

# Instrumental and Relational Understanding Analysis of 5<sup>th</sup> Grade Elementary School Students on Integers Addition

Dwi Priyo Utomo  
University of Muhammadiyah Malang  
dwi.priyo.umm@gmail.com

**Abstract.** This research aims to analyze and describe students' instrumental and relational understanding of the addition of integers. The research employs qualitative research with a descriptive approach. The subjects in the research were three students of the 5<sup>th</sup> grade at SD Insan Amanah, Malang, Indonesia: one subject with high mathematical abilities, one subject with moderate ability, and one subject with low ability. The instrument employed in this research is a test to measure subject's instrumental and relational understanding and interview guidelines to support the test data. The collected data was analyzed by using interactive techniques, which included data reduction, data presentation, and conclusion drawing. The results showed that students with high mathematical abilities had a high relational understanding because the student met all indicators of relational understanding, while the students who got good relational understanding met most of the indicators of relational understanding while low-ability students only met half the indicators of relational understanding. Students with average understanding is considered to wrongly understanding the conceptions while low-ability students have misconceptions and procedural errors.

**Keywords:** *Instrumental Understanding, Relational Understanding, Misconception, Procedural Error*

## INTRODUCTION

Skemp describes a relational understanding of knowing what and why it must be done, it can be interpreted that relational understanding is the understanding of students in knowing which procedures to be used and the reasons for each step in the procedure that will be carried out [1]. Relational thinking is not only focused on procedures for calculating answers but also in learning and teaching this understanding will increase students' understanding of Mathematics learning [2]. There are four benefits of relational understanding in Mathematics learning, including: (1) being facility in solving more complex problems; (2) making mathematical concepts easier to remember and understand; (3) making easier to understand and achieve learning goals; (4) becoming an understanding that is capable of creating original ideas [3].

Given the benefits of rational understanding, it is expected that students are able to develop that

understanding. In contrast, learning to foster relational understanding is very difficult because it must build a structure of concepts from a principle that produces many unlimited plans to produce a concept [4]. Growing relational understanding requires time, but relational understanding is able to develop four things including (1) developing a correct understanding of mathematical concepts; (2) training students to normally see the problem as a whole; (3) developing skills in using mathematical principles and concepts; (4) developing inductive abilities [2].

Research related to relational understanding includes the research carried out by [5] revealed that the high subject and the medium subject are having a relational understanding of the concept of limit, while the low subject has an instrumental understanding of the same concept. The research carried out by [6] revealed that Junior High School 2, Buduran, Sidoarjo students could solve the problem with the right concept but students did not know the reason or basis of each step taken. This shows that the relational understanding of students who are capable of high, medium and low-level mathematics is still lacking.

Some of the reasons why this research is important to be conducted include: (1) based on the exposure of aforementioned previous researches, no research has been found analyzing on a relational and instrumental understanding of integer addition in elementary schools, and (2) considering the importance of relational understanding and instrumental learning in Mathematics.

## METHOD

This research examined students' relational and instrumental understanding of the addition of integers in grade 5 elementary school. Data obtained during the research then analyzed and described in the discussion. The approach used in this research is a qualitative approach with a type of descriptive research. The subjects in this research were 3 students in grade 5 at SD Insan Amanah Malang Indonesia. One subject has high ability, one subject has a moderate ability, and one subject has low mathematical ability.

The data in this research are in the form of tests that are used to find out procedural and conceptual knowledge on the addition of integers as adaptations to the details of indicators of relational understanding presented by [7]. The

procedure of this research is divided into three stages, the first stage is preparation, the second stage is implementation and the third stage is preparing the report. Data processing techniques in this research is following three activities, namely data reduction, data presentation, and conclusion

## RESULT

This research aims to analyze and describe the relational understanding and instrumental understanding of students in integer addition operations. The indicators of relational understanding are (1) the ability to carry out the procedure as a whole; (2) the fluency in performing procedures; (3) the ability to obtain the right results; (4) the ability to show that they are capable of performing procedures; (5) the ability to knowing when to use the procedure; (6) The knowledge of the prerequisites needed in carrying out procedures; (7) the ability to know errors in procedures; (8) the ability to provide a reasonable argument in using procedures; (9) the ability to recognize the form of new questions that can be solved using the procedure.

### Student S1

Subject S1 is able to mention what is known, what is asked, how to solve the problem and make the Mathematical model. After that, the subject completes the model and then answers the problem well. This is also supported by interview data. The subject is able in performing the procedure and performing quick problem-solving. There are no obstacles when the subject is working on the problem. The subject also gets the correct results at each stage, both at the stage of delivering information that is known, the results of the settlement, as well as the answer to the problem. The subject showed the ability to explain how the procedure was resolved and seen in the interview conducted.

The subject knows when to use the procedure in the operation to calculate precisely that the initial temperature is Celsius then calculates the increase in temperature per 3 minutes by dividing 15 by 3 multiplied by 2. The mathematical model that the subject gets namely 3. In accordance with the question on the subject matter answering with 3 degrees Celsius. Subject S1 has the prerequisite knowledge needed when carrying out integer counting operations. It is seen that subjects have prerequisite knowledge, namely integers, time units, temperature units, addition, subtraction, multiplication, and division operations. According to the results of the interview also shows that the subject understands the knowledge of the prerequisites for solving the problem correctly. The subject did not make a mistake in the counting operation procedure. The results of the interview indicated that the S1 subject knew that there were no errors in the counting operation procedure performed. The subject is able to provide a logical argument in performing an integer addition procedure, which is to provide a logical explanation for asking the question what temperature is in the next 15 minutes. Then S1 explains the procedure. The

subject recognizes the form of a new question that can be solved using an integer operating procedure, according to him the question is a new problem that has never been encountered. Subject S1 recognizes that one question can be solved by counting operations because the question involves addition and multiplication. The results of the interview also showed that S1 subjects were able to recognize the form of a new problem that could be solved using an operating procedure to calculate integers correctly.

