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Open Problems in Applications of the Kalman Filtering Algorithm

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***Abstract*—In practical application, the Kalman filter (KF) still have technical problems which have not been solved in the LDS, such as the determination of filter initial values, the slight deviation of model coefficients, the outlier or systematic deviation of measurement data and the covariance estimation of model disturbance and measurement errors. Whether the above situations affect the KF estimation and its accuracy, it is a practical problem which is high-profile and unavoidable in the application of the KF. Therefore, in this paper, take a typical linear state-space model as object, Monte Carlo method is used to simulate and verify the above technical problems are not negligible under different bias conditions. The research results tell us that it is necessary to pay much attention to the influence of the initial deviation, the model coefficient deviation and outliers or systematic errors of measurement data on the KF in the LDS.**

Keywords*— *Kalman filter; dynamic system; initial deviation; model disturbance; outliers

I. INTRODUCTION

Kalman filter (KF) is proposed by R. E. Kalman [1] in the early 1960s. It is a time-domain filtering algorithm which is based on the state-space model, and uses the relationship between state equation and measurement equation to achieve the optimal recursive estimation of system states. The KF overcomes the shortcomings of classical Wiener filter theory and method, and is suitable for on-line estimation of state changes in multivariable time-varying systems and has laid the foundation for the development of modern control theory and real-time signal processing [2].

In the application of engineering, the KF has already expanded from the optimal state estimation to automation, target positioning and tracking, communication and signal processing, digital image processing, speech signal processing, earthquake prediction and many other fields. Meanwhile, it has become one of the most common tools in the fields of information, control and process automation [3-4].

Over the decades, many modifications to the KF have been proposed to improve performance. The most significant modifications are the extended KF (EKF) for nonlinear systems [5] and the unscented KF (UKF) for highly nonlinear systems [6]. Among other important modifications are the invariant EKF for nonlinear systems with symmetry [7], the assemble KF for high dimensional systems [8], the fast KF for systems with sparse matrices [9], the robust KF for models with heavy-tailed noise or Gaussian noise mixed with outliers [10], and some particular modifications are also of interest [11-18].

Although the application of the KF is expanding constantly, there are still a series of technical problems that have not been completely solved, for example, how to set the initial value of states during the recursive process? Similar problems also occur in slight deviation of model structures, the systematic deviation of measurement data and the covariance of measurement errors. Aim to these problems, many researchers have carried out and given the corresponding solutions, such as attempt to estimate the initial value of states by adding special conditions [19-20], focus on the influence of initial deviations on the convergence of the KF [21-23], however, these issues are still not well resolved, and some issues are even overlooked. Therefore, in this paper, the simulation of typical models and the analysis of simulation results are used to analyze the above problems, and illustrate that the above problems are assignable.

II. PROBLEM FORMULATION

For the stochastic system with random disturbances $\{\xi_k \in R^n\}$ and measurement errors $\{\varepsilon_k \in R^m\}$, the state-space model usually can be expressed as [24]

$$\begin{cases} X_{k+1} = A_k X_k + B_k U_k + \xi_k \\ Y_k = C_k X_k + \varepsilon_k \end{cases} \tag{1}$$

Where the input vector $\{U_k \in R^k\}$ and the measurement vector $\{Y_k \in R^m\}$ are known, $\{\xi_k \in R^n\}$ and $\{\varepsilon_k \in R^m\}$ are usually assumed to be zero-mean white Gaussian.

As an optimal estimation method, the KF is based on the state-space model (1) and measurement data $\{Y_1,...,Y_k\}$ to estimate the state vector X_k at different time and it contains two recursion formulas:

$$\hat{X}_{k+1|k+1} = A_k \hat{X}_{k|k} + B_k U_k + K_{k+1}\left(Y_{k+1} - C_{k+1}\left(A_k \hat{X}_{k|k} + B_k U_k\right)\right) \tag{2}$$

$$P_{k|k} = \left(I - K_k C_k\right)\left(A_k P_{k-1|k-1} A_k^{\tau} + Q_k\right) \tag{3}$$

