

Research on Existential Problems Based on Drawer Principle

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Abstract—The principle of drawer, also known as the "pigeon (nest)" principle, is a common method for solving existential problems. It was first proposed by the German mathematician Dirichlet. Therefore, it is also known as the Dirichlet principle in mathematics. In this paper, the combination of algorithms and the drawer principle is applied. Using this principle, we can solve some existential problems that are quite complicated and even impossible to start. Through the introduction of this thesis, using the drawer principle to solve the problem, deepen the understanding and understanding of the drawer principle.

Keywords—number theory; drawer principle; pigeonhole principle; existential problems; math

I. INTRODUCTION

A. Research Background

The drawer principle, also known as the principle of pigeonholes or the Dirichlet principle, was first discovered by the German mathematician Dirichlet (1805-1855). Drawer principle enjoys a very unique position in combinatorics. It is often used to prove some problems about existence, and it has also been widely used in number theory and algorithm. The key to using a drawer to understand the problem is to construct a "pigeon" or a drawer, that is, how to find the category that meets the conditions of the problem.

B. Problem Description

This paper selected a topic in the POJ (Peking University Online Judge) - POJ2356. This is a classic algorithm competition problem in the drawer principle. Title "Find a multiple", Original "The input contains N natural (i.e. positive integer) numbers ($N \leq 10000$). Each of that numbers is not greater than 15000. These numbers are not necessarily different (so it may happen that two or more of them will be equal). Your task is to choose a few of given numbers ($1 \leq \text{few} \leq N$) so that the sum of chosen numbers is multiple for N (i.e. $N * k = (\text{sum of chosen numbers})$ for some natural number k).".

The meaning of the Chinese title: input gives N positive integers ($N \leq 10000$), each number is not greater than 15000, these books can be the same. Please select some numbers in the given book ($1 \leq \text{few} \leq N$) so that the sum of the selected numbers is a multiple of N (ie $N * k = (\text{sum of the selected numbers})$, k is some Natural number).

Significance of research: The drawer principle is the simplest of combinatorial mathematics. It reflects the most basic nature of integers and is widely used in number theory and combinatorial mathematics. This simple principle has markedly improved the ability to think, and the problems solved are often highly skilled. Researching the principle of the drawer helps understand the title of the contest, provides people with ideas for solving the problem, and improves the efficiency of solving the problem.

II. DOMESTIC AND FOREIGN RESEARCH

A. Drawer Principle Form

Drawer principle 1: If $n+1$ (or greater than $n+1$) elements are split into n sets, then at least one set has 2 or more elements.

Drawer Principle 2: If we divide $qn+1$ ($q \in \mathbb{N}^+$) elements into n sets (drawers), then at least one set (drawer) has at least $q+1$ elements. Principle 2 is a generalization of principle 1. When the value of q is 1, it is the form of drawer principle 1.

Drawer Principle 3]: If you put m elements into n collections (drawers), there will be at least $[(m-1)/n]+1$ elements in at least one collection (drawer).

In the June to July 1958 issue of the American Mathematical Monthly, there was such a question: "At any of the six individuals, there were three people who knew each other before, or three people didn't know each other before." The six-member meeting was A simplest special case of the well-known Ramsey theorem in combinatorial mathematics, the proof of this simple problem can be used to draw conclusions for other problems. These results form an important part of combinatorial mathematics - Ramsey's theory. From the proof of the six-person problem, we once again saw the application of the drawer principle.

B. Advantages

Drawer principle can solve some existential problems that are quite tedious and even impossible to start. The simplification of complex issues is dealt with in detail in the next section. In addition, another advantage of the drawer principle is that it is simple and easy to understand. Unlike other theorems, principles, and other complex and difficult to understand, it can be obtained through simple analysis and inference. The principle of the drawer principle is that there are many things and drawers, and the number of things is more

than the number of drawers is the most basic situation when using drawers to understand the original question. The principle of drawers is easily proved by counter-evidence.

