of description of the structuralist methodological program.

First, we will briefly describe some theoretical presuppositions of this methodological program.

Then we will characterize some peculiarities of the conceptual apparatus used in the implementation of this program, avoiding as far as possible the cumbersome mathematical formulations.

II. Theoretical Bases of Structuralism Methodology

The idea of "absolute experience" which is independent of any theoretical presuppositions is flatly rejected in the structuralist methodological program. Respectively, the possibility of construction of a neutral observation language as a specific "technical epitome" of this idea is refuted as well.

The process of construction of a scientific theory usually begins with a compilation of a list of its fundamental concepts and principles. Each specific (empirical) theory is aimed at studying of a particular fragment of our experience; therefore, a necessary precondition for successful application of a certain theory to a specific subject area is the "conceptualization" of a given area, i.e., its description in a specific conceptual vocabulary.

Further, within the theory some general statements, containing concepts from the above-mentioned basic conceptual framework and referring to the field of study, are formulated. For example, the subject area of mechanics is interpreted using the concepts of "particle", "coordinate", "time", "velocity", "mass", etc. For the decision theory such basic concepts would be "action", "uncertainty," "expected utility," "subjective probability," and so on. [10].

In answering the question about the method of determining the senses and meanings of the basic concepts of a theory, it is necessary to take into account the following fact: each such concept takes its value only within the framework of a holistic conceptual scheme, have been correlated with other concepts. In other words, isolated scientific concepts, which are the conceptual basis of a certain theory, do not exist.

Therefore, the requirement to limit the ways of introduction of basic scientific concepts into theory by their explicit (so-called real) definitions is practically unrealizable: a strict definition of the meaning of a certain term t is equivalent to an exhaustive description of its relationship with other concepts of the corresponding conceptual framework. If in this case one is talking about a real definition, then its definable and defining parts (definiendum and definience) should be semantically equivalent. In some cases, such a formal procedure is feasible. For example, the concept of average velocity in classical mechanics can be correctly defined in this way via the concepts of "distance" and " time interval", since " average velocity" is semantically equivalent to the distance traveled divided by time. However, to give such definitions to all concepts of a certain theory is impossible technically: such an attempt would inevitably lead either to an infinite sequence of definitions or to a vicious circle [11].

That is why some concepts in any theory are accepted as indefinable — such concepts are called initial or basic. After compilation of a list of such indefinable concepts, other terms of the theory are introduced by means of the above-mentioned formally strict definitions. Thus, all concepts explicitly defined in a theory receive their meanings by means of sequences of definitions that depend substantially on indefinable concepts.

Relations between proper concepts that are explicitly definable in some theories are characterized (at least partially) by the fundamental postulates — the theory axioms, which include these concepts.

Obviously, with respect to natural science theories, the requirement for an exhaustive axiomatization of this type is excessive. The postulates of natural science theories, interpreted as axioms of a formal system, are not capable of fully defining the meanings of its concepts — otherwise, there would simply be no fundamental differences between mathematical and empirical theories.

Therefore, the characterization of the meanings of the fundamental concepts of the theory is carried out mainly with the help of models of the theory. The interpretation of the concept of a model of theory in the structuralist methodological program, as in other versions of the set-theoretic approach, goes back to the works of A. Tarski: the statement that the conceptual scheme of the theory can be interpreted in terms of some subject area is tantamount to statement that a given subject area (in a simplified, idealized form) can be understood as a model of the system of axioms of a given theory. It is in this way that the terms included in the axioms of the theory receive an empirical interpretation. Using a slightly different formulation, it can be said that the theory represents the corresponding subject area through the construction of a model.

The next fundamental problem of the philosophy of science, which has been raised within the framework of the structuralist approach, is the problem of the truth evaluation of empirical theories — if explanations, predictions, other "manipulations" with the objects of the theory domain are successful, it seems natural to call such theory true [12].

With regard to this subject, the question about the logical and methodological conditions for evaluating of empirical theory as true or false immediately arises.

According to supporters of the structuralist methodological program, neither the verificationist, nor the falsificationist ways of solution of this problem seem to be satisfactory.

The following fact became a crucial disadvantage of these "traditional" approaches in the philosophy of science: they all stem from the interpretation of scientific theory as a system of axioms and their logical consequences. The notion of a model of theory did not play any serious role in the traditional or so-called statement strategy of the analysis of scientific theory. A more flexible and adequate interpretation of the structure of the theory is achievable when the focus of meta-theoretical studies of the nature of science is shifted precisely to the concept of a model. From this point of view, theories should be considered as classes of structurally similar models, differing probably by some of their empirical interpretations, rather than sets of axioms, definitions, and theorems. Taking into account complex, mediated nature of the relationship between data models and models of the theory itself, we may get more plausible image of the structure and dynamics of the theory [13].