Where R_k and Q_k are the covariance matrices of ε_k and ξ_k, respectively, the filter gain K_k can be given

$$K_k = \left(A_k P_{k|k} A_k^{\tau} + Q_k\right) C_k^{\tau}\left(C_k\left(A_k P_{k-1|k-1} A_k^{\tau} + Q_k\right) C_k^{\tau} + R_k\right)^{-1} \tag{4}$$

As a recursion algorithm based on formula (2) and formula (3), the KF requires a prerequisite to know the initial value, $\hat{X}_{0|0}$ and $\hat{P}_{0|0}$, R_k and Q_k, $\{U_k\}$ and $\{Y_k\}$, and $\{A_k, B_k, C_k\}$. In general, the above problems can be divided into four basic questions:

A. Difficult to Determine Initial Values

The KF is based on recursive formulas (2) and (3) to iteratively calculate the system state. In order to ensure the accuracy of the KF, and must be set accurately. However, the initial value is usually set by experience or imagination, or assumes it is known, so as to avoid setting [25-27]. Therefore, it is necessary consider whether the size of initial values will affect the KF state in the LDS. If the initial value occurs deviation, will the difference between the KF state and the true state appear? Will the difference disappear rapidly?

B. Inaccurate System Model Structures

The KF implies an assumption that the mathematical model can describe the system process accurately. However, the mathematical model is often only the approximation of actual systems, the model coefficient matrix will be biased; there is a truncation error when nonlinear system model approximate into linear system model; it can bring about bias in the low order approximation of integral terms while analog control systems are transformed into digital control systems [28]. Therefore, it is necessary to consider that if the mathematical model is not accurate, will it affect the KF state? Will the KF state deviate from true states and produce the difference?

C. Measurement Data Outliers or Deviations

In the application of the KF, the relationship between the state vector and the measurement data is used to recursive the internal state. However, due to the complexity of measuring equipment and its working environment, the measuring equipment is difficult to measure and calibrate accurately, the measurement data will appear false data, outliers or systematic deviations inevitably. So, will outliers or deviations of the measurement data affect the effective of the KF? Will it result in appearing the difference between the KF state and true states?

D. Difficult to Determine Noise Covariance Matrices

The KF requires the noise covariance at each time index *k*. However, the noise covariance is usually difficult to determine in the actual dynamic system, therefore, it is not appropriate that the noise is assumed to be zero-mean white Gaussian roughly. If the noise, and are not satisfy the stationary white noise with Gaussian distribution, will the KF state deviate from the true state and produce the difference?

In this paper, we focus on the discussion and analysis of the first three basic issues, the last issue will be discussed in the subsequent study.

III. Analysis of Initial Deviation Effect on KF

A. Theoretical Analysis

Do not consider the control input vector $\{U_k \in R^k\}$, the formula (2) can be rewritten as:

$$\hat{X}_{k+1|k+1} = A_k \hat{X}_{k|k} + K_{k+1}\left(Y_{k+1} - C_{k+1} A_k \hat{X}_{k|k}\right) \tag{5}$$

The difference between the formula (5) and the true state X_{k+1} can be expressed as:

$$\hat{X}_{k+1|k+1} - X_{k+1} = A_k \hat{X}_{k|k} + K_{k+1}\left(Y_{k+1} - C_{k+1} A_k \hat{X}_{k|k}\right) - X_{k+1} \tag{6}$$

Assuming $\Delta\hat{X}_{k+1} = \hat{X}_{k+1|k+1} - X_{k+1}$, it can be obtained:

$$\Delta\hat{X}_{k+1} = \left(A_k - K_{k+1} C_{k+1} A_k\right)\Delta\hat{X}_k + \left(K_{k+1} C_{k+1} A_k - I\right)\xi_k + K_{k+1}\varepsilon_{k+1} \tag{7}$$

Introducing marks $M_k = A_k - K_{k+1} C_{k+1} A_k$ and $N_k = I - K_{k+1} C_{k+1} A_k$, the formula (7) can be written:

$$\begin{aligned}\Delta\hat{X}_{k+1} = &\prod_{i=0}^{k} M_{k-i}\left(\hat{X}_{0|0} - X_0\right) \\ &- \sum_{i=0}^{k-1}\prod_{j=i+}^{k} M_{k+1-j}\left(N_i \xi_i\right) - N_k \xi_k + \sum_{i=0}^{k}\prod_{j=0}^{k} M_{k-j}\left(K_i \varepsilon_i\right) + K_{k+1}\varepsilon_{k+1}\end{aligned} \tag{8}$$

