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Study on Time-of-Use Power Price on Demand Side Based on Game Theory

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Abstract—**Demand response plays an important role in realizing power supply and demand balance as well as optimizing resource allocation. In an increasingly open market environment, it is necessary to study the interests of each participant in demand response and formulate a reasonable and effective response mechanism. In this paper, the incomplete information game relationship between the Power Company and its users is established to solve the problem of Time-of-Use(TOU) power price on the demand side. According to the relationship between electricity consumption and electricity price featuring multiple feedbacks, the Power Company can gradually determine the responsiveness of its users based on Bayesian law, and obtain the optimal TOU price with the goal of maximizing revenue. Finally, the rationality of the model and its advantages in fully mobilizing user enthusiasm for participation and optimizing the load curve can be verified through analysis and comparison of examples.**

Keywords—Demand response; TOU price; game theory; energy Internet strategy

I. INTRODUCTION

In the early 1980s, MIT scholars led by Fred C. Schweppe put forward the theory of the real-time power price, but the realization of the real-time power price has higher requirements on the system. The peak-valley TOU price can provide an effective guide to help users adopt reasonable power consumption, thus avoiding peak power load under the current market environment, and therefore playing an important role in reducing peak load demand. Literature [1] establishes a power market simulation model of user response, introduces lag elasticity coefficient and non-price factors aimed toward maximizing social welfare and obtains the optimal peak-valley TOU price based on user response. Literature [2] proposes a user response model based on the price elasticity matrix quantitatively calculates the comprehensive satisfaction of users and establishes a multiple evaluation index model of peak-valley TOU price and peak-to-valley TOU price.

In the development of the TOU price, it is necessary to consider the relationship between the government, the Power Company, the users and other stakeholders. Game theory is a good analytical tool [3]. Literature [4] indicates that the Power Company should focus on the demand side when formulating the demand side power price structure, and comprehensively consider the power generation cost in the power generation side. Literature [5] presents a dynamic game linkage model of TOU price on the generation side and the demand side, which can objectively distribute the peak-cutting and valley-filling benefits of the TOU price according to the market power of each link in the system.

This paper mainly considers users who do not fully disclose true information in the electricity market, and uses the dynamic game model of incomplete information to study possible TOU price changes.

II. DYNAMIC GAME OF INCOMPLETE INFORMATION BASED ON TOU PRICE

A. User Response Model

User resilience to price changes differs, that is, when the price rises in peak periods and decreases in trough periods, users adjust the structure and mode of electricity consumption. Because of varying user requirements and preferences, the adjustment space and range are different. Therefore, according to the resilience of electricity price, users are divided into three types: high resilience, medium resilience and low resilience. Users are aware of their resilience, but this information is unknown to the Power Company. In combination with the initial judgment of user resilience, the Power Company regards the power price as a transmission mechanism to guide users to adjust their electricity consumption. According to user feedback information, the Power Company revises the judgment of user resilience, and adjusts the price in combination with such judgments. The purpose of such multiple interactions is to enable the Power Company to grasp the true information of user resilience, avoid setting prices too high or too low because of ignorance of user information, and failing to give full play to the enthusiasm of users to participate in TOU pricing or reduce the electricity sales revenue of the Power Company.

Combining the users' price elasticity matrix of demand consisting of self-elasticity and cross-elasticity, electricity demand following the execution of the peak-valley price response is obtained, which is formula (1).

$$\begin{bmatrix} Q_1^n \\ Q_2^n \\ Q_3^n \end{bmatrix} = \frac{1}{3}\begin{bmatrix} Q_1 & & \\ & Q_2 & \\ & & Q_3 \end{bmatrix} E \begin{bmatrix} \frac{\Delta P_{f1}}{P_{f1}} \\ \frac{\Delta P_{f2}}{P_{f2}} \\ \frac{\Delta P_{f3}}{P_{f3}} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} \tag{1}$$

In the formula: P_{f1}, P_{f2}, P_{f3} represent prices of demand side in peak, valley and flat periods, respectively; E is price elasticity matrix;Q_1,Q_2 andQ_3 represent loads during peak, valley and flat periods before implementing the TOU price, respectively.Q^n_1,Q^n_2,Q^n_3 represent loads during peak, valley and flat periods following the nth adjustment of the power price, respectively.