Any mature scientific theory is capable of "embracing" an open (incomplete) set of models, which are determined by one and the same set of axioms but differ in empirical interpretations — features of subject areas, for the representation of which some theory is used. As a result, the theory may contain models, successfully representing certain classes of phenomena, models that represent some empirical data less successfully and, finally, models which prove completely inadequate. Given this fact, the assessment of the adequacy of a scientific theory cannot be implemented in binary terms "all or nothing". Some theories are more successful than others in the sense that they "cover" a larger number of data models and do it more efficiently than their "rivals" do. Of course, if a theory is not able to represent successfully any of the data models for which representation it had been originally formulated, it should be rejected completely and unconditionally. This case, however, is not typical of scientific practice. On the other hand, it is difficult (if not impossible) to find a theory in the history of science, all theoretical models of which ideally represent the corresponding data models. In this sense, the theory should be characterized not as true/false in general, but as "completely true/adequate" with respect to some of its models, "partially true/adequate" with respect to others, and "completely inadequate/false" with respect to the rest. In other words, epistemic assessment of scientific theories should be graded, which, however, does not imply its literal truth-functional many-valuedness.

The next methodological problem that essentially characterizes the structure of a scientific theory is the search for the grounds for classifying of various types of its models.

From this point of view, all models of the theory may differ, first of all, according to the following significant criteria:

- Models of the theory that satisfy the same system of fundamental axioms (theory postulates), but differ in specific empirical interpretations (features of the data, for representation of which the theory is used — "horizontal" classification).
- Models of the theory that have identical empirical interpretations (representing the same data models), but differing in the degree of universality ("vertical" classification).

Thus, in the most general form, within the framework of the structuralist methodological program, natural science theory is understood as a hierarchically organized set of axioms and corresponding to them classes of models. Some axioms are virtually universal and are satisfied by every models of the theory. Others have lesser degree of generality; finally, some are extremely specific and belong to particular engineering applications of the theory [14].

III. Conceptual Apparatus of Structuralist Methodology

The structuralist strategy of the axiomatization of scientific theories is a variant of the *metatheoretical* approach to the analysis of scientific knowledge — the linguistic formulation of the fundamental postulates of a theory is considered essential (although not decisive) in description and identification of the theory.

As in the "classical" versions of a set-theoretic program, theories are considered as ordered sets of models in the sense of Tarski. A model of a theory is its possible realization that satisfies all the axioms of a theory. Characterization of a theory is understood as a description of the class of its models, the features of which are (partially) determined by the list of its axioms. At the same time, any set of statements of the theory defining the same class of models can be chosen as a list of its axioms.

In this approach, the so-called framework conditions/axioms and proper, substantial conditions/axioms are distinguished. Structural axioms define the "background" or "paradigmal" parameters of the construction of the corresponding theory — the properties of the base sets of elements, included in the models of the theory. In the set-theoretic terms, structural axioms determine the number and the admissible type (order) of the elements of the domain of the theory, the number and the admissible type of the functions and relations defined on these elements. The proper axioms of the theory are "law-like" statements about the relationship between these elements of the theory domain. For example, in Newton's mechanics, its second law is a theory's proper axiom, and the implicitly assumed precondition of the differentiability of a coordinate function is a structural axiom.

Thus, the theory's proper axioms describe the conditions that a certain potential model of a theory must satisfy in order to be its actual (real) model. All the theory's proper axioms can be represented as a finite conjunction of the statements of the theory [15].

Set-theoretic structures that are models of the structural conditions of a certain theory T (which satisfy *only* these conditions) are called potential models Mp of the corresponding theory.

Set-theoretic structures that satisfy both structural and proper axioms of a certain theory T are called its actual or real models M.

For these classes of models, the $M \subseteq Mp$ relation naturally holds. The main stages of characterization (identification) of a certain theory T are the description of the class of its potential models, fixing type-theoretic order and properties of the basic sets of the theory, and also of relations and functions, defined on these sets; description of the class of its real models; description of the admissible relationships between models belonging to the same theory and to some different theories; description of the grounds for distinguishing between theoretical and non-theoretical terms of the theory; description of the scope of possible applications of the theory; formulation of the fundamental empirical hypothesis of the theory about the degree of correspondence between the models of the theory and the elements of its domain.

It should be noted that the method of distinguishing between theoretical and non-theoretical terms in structuralist strategy radically differs from such methods, received in the neo-positivistic philosophy of science. In structuralist methodological program some notion ought to be attributed to theoretical terms, if all logically correct ways of its definition in the language of some theory T necessarily presuppose the existence of at least one *actual* model of this theory.

In the equivalent syntactic terms, the above-stated "condition of theoreticity" will look as follows: a concept is theoretical in the context of a certain theory T, if all logically appropriate ways of its definition necessarily presuppose the truthfulness of (at least some) of the proper axioms (fundamental laws) of T.

Hence, the concept is not theoretical, if it can be correctly defined without resorting to any actual models of the theory.

For example, in Newtonian mechanics, the concepts of mass and force are theoretical, since all their logically correct definitions necessarily imply the use of some of the proper laws of this theory. The concepts of a set of particles, a set of moments of time, coordinates and distances are not theoretical since their definition does not necessarily imply the use of the proper laws of this theory. It is also obvious that the concept cannot be theoretical or non-theoretical "by itself", being considered in isolation, independently of the context of certain theory — for example, in thermodynamics the concepts of mass and force turn out to be not theoretical.