The average of the formula (8) can be obtained:

$$E\left(\Delta\hat{X}_{k+1}\right) = \prod_{i=0}^{k} M_{k-i}\left(\hat{X}_{0|0} - X_0\right) \tag{9}$$

The formula (9) shows that the filter initial deviation can obviously affect the KF state at the initial time, and the KF state will converge to the true state after the moment $M_k = 0$.

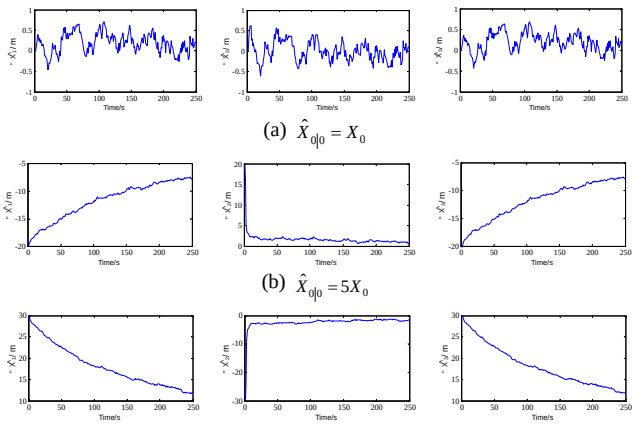
B. Analysis of Simulation Results

To ensure the typical and general of system process and simulation results, take the model (1) of the LDS as the object, select the state-space model as follow:

$$\begin{cases} X_{k+1} = \begin{bmatrix} 1+a & b & 0 \\ 0 & 1+a & -b \\ 0 & 0 & 1+a \end{bmatrix} X_k + \begin{bmatrix} 0.5 & 0.1 & -0.6 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} \\ Y_k = \begin{bmatrix} c & 0 & -c \\ 0 & d & 0 \end{bmatrix} X_k + \begin{bmatrix} 0.1 & -0.1 \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \end{cases} \tag{10}$$

In order to well reflect the influence of filter initial deviations on the KF in the LDS, assume the initial value, $X_0 = (5.0,-5.0,5.0)^T$ and $\hat{P}_{0|0} = I_3$, select different filter states value ($\hat{X}_{0|0} = X_0$, $\hat{X}_{0|0} = 5X_0$ and $\hat{X}_{0|0} = -5X_0$), compare the difference between KF states and true states in four conditions.

1) Effect of Filter Initial Deviations on KF in Stable Systems: Setting model parameters $(a,b,c,d) = (-0.01,0.01,0.9,0.05)$, the result is shown in Fig. 1.



(a) $\hat{X}_{0|0} = X_0$

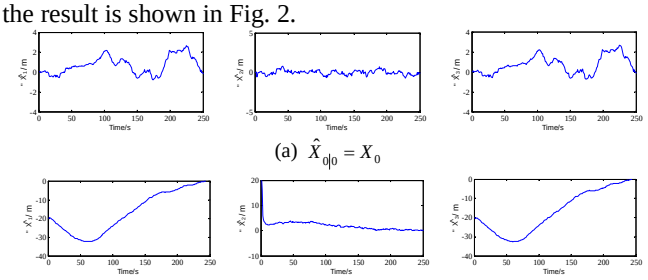
(b) $\hat{X}_{0|0} = 5X_0$

(c) $\hat{X}_{0|0} = -5X_0$

FIGURE I. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF FILTER INITIAL STATES IN THE STABLE SYSTEM.

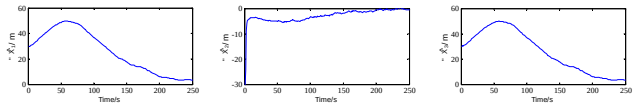
Fig. 1 shows that the initial deviation of filter states will lead to the KF state deviate from the true state at the initial time, and the larger the initial deviation is, the larger the initial difference; the KF state will continue to deviate for a long time.

