Oscillatory Correlations of Nonlinear Stochastic Energy Harvesting System

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***Abstract*—The auto-correlation and cross-correlation of a nonlinear stochastic energy harvesting system are studied by stochastic simulation method, which show periodic oscillation and gradual attenuation, implying that the nonlinear stochastic energy harvesting system possesses complex dynamical behaviors. Among correlations of the dynamical variables, the cross-correlation strength of system variables x and *V* is largest, which possess the important effect on the system output. The coupling coefficient of the voltage K_v, the capacitance coupling constant K_c, and the time constant τ_p, enhance the strength of system correlations, and increase the system output. However, the roles of the viscous coefficient γ and the noise strength D is just opposite, decrease the system output and increase the output stability of the system.**

***Keywords*—oscillatory; auto-correlation; cross-correlation; nonlinear; stochastic; energy harvesting system**

I. Introduction

In modern society, remotely distributed wireless microsensors have been widely popular and widely used. It is desirable extensively that when people need electricity, they can get it anywhere and anytime. Namely, one can easily convert the almost ubiquitous ambient energy into electricity, such as the electromagnetic energy, the mechanical energy, the thermal energy, and so on. Adopting the nonlinear stochastic energy harvesting model instead of the linear model is an effective way of enhancing efficiency of ambient energy harvesting [1-13]. For a nonlinear energy harvesting system, in order to gain further the maximum output power, the mean value of the dynamical variable of the system has to be larger, and has to be transduced into the mean value of the output voltage of the system with minor losses, implying that correlation strengths of dynamical variables possesses the important role on the system output[14].

Correlation functions of the stochastic system can indicates the related activity of the dynamical variables between various times. Correlation functions can be classified into the uato-correlation function and the cross-correlation function. The uato-correlation function exhibits the related activity of the same variable at various time, and the cross-correlation function exhibits the related activity between different variables at various time. It is a useful method to discuss the characteristics of the system through the analysis of the correlation function. Over the past decades, many interesting and important research results of correlations of the stochastic system have been given in a series of papers [16-30].

Investigating the auto-correlation and the cross-correlation of dynamical variables of stochastic dynamic variables of the nonlinear stochastic energy harvesting system, we further point out dynamics characteristics of the system, and show effects of parameters on the output power of system, which is interesting and important.

This paper is arranged as follows: In section 2, we present the theoretic analysis of the auto-correlation and cross-correlation functions of the dynamics variables x, y, and V of the nonlinear stochastic energy harvesting system. In section 3, using the numerical simulation method we present the auto-correlation and the cross-correlation function distributions, and show oscillatory correlations of nonlinear stochastic energy harvesting system. Finally, summaries and conclusions conclude the paper.

II. Theoretic Analysis of Auto-correlation and Cross-correlations

Dynamic differential equations of a nonlinear energy harvesting system driven by stochastic force can be denoted by [13-15]

$$\begin{aligned}
\frac{dx}{dt} &= y \\
\frac{dy}{dt} &= -\frac{dU}{dx} - \gamma y - K_v V + \zeta(t) \\
\frac{dV}{dt} &= K_c \frac{dx}{dt} - \frac{1}{\tau_p} V
\end{aligned} \tag{1}$$

where x presents the dynamical variable of the system, y is the oscillator velocity, *V* is the output voltage, γ is the viscous coefficient, K_V is the voltage coupling coefficient, $\zeta(t)$ is the ambient stochastic force, K_C is the capacitance coupling constant, τ_P is the time constant , and *U(x)* is the potential function.

We consider the random force $\zeta(t)$ to be Gaussian white noise, whose statistical properties satisfy

$$\langle \zeta(t) \rangle = 0, \\ \langle \zeta(t)\zeta(t') \rangle = 2D\delta(t-t'), \tag{2}$$

where D is the noise intensity.