C. Disadvantages

Of course, everything has two sides, and the drawer principle also has its shortcomings. Using him can only prove the "existence" of objects of a certain nature, but it does not necessarily know exactly who these objects are.

III. DRAWER PRINCIPLE SOLVES THE PROBLEM

A. General Steps

- Read and understand the meaning of the questions, clarify what is the "element", what is the "drawer", that is, what can be "elements", what can be "drawer."
- Make drawers. This is an important step. This step is how to make a drawer. According to the problem conditions and known results, combined with the relevant and learned knowledge, grasp the most basic quantitative relationship, design and confirm the number of drawers and the number of drawers needed to solve the problem, and pave the way for the next use of the drawer principle.
- Apply the principle of different drawers: Observe the conditions of the subject, combine steps 2 and apply the principles given in the questions to solve the problem.

B. Title Requirements

Input: The first line of the input gives the integer N, and the next N lines each give an integer of the set.

Output: This problem is not set to output the number 0. The first line in the output gives the number of selected digits, and these selected digits are then output in an arbitrary order. If there are more than one set of numbers that have the required properties, just output one.

C. Chart

TABLE I. THIS TABLE IS FOR SAMPLE INPUT AND OUTPUT

Input	Output
5	2
1	2
2	3
3	
4	
1	

D. Ideas

For the sequence a_i , there must be a contiguous segment of $a_1 \dots a_r$ so that $a_1 + \dots + a_r$ can be divisible by n . We abstract the remainder into drawers, abstract b_i as an element, and get n elements and $n-1$ drawers. According to drawer principle 1 (that is to say put more than n objects in n drawers, there are at least one drawer with two or more objects.) There must be two equal remainders. As long as we differentiate the two ends, we can construct a continuous set of a_i , and its sum can be divisible by n .

E. Code Fragments

```

For(int i=1;i<=n;i++) //Record the number of drawers for each
drawer. If there are several, direct difference structure
{
    Scanf("%d",a+i);
    s[i]=s[i-1]+a[i]; //sum of the sum of the previous i numbers
    If(s[i]%n==0) { //If the sum of the first i integers is
divisible by n, the first i number is output and the program exits
        Print(1,i);
        Break;
    }
    Else if (! mod [s [i]% n]) { // Otherwise if the sum of the
first i number does not generate a mantissa for n, then the
subscript i enters the "remainder drawer"
        Mod[s[i]%n]=i; //otherwise output a[subscript
+1]·a[i] in "remainder drawer" and exit the program
    }
    else{
        Print(mod[s[i]%n]+1,i);
        Break;
    }
}

```

IV. RESULT ANALYSIS

Problem solving requirements:

Time Limit: 1000MS

Memory Limit: 65536K

A. Running Time Comparison

The violence solution takes 0.249s;

The principle of using drawers takes 0.015s;

B. Memory Comparison

Violence solution: 368Kb

Drawer principle solution: 296KB

C. Chart

TABLE II. THE MEMORY AND OPERATING SCHEDULE

	Title Requirement	Violence	Drawer
Memory/KB	65536	368	296
Running Times/ms	1000	249	15