2) Effect of Filter Initial Deviations on KF in Unstable Systems: Setting model parameters $(a,b,c,d) = (0.01,0.01,0.9,0.05)$, the result is shown in Fig. 2.



(a) $\hat{X}_{0|0} = X_0$

(b) $\hat{X}_{0|0} = 5X_0$

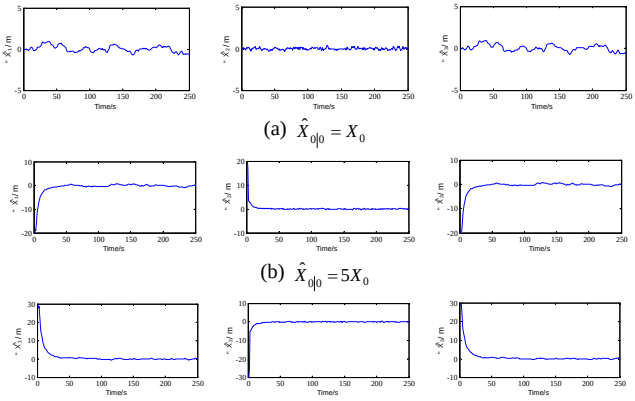


(c) $\hat{X}_{0|0} = -5X_0$

FIGURE II. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF FILTER INITIAL STATES IN THE UNSTABLE SYSTEM.

Fig. 2 shows that the initial deviation of filter states will lead to KF states deviate from true states at the initial time, and after the difference reaches the maximum, the KF state starts to tend to the true state gradually.

3) Effect of Filter Initial Deviations on KF in Observable System: Setting the model parameters $(a,b,c,d) = (-0.01,0.1,0.9,0.05)$, the result is shown in Fig. 3.



(a) $\hat{X}_{0|0} = X_0$

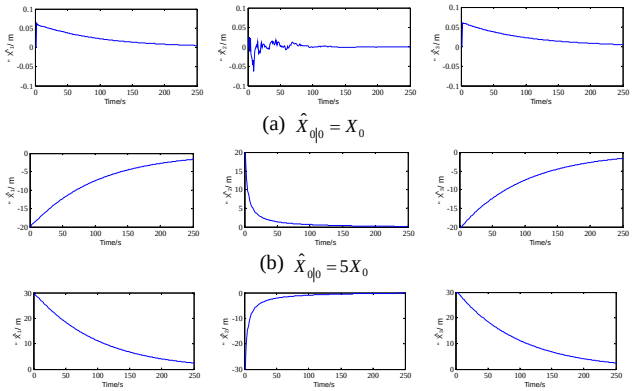
(b) $\hat{X}_{0|0} = 5X_0$

(c) $\hat{X}_{0|0} = -5X_0$

FIGURE III. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF FILTER INITIAL STATES IN THE OBSERVABLE SYSTEM.

Fig. 3 shows that the initial deviation of filter states will result in the KF state diverging the true state at the initial time; the KF state will continue to deviate for a long time, and then converge to the true state quickly.

4) Effect of Filter Initial Deviations on KF in Unobservable System: Setting the model parameters $(a,b,c,d) = (-0.01,0.0,0.9,0.05)$, the result is shown in Fig. 4.



(a) $\hat{X}_{0|0} = X_0$

(b) $\hat{X}_{0|0} = 5X_0$

(c) $\hat{X}_{0|0} = -5X_0$

FIGURE IV. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF FILTER INITIAL STATES IN THE UNOBSERVABLE SYSTEM.

Fig. 4 shows that the initial deviation of filter states will result in the KF state deviating from the true state at the initial time, the KF state will continue to deviate for a long time, and then start to converge to the true state gradually.