B. Formulation of TOU Price on the Demand Side

The Power Company formulates the TOU price on the demand side based on the TOU price and historical load conditions, aimed at maximizing its own interests, without considering costs in transmission and distribution links. Its objective function is shown in the equation (2).

$$\max F = \sum_{i=1}^{3} P_{fi} Q_i + I_g \tag{2}$$

In the formula: i= 1, 2, 3, representing peak, valley and flat periods, respectively.Ig represents the benefit in delaying grid construction brought by the TOU price, shown in RMB.

$$I_g = \frac{L_{\max} - L_{\max}^n}{1-\tau} J\sigma \tag{3}$$

In the above formula, $L_{\max}^n$ and $L_{\max}$ represent the maximum loads of the nth and initial load curves, respectively, in MW. τ —Standby rate;J—Unit cost of power grid, RMB/MW; σ —The annuity coefficient of the power grid operation cycle.

The power price in the flat period is regarded as the average power price, and the power price in the peak period and the valley period can be expressed as:

$$P_{f1} = P_{av}(1+\mu) \tag{4}$$

$$P_{f2} = P_{av}(1-\varpi) \tag{5}$$

In the formula μ , ϖ represent the proportion of the power price rising and falling on the basis of average power price, respectively.

Meanwhile, the Power Company should meet the maximum price constraints when formulating TOU price:

$$P_{fi} \le P_{fi.\max} \tag{6}$$

In the formula: $P_{fi.\max}$ represents the maximum price limit in i period.

C. Dynamic Game Model in the Incomplete Information Market

The Power Company is faced with users who do not disclose true elasticity information, and the Power Company needs to know whether the elasticity coefficient of the power price is high or low, in order to formulate different peak and valley prices according to the elasticity. The way in which the Power Company handles incomplete user information is mainly based on Bayesian law. The specific process is as follows:

(1) According to historical information, the Power Company first forms an initial judgment on the size of user elasticity, i. e. a prior probability, which is assumed to be $P_{rob}(h)$, $P_{rob}(m)$, $P_{rob}(l)$. Among them, P_{rob} represents the probability that user elasticity belongs to a certain type, and h, m and l correspond to high elasticity, medium elasticity and low elasticity, respectively.

(2) The Power Company first gives a set of initial peak-valley level power prices. After obtaining price information, the users adjust the power consumption mode according to formula (1), and the load changes to obtain a new set of load values. The load that changes in each period includes the load that responds to the change of power price in the original period, and the load that transfers from other periods. The Power Company judges which category the user's electricity price elasticity belongs to according to the load change rate of users during each period and the change rate of the power price during each period, that is, the Power Company judges the user type according to formula (7).

$$\gamma = \frac{\sum_{i=1}^{n} \left| \frac{\delta Q_i}{\delta P_{fi}} \right|}{\mathrm{T}} \tag{7}$$

In the formula: δQ_i is the change rate of the load in the ith period;δP_{fi} is the change rate of price in the ith period; T refers to the time of day divided into T periods. Here, T is regarded as 24.

(3) Because we are not aware of the elasticity of the power price, there remains the probability that users do not really express the elasticity of electricity due to fluctuation in power consumption arrangement and electricity demand. That is, high elasticity users may also behave in a similar manner to medium elasticity ones, leading to a mistaken belief at the Power Company that they are in fact medium elasticity users. Therefore, revision of user elasticity judgment is necessary in

accordance with Bayesian law. Following the first round of power price and electricity consumption response, the Power Company can obtain the probabilities that users show high elasticity, medium elasticity and low elasticity, respectively, as follows:

$$P_{rob}(h\varepsilon)=P_{rob}(\mathrm{h}\varepsilon \mid h)*P_{rob}(\mathrm{h})+ P_{rob}(\mathrm{h}\varepsilon \mid m)*P_{rob}(\mathrm{m})+P_{rob}(\mathrm{h}\varepsilon \mid l)*P_{rob}(\mathrm{l}) \tag{8}$$

$$P_{rob}(m\varepsilon)=P_{rob}(\mathrm{m}\varepsilon \mid h)*P_{rob}(\mathrm{h})+ P_{rob}(\mathrm{m}\varepsilon \mid m)*P_{rob}(\mathrm{m})+P_{rob}(\mathrm{m}\varepsilon \mid l)*P_{rob}(\mathrm{l}) \tag{9}$$

$$P_{rob}(l\varepsilon)=P_{rob}(\mathrm{l}\varepsilon \mid h)*P_{rob}(\mathrm{h})+ P_{rob}(\mathrm{l}\varepsilon \mid m)*P_{rob}(\mathrm{m})+P_{rob}(\mathrm{l}\varepsilon \mid l)*P_{rob}(\mathrm{l}) \tag{10}$$

In the formula: $P_{rob}(h\varepsilon)$refers to the probability that elasticity that is displayed by users is high elasticity; $P_{rob}(h\varepsilon)$ refers to the probability that elasticity that is displayed by users is medium elasticity; $P_{rob}(l\varepsilon)$ refers to the probability that elasticity that is displayed by users is low elasticity; $P_{rob}(h\varepsilon|h)$, $P_{rob}(h\varepsilon|m)$ and $P_{rob}(h\varepsilon|l)$ refer to the conditional probabilities of high elasticity users, medium elasticity users and low elasticity users displaying high elasticity in terms of power price.