The potential function of the system *U(x)* is given by a quadratic bistable potential [15]

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{3}$$

For a three-dimensional stochastic process with a steady state, the normalized auto-correlation function of dynamic variables $x_i (i=1,2,3)$ is given by [31-33]

$$C_{x_i}(s) = \frac{\langle \delta x_i(t+s)\delta x_i(t) \rangle_{st}}{\langle (\delta x_i(t))^2 \rangle_{st}} = \lim_{t\to\infty} \frac{\langle \delta x_i(t+s)\delta x_i(t') \rangle - \langle x_i(t) \rangle^2}{\langle (\delta x_i(t))^2 \rangle}, \tag{4}$$

where $\delta x_i(t+s) = x_i(t+s) - \langle x_i(t+s) \rangle$,

and $\delta x_i(t) = x_i(t) - \langle x_i(t) \rangle$.

The normalized cross-correlation function between system variables x_i and x_j ($i, j = 1,2,3$ and $i \neq j$) are given by[34]

$$C_{x_i,\, x_j}(s) = \frac{\langle \delta x_i(t+s)\delta x_j(t) \rangle_{st}}{\sqrt{\langle (\delta x_i(t+s))^2 \rangle_{st}}\sqrt{\langle (\delta x_i(t))^2 \rangle_{st}}} = \lim_{t\to\infty} \frac{\langle \delta x_i(t+s)\delta x_j(t) \rangle}{\sqrt{\langle (\delta x_i(t+s))^2 \rangle}\sqrt{\langle (\delta x_j(t))^2 \rangle}}. \tag{5}$$

To discuss characteristics of the auto-correlation and the cross-correlation, and to present the effects of the system parameters D, γ, K_C, K_V, τ_p on correlations, we should first solve the Fokker-Planck equation corresponding Eq. (1), so that can get the probability density function $p(x, y, V, t)$, and then we follow to make other relative work. However, for us to obtain the analytic solution of Eq. (1) under arbitrary values of noise intensity D utilizing the analytic method is extremely difficult or even impossible. Here using the stochastic simulation method we discuss relative problems.

The data cited by the correlation function of dynamic variables are derived from the stochastic simulation of Eq.(1) , where the Euler forward program is used, in which the Box-Muller algorithm generates Gauss noise from two uniformly distributed independent random numbers between 0 and 1. The correctness of the following results is verified by adjusting the time step and the total time.

III. OSCILLATORY CORRELATIONS

A. *Oscillatory Auto-correlation*

Through the stochastic simulation, we plott auto-correlation distribution graphs of the dynamics variables of the nonlinear energy harvesting system driven by white noise in Figs. I-III, where $C_X(s)$, $C_y(s)$, and $C_V(s)$ as functions of the decay time s stand for the auto-correlation functions of dynamical variables $x, y,$ and V , respectively.

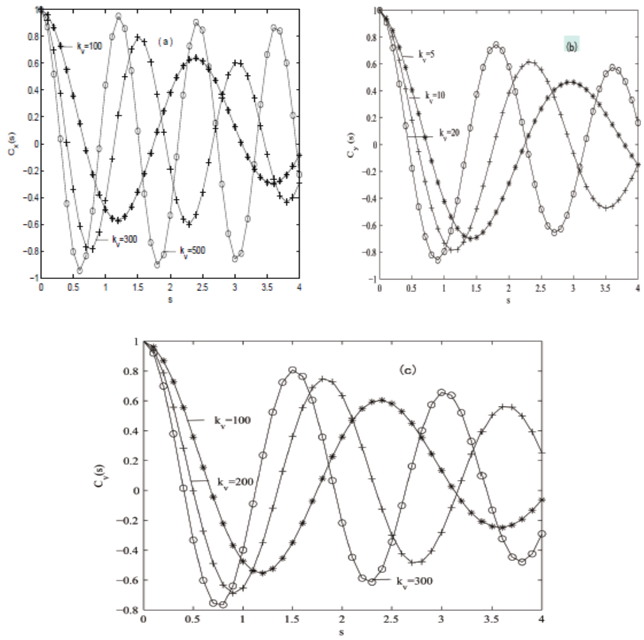

FIGURE I. AUTO-CORRELATION FUNCTIONS $C_x(s)$, $C_y(s)$, AND $C_v(s)$, VERSUS THE DECAY TIME S, FOR $a=2$, $b=1$, D=0.05, γ=0.6 , τ_p=100 , K_C=0.05 , AND K_v TAKING VARIOUS VALUES. (a) THE AUTO-CORRELATION FUNCTION $C_x(s)$. (b) THE AUTO-CORRELATION FUNCTION $C_y(s)$. (c) THE AUTO-CORRELATION FUNCTION $C_v(s)$