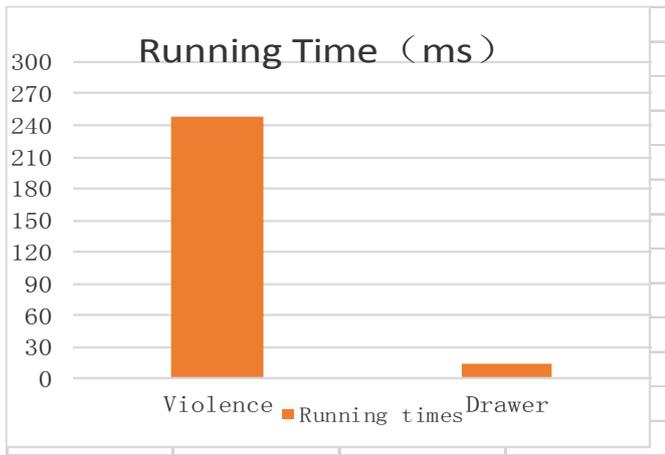


FIGURE I. TWO METHODS RUNNING TIME



FIGURE II. TWO METHODS USING MEMORY

7881574	08:53:45 14 May 2018	robustll	1032. Find a Multiple	Visual C++ 2017	Accepted	0.249	368 KB
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FIGURE III. RECORD OF VIOLENCE ACCEPTED

ID	Date	Author	Problem	Language	Judgement result	Test #	Execution time	Memory used
7881573	08:51:45 14 May 2018	robustll	1032. Find a Multiple	Visual C 2017	Accepted		0.015	296 KB

FIGURE IV. RECORD OF THE DRAWER PRINCIPLE ACCEPTED

Figure III shows the record of violence accepted, and figure IV shows the record of the drawer principle accepted. From the above several charts, it is clear that the drawer principle is more convenient than the violence solution, and the running time is far less than the ordinary method.

V. KEY ISSUES AND SOLUTIONS

The key issue: The principle of the drawer as its name implies, the main problem is to find a good drawer. Finding the right drawer can make it easier to solve the problem.

Solution: Carefully observe the number in the problem, analyze the existing rules in the number, and apply the drawer principle to solve the problem. The key lies in “manufacturing the drawer”, that is, find out the classification criterion [8] that meets the conditions of the problem. According to the characteristics of the object, scientific classification should be used to construct the drawer. Different issues should be treated

differently. Do not read it according to the original. Drawers are made well and made skillfully. Not only do they provide wonderful answers, they give people a sense of enjoyment, but they also measure thinking.

For this question, we assume: $s[1]=a[1]$, $s[2]=a[1]+a[2]$, $s[3]=a[1]+a[2]+a[3]$, $s[n]=a[1]+a[2]+...+a[n]$ [12]; if there is $s[i]$ that is exactly a multiple of n , the direct output is, but if it does not exist, then $s[i]\%n$ must belong to $[1,n-1]$, because $s[i]$ has n items, then the principle of the drawer is applied. It must be known from the drawer principle that there is a pair of $s[i]=s[j]$ and $i \neq j$, and the difference between the two s is also a multiple of n ; so only the value of $i+1$ to j need be output.

The construction of the drawer principle: The content of the drawer principle is simple to understand and easy to accept. It has a unique position in problem solving. Many problems with existential problems can be solved with it. When using the drawer principle to solve the competition problem, the most

important thing is to determine which are the "balls" and which are the "drawers." Many times we need to select and manufacture "appropriate and appropriate" drawers. "Suitable" - the "specification" of each drawer is required to be the same, because the elements are put in any way, the possibility of putting elements in each drawer is the same; "appropriate" - the number of drawers is less The number of elements, and to meet the conclusions.

VI. SUMMARY

The application of the principle of drawers is very extensive. I just mentioned one example that I think is more classic. There are still many practical problems that are solved by using the drawer principle in life. In the manufacture of drawers for the same topic, there is not only one construction method, only the contact with different forms of problems can make good use of the drawer principle. The key to using the drawer to understand the problem is to construct the "pigeon nest" or the drawer in a subtle way, that is, how to find the classification principle that meets the problem conditions.

In this question POJ2356, if the conventional method is not only long, but also takes a long time, the program competition, as the name suggests, you have to find the best solution to win, otherwise the same correct, how to judge who is the idea or solution is What's the best?

In summary, the drawer principle is one of many methods for solving a problem. The purpose of the program design is to achieve the optimal solution to the problem. The drawer principle is suitable for solving the problem, such as the selected questions. In the actual problem solving process, you need to carefully examine the problem, find the best algorithm, the best way to achieve the purpose of Accepted.

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