The definitions of conditional probability in formulas (9) and (10) are similar to those in formula (8); therefore, we will not elaborate on them here.

According to Bayesian law, the posterior probability of the category to which the user belongs can be obtained when the Power Company judges the elasticity of the user by observing the user's response, as indicated in formula (11).

$$\begin{cases} P_{rob}(h \mid h\varepsilon)=\dfrac{P_{rob}(\mathrm{h}\varepsilon \mid h)*P_{rob}(\mathrm{h})}{P_{rob}(h\varepsilon)} & \varepsilon \in h\varepsilon \\ P_{rob}(h \mid m\varepsilon)=\dfrac{P_{rob}(\mathrm{m}\varepsilon \mid h)*P_{rob}(\mathrm{h})}{P_{rob}(m\varepsilon)} & \varepsilon \in m\varepsilon \\ P_{rob}(h \mid l\varepsilon)=\dfrac{P_{rob}(\mathrm{l}\varepsilon \mid h)*P_{rob}(\mathrm{h})}{P_{rob}(l\varepsilon)} & \varepsilon \in l\varepsilon \end{cases} \tag{11}$$

In the formula, $P_{rob}(h \mid h\varepsilon)$, $P_{rob}(h \mid m\varepsilon)$ and $P_{rob}(h \mid l\varepsilon)$ refer to conditional probabilities that the users actually belong to the high elasticity category, however the Power Company considers them to be high elasticity users, medium elasticity users and low elasticity users.

The analysis of the other two cases is similar to that of high elasticity users; therefore, we will not elaborate on them here.

(4) The obtained posterior probability is taken as the prior probability of setting the next price mechanism. According to the judgment of user elasticity, the space for upward and downward fluctuations in terms of peak-valley price on the basis of parity price is set. Users with greater elasticity can have smaller upward and downward fluctuations, and users with smaller elasticity can increase their upward and downward fluctuations to stimulate users to adjust more in the power consumption structure. For the price set by the Power Company each time, the company should be aimed toward maximizing its own benefits, that is, to meet the target of Formula (2).

(5) After the users obtain price information, they shall react with their own elasticity, adjust the mode of electricity consumption, and according to the reactions of users, based on which the Power Company shall revise the judgment of the type of users, they are to readjust the price to the extent that the optimal price is determined.

III. Example Analyses

Taking residential users as an example, daily load data are shown in Table 1. The time division is as follows, in which 9:00-13:00 and 17:00-20:00 are regarded as peak periods, 13:00-17:00, 20:00-24:00 are regarded as flat periods and 24:00-9:00 is considered the valley period.

TABLE I. THE PROBABILITY OF POWER COMPANY JUDGMENT OF USER ELASTICITY

Prior Probability	$P_{rob}(h)$	$P_{rob}(m)$	$P_{rob}(l)$
	0.2	0.5	0.3
Conditional Probability	$P_{rob}(h\varepsilon\|h)$	$P_{rob}(m\varepsilon\|h)$	$P_{rob}(l\varepsilon\|h)$
	0.6	0.2	0.1
	$P_{rob}(h\varepsilon\|m)$	$P_{rob}(m\varepsilon\|m)$	$P_{rob}(l\varepsilon\|l)$
	0.3	0.6	0.3
	$P_{rob}(h\varepsilon\|l)$	$P_{rob}(m\varepsilon\|l)$	$P_{rob}(l\varepsilon\|l)$
	0.1	0.2	0.6

The basic data of the power supply side are: J= 9200rmb/kW, σ=0.0333, τ = 10%, the maximum price of TOU price =1.2rmb/kWh and the average price Pav = 0.4rmb/kWh. Residential users' electricity price elasticity coefficients at different periods are shown in Table 2, respectively.