In Fig. I, we plot distribution curves of auto-correlation functions $C_x(s)$, $C_y(s)$, and $C_V(s)$ for various voltage coupling coefficients. Fig.I (a) presents the auto-correlation function $C_x(s)$ of the variable x for various values of K_V . Fig.I (b) presents the auto-correlation function $C_y(s)$ of the variable y for various values of K_V . Fig. I (c) presents the normalized auto-correlation functions $C_V(s)$ of the variable *V* for various values of K_V . From Fig. I, we see that auto-correlation functions $C_x(s)$, $C_y(s)$, and $C_V(s)$ exhibit alternate periodically oscillations between the positive correlation and the negative correlation when the decay time increase, the peak value of which decreases gradually with the decay time s, and finally, the auto-correlation function tends to zero; the evolution period decreases and the oscillation amplitude increases of the auto- correlation as the voltage coupling coefficient K_V increases.

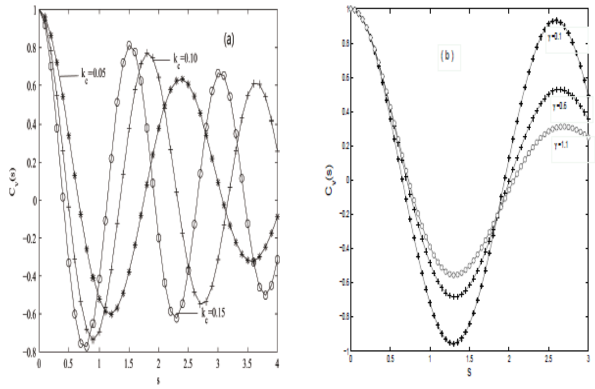

FIGURE II. THE AUTO-CORRELATION FUNCTION $C_V(s)$ VERSUS THE DECAY TIME S FOR $a=2$, $b=1$, D=0.05, K_V =100 , $\tau_p=100$, AND K_c AND γ TAKING VARIOUS VALUES. (a) $\gamma=0.8$, AND K_c TAKING 0.05, 0.10, AND 0.15, RESPECTIVELY. (b) $K_c=0.08$, AND γ TAKING 0.1, 0.6, AND 1.1, RESPECTIVELY

Effects of the capacitance coupling constant K_c and the viscous coefficient γ on the auto-correlation of system variable *V* are given in Fig.II. Fig.II (a) shows the auto-correlation function $C_V(s)$ for various capacitance coupling constant K_c . Fig.II (b) shows the normalized auto-correlation function $C_V(s)$ for various viscous coefficients. The capacitance coupling constant K_c decreases the evolution period and enhances the amplitude of the auto-correlation of the output voltage *V*. The viscous coefficient γ enhances the amplitude of the oscillatory auto-correlation.

Effects of the time constant τ_p and the noise strength D on the auto-correlation of system variable *V* are given in Fig. III. Fig .III (a) shows the normalized auto-correlation function $C_V(s)$ versus the decay time s for various the time constant τ_p . Fig. III (b) shows the normalized auto-correlation function $C_V(s)$ versus the decay time s for various noise strength D. The oscillation amplitude of the oscillatory auto-correlation increases and the oscillatory period keeps unchanged as the time constant τ_p . However, The oscillation amplitude of the oscillatory auto-correlation decreases as the noise strength D increases.