So the criteria of theoreticity, proposed in the structuralist methodological program, are pragmatic, since they depend on the contexts of particular theories, do not imply the formulation of " universal" principles of distinguishing between observable and non-observable terms, and also do not refer to the verification theory of meaning and the technique of correspondence rules, elaborated in the methodology of logical positivism.

The next tool of theory identification is the description of its empirical domain I (the intended applications).

A successful application of a certain theory T to some set of empirical data is preconditioned by its preliminary "conceptualization", i.e. description of this set in terms of potential models of T. The next step of application T to I is the formulation of the fundamental hypothesis of a theory that has empirically verifiable consequences. This hypothesis asserts, roughly speaking, that the resulting potential model T is also its actual model (satisfies proper axioms of T). If this assertion proves to be true, the application of T to I can be considered successful.

From a formal point of view the set I of intended applications of theory T is a set of *partial potential models of* T, which is a subset of the set of its potential models: $I=Mpp\subseteq Mp$.

Formal reconstruction of the theory begins with the compilation of a list of terms and concepts indefinable in the theory. These concepts are elements of the basic sets of the theory models; they form the ontological basis of the theory. They are semantically independent of the theory's proper axioms. This statement is true for all empirical theories without exception: the senses and meanings (at least some) of the concepts included in the basic sets of models of empirical theories can be determined independently from theory’s proper axioms. This is a necessary condition for the theory to have any empirical content at all [16].

Consequently, the elements of the basic sets, at least of some models of the theory, must either be formulated in ordinary language or be "borrowed" from the conceptual arsenal of other theories.

Those potential models of a theory whose basic sets contain as their elements only concepts that are not definable in a given theory and do not contain functions and relations defined on these elements are called partial potential models of this theory.

Since $I=Mpp\subseteq Mp$, $M\subseteq Mp$, between the set of possible applications of the theory I and the set of its actual models M the following relations are possible:

- $I\subseteq M$
- $I \not\subset M$, $I\cap M\neq\varnothing$
- $I\cap M=\varnothing$

The first option is ideal and means "total triumph" of the theory: all the intended applications of the theory are correctly represented in its actual models. The second option represents a "partially" successful theory; it is clear, however, that the larger the intersection of I, and M is, the more successful the corresponding theory is. Finally, the third variant corresponds to the meaningless theory: none of the intended applications of the theory can be represented in any of its actual models.

Conceptually speaking, I is a complete set of all possible applications of the theory. Therefore, although the relationship between classes I, M should be determined with the utmost possible precision, this procedure cannot be carried out by purely formal semantic methods and I cannot be characterized (only) as a set of models, that is, as a formal object in the strict sense of the word. Any "practically acceptable" version of the definition of the relationship between I, M, corresponding to the actual practice of scientific knowledge, should include pragmatic and diachronic considerations, since the definition of specific "boundaries" of I is the result of the choice of the scientific community using the theory T. It is possible to say that the domain of I always remains in Mp, however its specific "contours" can change according to the particular interests of the scientific community, certain peculiarities of experimental procedures etc.

The so-called *formal core* K of some theory T includes the following components: structural axioms and the corresponding classes of potential models of the theory; proper axioms of the theory and classes of its actual models determined by them; constraints characterizing the relationship between various possible applications of theory T (constraints are understood as relationships between models of the same theory — admissible combinations of its potential models); inter-theoretical links which include the relations of reduction, equivalence, approximation; the set of partial potential models of the theory, which is a tool of the conceptualization of its empirical domain; the fundamental empirical hypothesis of the theory, which states that the set of partial potential models of the theory, describing I, is compatible with the laws, restrictions, and connections that characterize the sets Mp, M of potential and actual models of the theory. In a purely extensional, set-theoretic sense, the fundamental empirical hypothesis of the theory has the following form: $I \subseteq \langle Mp, M \rangle$.

An ordered set $\langle K, I\rangle$ is called a *theory-element*. The theory-element is the minimal structural unit for the analysis of scientific knowledge, permitting the formulation of the fundamental empirical hypothesis of the theory.

Typical examples of isolated theory-elements are applied empirical theories, determined by some particular laws. For example, by means of a single theory-element, the theory of elasticity, the classical theory of gravity, the gas theory of van der Waals can be represented. Theories in the proper sense (for example, classical mechanics) can be represented as ordered sequences of "atomic" theory-elements.

IV. Conclusion

It seems to us that the structuralist program of analysis of scientific knowledge is a unique example of a successful combination of formal, "static" and descriptive, "dynamic" methods of analysis of scientific knowledge. The first component of this program provides possibility of correct clarification of the concept of truth in relation to the theories of the natural and social sciences. The second one provides the possibility of taking into account pragmatic, contextual factors of the development of scientific knowledge. All this makes the structuralist methodological program a very effective instrument of modeling of the theoretical activity of cognitive agents of various types.

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