IV. Analysis of Slight Deviations from Model Structure Effect on KF

A. Theoretical Analysis

Suppose the measurement data is constant, set the initial value $\hat{X}_{0|0} = X_0$ and $P_{0|0} = P_0$, take two sets of model coefficient are $\{A_k^a, C_k^a\}$ and $\{A_k^b, C_k^b\}$, assume $\Delta\hat{X}_{k|k} = \hat{X}_{k|k}^a - \hat{X}_{k|k}^b$, $\Delta A_k = A_k^a - A_k^b$, $\Delta C_k = C_k^a - C_k^b$ and $\Delta K_k = K_k^a - K_k^b$, the difference can be given.

$$\Delta\hat{X}_{k+1|k+1} = \left(A_k^a - K_{k+1}^a C_{k+1}^a A_k^a\right)\Delta\hat{X}_{k|k} + \left[\left(I - \Delta K_{k+1} C_{k+1}^b\right)\Delta A_k + K_{k+1}^a \Delta C_{k+1} A_k^a\right]\hat{X}_{k|k}^b + \Delta K_{k+1} Y_{k+1} \tag{11}$$

Take the sign $M_k^a = \left(A_k^a - K_{k+1}^a C_{k+1}^a A_k^a\right)$ and $N_k^b = \left(I - \Delta K_{k+1} C_{k+1}^b\right)$, the formula (13) can be rewritten as

$$\Delta\hat{X}_{k+1|k+1} = \prod_{i=0}^{k} M_{k-i}^a \left(\hat{X}_{0|0}^a - \hat{X}_{0|0}^b\right) + \sum_{i=0}^{k-1}\prod_{j=i+1}^{k} M_{k+1-j} W_i + W_k \tag{12}$$

Where $W_k = \left[N_k^b \Delta A_k + K_{k+1}^a \Delta C_{k+1} A_k^a\right]\hat{X}_{k|k}^b + \Delta K_{k+1} Y_{k+1}$

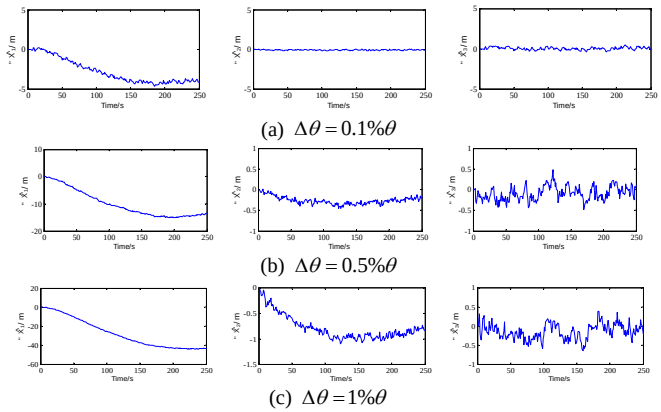
$$\Delta\hat{X}_{k+1|k+1} = \sum_{i=0}^{k-1}\prod_{j=i+1}^{k} M_{k+1-j} W_i + W_k \tag{13}$$

In the formula (13), the W_k is the function of model coefficient deviation $\{\Delta A_k, \Delta C_k\}$, thus, the deviation of model coefficients will affect the estimation of the KF.

B. Analysis of Simulation Results

In order to show the effect of slight deviations from model structures on the KF in the LDS, assume the initial value $\hat{X}_{0|0} = X_0 = (5.0, -5.0, 5.0)^\tau$ and $\hat{P}_{0|0} = I_3$, select different slight deviations ($\Delta\theta = 0.1\%\theta$, $\Delta\theta = 0.5\%\theta$ and $\Delta\theta = 1\%\theta$), compare the difference between KF states and true states in different conditions.

1) Effect of Slight Deviations from Model Structure on KF in Stable Systems: Setting model parameters $\theta = (a, b, c, d) = (-0.01, 0.5, 0.8, 0.5)$, the result is shown in Fig. 5.