### Student S2

The S2 subject is able to perform the procedure correctly. The subject mentioned what was known but did not mention what was asked, but able to make mathematical models. After that, S2 completes the model and then answers the problem correctly, reinforced by interview data. The subject is also fluent in performing procedures. Problem-solving is relatively fast with no obstacles when working on the problem. The subject mentioned what was known but did not mention what was asked and made the mathematical model. After that, the subject completes the model and then answers the problem reinforced by interview data which shows that the subject has no problem in performing the procedure. The subject also gets the correct results at each stage, both at the stage of the information that is known, the results of the settlement, and the answer to the problem and can demonstrate its ability to explain how the procedure is resolved. The subject already knows when to use the procedure in the operation to calculate exactly that the initial temperature is  $-7^{\circ}\text{C}$  then calculate the increase in temperature per 3 minutes by dividing 15 by 3 multiplied by 2. The mathematical model that the subject gets is  $-7 + 10$  that is 3. In accordance with the question on the subject answered with 3 degrees Celsius. The subject shows the prerequisites knowledge when performing an integer counting operation as seen from the interview conducted. The subject did not show any errors in performing the counting operation procedure and was able to provide a logical argument in carrying out the integer count operation procedure, which provided a logical explanation for the question asking what temperature was in the next 15 minutes. Then the subject explains the procedure.

The subject is able to recognize the form of a new question that can be solved using an integer operating procedure, according to the subject, the question given is a new problem that has never been encountered previously. The subject recognizes that the question can be solved by a calculation operation because the question concerns the addition and multiplication supported by the interview that has been conducted.

### Student S3

S3 subject does not show the ability to perform the procedure correctly. The subject was also unable to mention what was known, what was asked, how to solve the problem. The subject tried to make a model but was wrong. This is confirmed by interview data. Subjects are

also not fluent in performing procedures. S3 does not specify what is known, what is asked, and how to solve the problem as supported by interview data which shows that the subject is not fluent in performing procedures. The subject gets results that are incorrect at each stage, both at the stage of delivering information that is known, the results of the settlement, and the answer to the problem also shows his inability to explain how the procedure is resolved.

The subject does not know when to use the procedure in the operation to calculate exactly that the initial temperature is  $-7^{\circ}\text{C}$  then calculates the increase in temperature per 3 minutes. The subject adds the calculation to the first temperature  $-7$  and continuously adding  $-7$  for  $15/2$  times (normally  $15/3$  times). Based on the aforementioned facts, it can be seen that the subject does not have the necessary prerequisite knowledge when carrying out integer addition operations and makes mistakes in performing the count operation procedure. The subject gives an illogical argument in performing an integer addition operation procedure. The subject is also not able to recognize the form of a new question that can be solved using an integer addition operation procedure. The subject is not able to recognize the form of a new problem that can be solved using an integer addition operation procedure. Subjects do not have a relational understanding of integer operations. This conclusion is based on the results of student work analysis on the answer sheet. Almost all indicators of relational understanding are not fulfilled. The evidence of subject's error on the answer sheet is supported by the results of interviews with the subject. The inability of subjects to meet indicators of relational understanding provides evidence that S3 is included in the category of instrumental understanding.

#### CONCLUSION

Based on the research findings, S1 subject has excellent relational understanding, while S2 subject has good relational understanding, and S3 subject does not have adequate relational understanding. A student with high mathematical abilities has excellent relational understanding and able to perform operating procedures to calculate integers accurately and in the end can solve the problem correctly. The student has the ability in knowing when to use calculating operating procedures appropriately, knowing the prerequisite knowledge needed in performing procedures, knowing errors in procedures, giving logical arguments in performing procedures, and recognizing new forms of problems that can be solved using integer counting operations procedures correctly.

Student with moderate mathematical abilities has a good relational understanding and the ability to carry out accurate operating procedures, is not fluent in performing procedures, gets the right results, shows the able to perform procedures well, knows when to use procedures,

knows the prerequisite knowledge needed to perform procedures, knows errors in the procedure inappropriately, gives a logical argument in conducting an induction procedure, and recognizes the form of a new question that can be solved using an induction procedure with less degree than the student with high ability.

Student with low mathematical abilities has a low relational understanding and showing the inability to do an induction procedure, is not fluent in performing induction procedures, unable to obtain results, unable to perform procedures, inaccuracy in using the induction procedure, incorrectly perform the prerequisite knowledge needed to do induction procedure, knowing errors in induction procedures, giving illogical arguments in conducting induction procedures, and incorrectly recognizing the shape of new questions that can be solved by using the induction procedure.

For researchers who want to conduct research on relational understanding are expected to be able to integrate the research with other variables such as methods that can improve students' relational understanding or use media to improve relational understanding. This research is not yet perfect, so the researcher expects the future researchers interested in discussing relational understanding to take larger samples.

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