TABLE II. RESIDENTIAL USER ELASTICITY COEFFICIENT

Elasticity Coefficient	Peak	Valley	Flat
Peak	-0.396	0.590	0.270
Valley	0.320	-0.605	0.354
Flat	0.060	0.910	-0.034

The initial peak-valley price is 0.2 higher than the peak price and 0.1 lower than valley price. According to the Power Company's judgment of user elasticity, if the probability that a given user belongs to the high elasticity category is greater than 0.5, with the increase of iterations, the peak price will increase by 0.01 on the basis of the previous price, and the valley price will decrease by 0.01 each time; if the probability that the users belong to medium elasticity category is greater than 0.5, the peak price will increase by 0.015 on the basis of the previous price with each iteration, and the valley price will decrease by 0.015. If the probability that the user belongs to the high elasticity category is greater than 0.5, the peak price will increase by 0.02 on the basis of the previous price, and the valley price will decrease by 0.02. According to formulas (4-12), user elasticity is judged. It is stipulated that when γ > 0.18, it indicates a high elastic coefficient, when γ < 0.1, it indicates low elastic coefficient and when 0.1<γ < 0.18, a medium elastic coefficient is indicated.

Following a large number of price adjustments, the fluctuation of peak and valley price of residential users is shown in Figure 1.

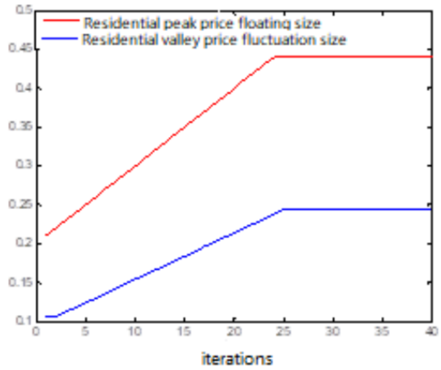

FIGURE I. CHANGE OF TOU PRICE ADJUSTMENT RATIO OF RESIDENTS WITH INCREASED NUMBER OF ITERATIONS

As can be seen from Figure 1, as the number of iterations increases, the price no longer features a trend of adjustment, and basically remains unchanged following the 25th iteration, reaching a peak price that is 0.44 higher than the parity price, while the valley price is 0.244 lower than the parity price, indicating that at about the 25th adjustment, the Power Company has clearly identified user types, and reached the value of electricity that maximizes Power Company revenue. The obtained optimal peak-valley price of residential users is shown in Table 3.

TABLE III. OPTIMAL TOU PRICE FOR RESIDENTIAL USERS

Peak period /(RMB/kWh)	Valley period /(RMB/kWh)	Flat period /(RMB/kWh)
0.576	0.302	0.400

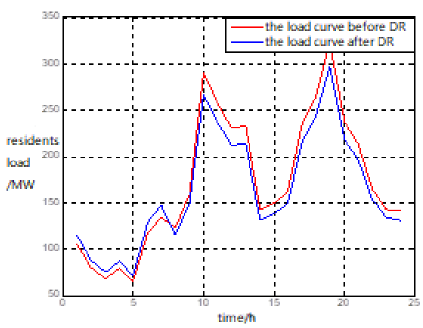

FIGURE II. LOAD CURVE BEFORE AND AFTER IMPLEMENTING PEAK-VALLEY TOU PRICE OF RESIDENTIAL LOAD

From Figure 2, it can be seen that following the implementation of the TOU price, the stability of the load curve is improved, the load during peak period is reduced and the maximum load is reduced from 322.5 MW to 296.7 MW. The reduction of the maximum load has a positive effect in reducing supply pressure on the power generation side. Resident load drops by 4 percentage points, and the load curve becomes more stable.

It can be seen that following the implementation of the peak-valley TOU price, a certain peak load shifting effect is generated, which has a certain effect on improving the utilization rate of power generation equipment and improving the safety and economic level of power grid operation.

IV. CONCLUSION

Based on TOU price formulation in peak and valley periods on the demand side, this paper discusses how a Power Company can formulate a price mechanism that maximizes user response potential, while also maximizing its own benefits, all of which is achieved without knowing the elasticity of user response. Firstly, the basic idea and solution method of dynamic game of incomplete information are introduced, providing theoretical basis for solving the latter model. The Power Company is not aware that the elasticity of each user's response to the TOU price is different. According to the prior judgment of user elasticity, the Power Company revises its judgment of user elasticity by adjusting the power price on a one-by-one basis, while also changing the load change rate and power price change rate after the user responds. In this process, the power price formulated should be able to achieve optimal Power Company revenue. This method is realized by using Matlab programming, and the validity of this method is verified by an example, which is compared with the unified TOU price without considering the difference of user elasticity, proving that the method in this paper can better tap the potential of user participation in TOU pricing, while also improving the effect of peak load shifting.

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