(a) $\Delta\theta = 0.1\%\theta$

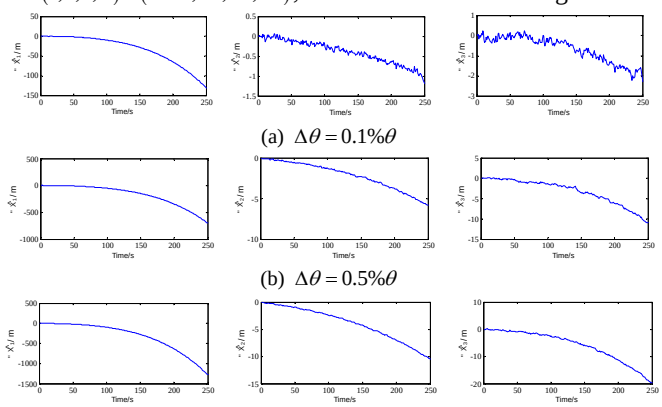
(b) $\Delta\theta = 0.5\%\theta$

(c) $\Delta\theta = 1\%\theta$

FIGURE V. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF MODEL STRUCTURE DEVIATIONS IN THE STABLE SYSTEM.

Fig. 5 shows that the slight deviation of model structures may lead to the difference reaches the maximum over time, and then KF states start to converge to true states slowly; the larger the slight deviation is, the more obvious the departure degree.

2) Effect of Slight Deviations from Model Structure on KF in Unstable Systems: Setting the model parameters $\theta = (a, b, c, d) = (0.005, 0.5, 0.8, 0.5)$, the result is shown in Fig. 6.



(a) $\Delta\theta = 0.1\%\theta$

(b) $\Delta\theta = 0.5\%\theta$

(c) $\Delta\theta = 1\%\theta$

FIGURE VI. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER THREE CASES OF MODEL STRUCTURE DEVIATIONS IN UNSTABLE SYSTEMS.

Fig. 6 shows that the slight deviation of model structures may lead to KF states deviate from true states slowly; with the

passage of time, the difference is getting bigger and bigger, and finally will never converge to zero.

V. Analysis of Outliers and Deviations from Measurement Data Effect on KF

A. Theoretical Analysis

Take two sets of measurement data $\{Y_k^a, Y_k^b\}$, assume $\Delta Y_k = Y_k^a - Y_k^b$, the difference can be given

$$\Delta\hat{X}_{k+1|k+1} = (A_k - K_{k+1}C_{k+1}A_k)\Delta\hat{X}_{k|k} + K_{k+1}\Delta Y_{k+1} \tag{14}$$

$$\Delta\hat{X}_{k+1|k+1} = \sum_{i=0}^{k}\prod_{j=0}^{k} M_{k-j}(K_i\Delta Y_i) + K_{k+1}\Delta Y_{k+1} \tag{15}$$

The formula (15) shows that the inaccurate measurement data will affect the estimation of the KF, and its different sizes will cause the different degree of estimation errors.

B. Analysis of Simulation Results

In order to well reflect the influence of inaccurate measurement data on the KF, the model (10) is used to simulate and analyze in two cases of outliers and systematic deviations.

1) Effect of Outliers from Measurement Data on KF in the LDS

Setting model parameters $(a,b,c,d) = (-0.02, 0.5, 0.8, 0.5)$, $Q = diag(0.5^2, 0.1^2, 0.6^2)$ and $R = diag(0.01^2, 0.1^2)$. Select different time points $(Y_{60}, Y_{140}, Y_{220})$, and set the abnormal amplitudes ($\Delta Y_{60} = (-20, 20)^\tau$, $\Delta Y_{140} = (-50, 50)^\tau$ and $\Delta Y_{220} = (-70, 70)^\tau$). The simulation result is shown in Fig. 7~8.

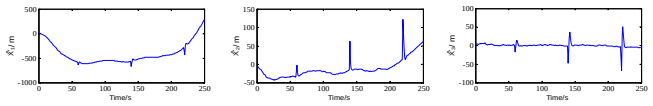


FIGURE VII. COMPUTATION RESULT OF KF STATES UNDER OUTLIERS

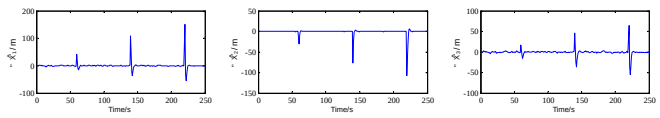


FIGURE VIII. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER OUTLIERS FROM MEASUREMENT DATA

Fig. 7~8 show that outliers will lead to KF states deviate from true states in a period of time, and the difference will reach the maximum at outliers' point; the larger the outliers is, the longer the influence of the KF state.