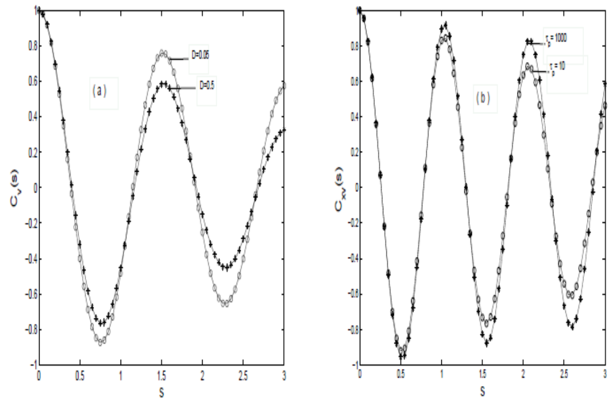

FIGURE III. THE AUTO-CORRELATION FUNCTION $C_V(s)$ VERSUS THE DECAY TIME S FOR $a=2$, $b=1$, K_C=0.05 , $K_V=100$, γ=0.6 , AND τ_p AND D TAKING THE VARIOUS VALUES. (a) τ_p=100 WITH D=0.005 AND D=0.5. (b) D=0.05 WITH τ_p=10 AND τ_p=1000

B. *Oscillatory Cross-correlations*

By the stochastic simulation, we plot cross-correlation distribution graphs of the dynamical variables of the nonlinear energy harvesting system with white noise in Figs. IV-VI.

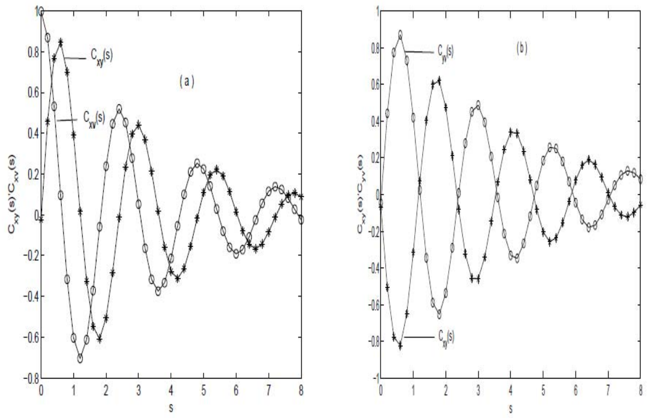

FIGURE IV. THE CROSS-CORRELATION FUNCTIONS $C_{xy}(s)$, $C_{xv}(s)$ AND $C_{yx}(s)$ VERSUS THE DECAY TIME S FOR $a=2$, $b=1$, D=0.05, γ=0.6 , $K_v=100$, K_c=0.05 , AND τ_p=100 . (a) THE CROSS-CORRELATION FUNCTIONS $C_{xy}(s)$ AND $C_{xv}(s)$. (b) THE CROSS-CORRELATION FUNCTIONS $C_{xy}(s)$ AND $C_{yv}(s)$.

Figure IV displays cross-correlation function distributions $C_{xy}(s)$, $C_{xv}(s)$, and $C_{yv}(s)$ and gives comparisons between cross-correlations. Fig.IV (a) exhibits the comparison

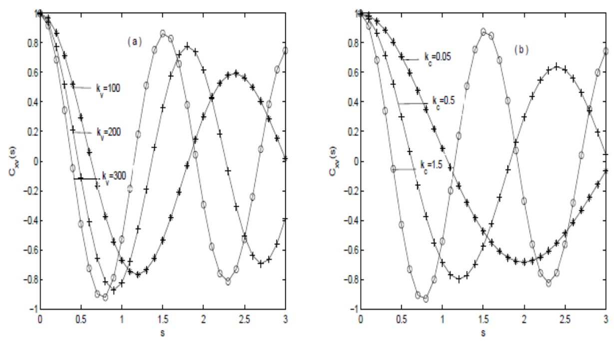

FIGURE V. THE CROSS-CORRELATION FUNCTIONS $C_{xv}(s)$ VERSUS THE DECAY TIME S FOR $a=2$, $b=1$, D=0.05, γ=0.5 , AND τ_p=100 . (a) K_c=0.05 , AND K_v TAKING 100, 200, AND 300, RESPECTIVELY. (b) K_v=10 , AND K_c TAKING 0.05, 0.5, AND 1.5, RESPECTIVELY