2) Effect of Systematic Deviations from Measurement Data on KF in Observable Systems

Setting model parameters $(a,b,c,d) = (-0.02, 0.5, 0.8, 0.5)$, select different systematic deviations, the simulation result is shown in Fig. 10.

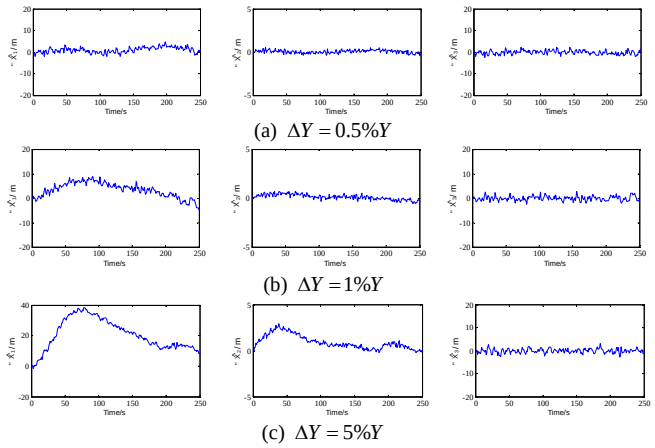


FIGURE IX. THE DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER DIFFERENT DEVIATION OF MEASUREMENT DATA IN OBSERVABLE SYSTEMS.

Fig. 9 shows that systematic deviations will lead KF states to deviate from true states, the larger the systematic deviation is, the more obvious the departure degree and the longer the departure time.

3) Effect of Systematic Deviations from Measurement Data on KF in Unobservable Systems: Setting model parameters $(a,b,c,d) = (0.005, 0., 0.8, 0.5)$, the result is shown in Fig. 11.

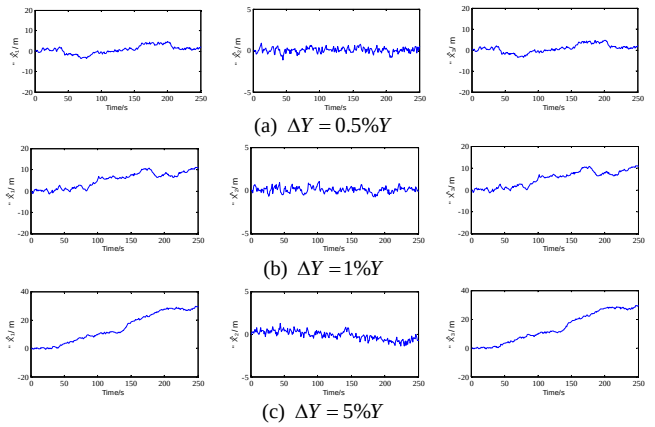


FIGURE X. DIFFERENCE BETWEEN KF STATES AND TRUE STATES UNDER DIFFERENT DEVIATION OF MEASUREMENT DATA IN UNOBSERVABLE SYSTEM.

Fig. 10 shows that the systematic deviation will result in the KF state obviously deviating from the true state, and the KF state will never converge to the true state over time.

VI. Conclusion

In this paper, from the practical application of the KF, a brief description of the four basic problems, take the LDS with three-dimensional state and two-dimensional observation as an object, set different initial deviations, the state-space model of the LDS is adopted to simulate and analyze the influence of the first three basic problems on the KF state.

The comparison and analysis of simulation results can be obtained as follow: (1) the initial deviation of filter states may lead to the KF state deviate from the true state for a long time; (2) when the slight deviation of model structures causes the KF state to deviate for a long time, it will slowly converge in the stable system; (3) outliers may lead to the KF state deviate from the true state after the presence of outliers, and with the increase of the outlier, the time that affects the KF state may become longer and longer; (4) when the systematic deviation of measurement data cause the KF state to appear obviously deviate, it will never converge in unobservable systems.

Therefore, in the actual application of the LDS, we must pay attention not to blindly use the KF algorithm and ignore the above four basic problems, otherwise, it can not only obtain the well effective of the filter state estimation, but also affect its reliability and precision.

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