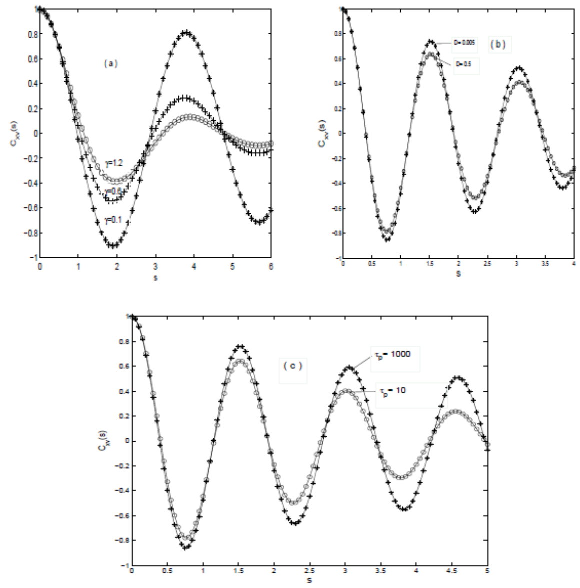

FIGURE VI. THE NORMALIZED CROSS-CORRELATION FUNCTION $C_{xv}(s)$ VERSUS THE DECAY TIME S FOR $a=2$, $b=1$, K_v=10 , K_c=0.05 , AND γ , D , AND τ_p TAKING VARIOUS VALUES. (a) D=0.05, AND τ_p=50 WITH γ=0.1 , 0.6, 1.2. (b) γ=0.6 , AND τ_p=100 WITH D=0.005, 0.5. (c) D=0.05, AND γ=0.8 WITH τ_p=10 , 1000.

between $C_{xy}(s)$ and $C_{xv}(s)$, and Fig.IV (b) gives the comparison between $C_{xy}(s)$ and $C_{yv}(s)$. Figs.V-VI also show the effects of system parameters on the cross-correlation function distribution $C_{xv}(s)$.

The cross-correlation function, like the auto-correlation function, possesses the same characteristics which periodically oscillates, gradually attenuates, finally tends to zero when the decay time increases. However, In Fig. IV (b), we see that the oscillatory phases of $C_{xy}(s)$ with $C_{xv}(s)$ are the opposite, and the oscillatory period are the same. $C_{xy}(s)$ begins at the initial value 0, monotonically reduces to a negative extremum, and then periodically oscillates and gradually attenuates with the decay time; in contrast, $C_{yv}(s)$ starts from the initial value zero, monotonically increases to a positive extremum, and then periodically oscillates and gradually attenuates with the decay time

From Figs. V-VI, we see that the voltage coupling coefficient K_v and the capacitance coupling constant K_c decrease the oscillatory period, but enhance the oscillatory amplitude of the cross-correlation; the viscous coefficient γ

and the noise intensity D decrease the oscillatory amplitude of the cross-correlation, and the oscillatory period keeps unchangeableness; however, the effect of time constant τ_p is the opposite, enhancing the oscillatory amplitude of the cross-correlation.

IV. Summaries and Conclusions

In the previous discussions, we can further see the auto-correlation and the cross-correlation of the nonlinear stochastic energy harvesting system, and effects of the system parameters on correlations and the output power of the system.

1) The auto-correlation and the cross-correlation of the nonlinear energy harvesting system with white noise show correlations of periodic oscillation and gradual attenuation, implying that the mutual effect of dynamical variables at the various time and the mutual roles between various variables at various time possess periodic change and considerable complexity.

2) Comparing the cross-correlations between $C_{xy}(s)$, $C_{xv}(s)$, and $C_{yv}(s)$, we find that the oscillatory phases of $C_{xy}(s)$ and $C_{yv}(s)$ are the opposite, however, the oscillatory period are the same. $C_{xv}(s)$ has the strongest cross-correlation strength, namely, the contact of the variables x and V is most close, so the effect of $C_{xv}(s)$ on the system output is the important.

3) The voltage coupling coefficient K_V, the capacitance Coupling K_c, and the time constant τ_p, enhance the oscillation amplitudes of the system correlation, and increase the strength of system correlations, implying that those parameters increase the system output. However, the roles of the viscous coefficient γ and the noise strength D is just opposite, decrease the system output and increase the stability of the system output.

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