

Determination of Distribution Route using Linear Programming Model (Case Study at Washing Jeans Company)

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Abstract. Distribution activity is important factor to determine service performance from producer to customer. One of distribution problems is about route determination. Some researches explain about Vehicle Routing Problem (VRP). VRP is a development of Traveling Salesman Problem (TSP) with various modifications such as capacity and time constraints. This research try to use linear programming model to solve the problem of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) based on Dell'Amico, et al (2007) model with some adjustments such as the index period and some functions in cost. Branch and bound method will be used to solve linear programming model and will give the best route with minimum total distribution cost. Case study at washing jeans company which is used for numerical example aim to determine distribution route which could give minimum total distribution cost and time to company. Distribution costs consist of travel cost and overtime cost. Based on calculation process, washing jeans company will get total distribution cost about IDR 160927 per week and actual route which give cost about IDR 181525 per week. The total saving cost about 20598 IDR per week or 11.35% per week.

Introduction

Supply chain management is a complete cycle chain management starting from raw materials from various suppliers, followed by operational activities in the company, and continues with distribution to reach the customers. In supply chain management, one factor that creates competitive advantage is distribution activity. Distribution refers to the steps taken to move and store a product from supplier stage to a customer stage in the supply chain. Distribution is a key driver of the all profitability of a firm because it affects both the supply chain cost and the customer experience directly [1].

Market competition and changes in customer demand make distribution and transportation networks important to be noticed. There are three product distribution strategies from producers to customers: direct shipment, shipping through warehouse, and cross-docking [2]. Determination of distribution routes will determine the length of time and the amount of distribution costs that must be spent by the company. There are many obstacles in distribution process such as variable demand, limited capacity of vehicle, and different geographic customers. In this study we will discuss the problem of determining the distribution route between two entities, company and customer, using one of the VRP models. The model is CVRPTW which is described in the linear programming model and solved using branch and bound optimization methods. The aim to be achieved in this study is to minimize the distance and time needed to carry out distribution activities with minimum total distribution cost.

The following part will describe about methodology which explain about VRP research from time to time and model with adjustment that used in this research. The next part explain about problem formulation which consists of mathematical model and numerical example with calculation process. The last part describe about conclusion of this research.

Methodology

VRP was first introduced by Dantzig and Ramser [3]. VRP is defined as a way of searching for efficient use of a number of vehicles that must travel to visit a number of places to deliver and pick up people or goods [4]. VRP is a development of TSP and has four main components, work networks (links), costumers, depots, vehicles, and drivers. VRP problems are combinatorial problems that are

included in the NP-Hard Problem (Non Polynomial Hard Problem) category, which means that increasing the size of the problem will increase the level of difficulty in computing and problem solving. In general, the purpose of solving this VRP problem is to minimize vehicle mileage and the total time needed for each vehicle to carry out distribution activities [5].

One of the VRP models is Capacitated Vehicle Routing Problem (CVRP). CVRP is a VRP which is given a number of vehicles with separate capacities that must serve a number of known customer requests for one commodity from a depot with minimum transit costs. Another model of VRP is Vehicle Routing Problem with Time Window (VRPTW), almost the same as VRP but has an additional limit, such as period of time which relates to each customer, which defines a period of time in which the customer must be supplied, the time interval at the depot as a limit scheduling. The aim of this model is to minimize the number of vehicles and the total travel time and waiting time needed to supply all customers at certain hours.

The combination of CVRP and VRPTW is called Capacitated Vehicle Routing Problem with Time Window (CVRPTW) which has one central and one or more identical vehicles. The main difference with CVRP is the time limit in the form of working hours available from the company or opening and closing hours of customers served, so that in CVRPTW problems there are two objectives, to minimize the distance and minimize the time needed to carry out distribution activities [5]. The mathematical model used in this study is model about Fleet Size and Mix Vehicle Routing Problem which is part of the CVRPTW [6], but some adjustments have been made such as the period index and some functions. The model is a linear programming model that has objective functions and constraint functions. Linear programming can be applied in modeling for various types of problems in planning, route design, scheduling, assignment, and design.

There are many methods that can be used to solve VRP problems in either heuristic or optimization ways. Kolen et al (1987) used the branch and bound method to solve VRP problems [7]. Branch and bound method is an algorithm used in optimization problems. The branch and bound method was first introduced by Land and Doig (1960). The basic idea is to divide the feasible solution area into smaller feasible solution areas. The solution search for this algorithm starts by describing the candidate solutions to be branched using a search tree where each node describes a possible solution. These smaller nodes can then be systematically evaluated until the best solution is found [8].

This research try to use linear programming model to solve the problem of Capacitated Vehicle Routing Problem with Time Windows (CVRPTW) based on Dell'Amico, et al (2007) model with some adjustments such as the index period and some functions in cost. The adjustments are done because of company's characteristic which have demand, distribution route, and working hours per period. Basic Dell'Amico, et al (2007) model considered about handling time, the starting time and ending time of its route. To simplify the adjustments to the Dell'Amico et al (2007) mathematical model used, an influence diagram is made which can be seen in Figure 1.

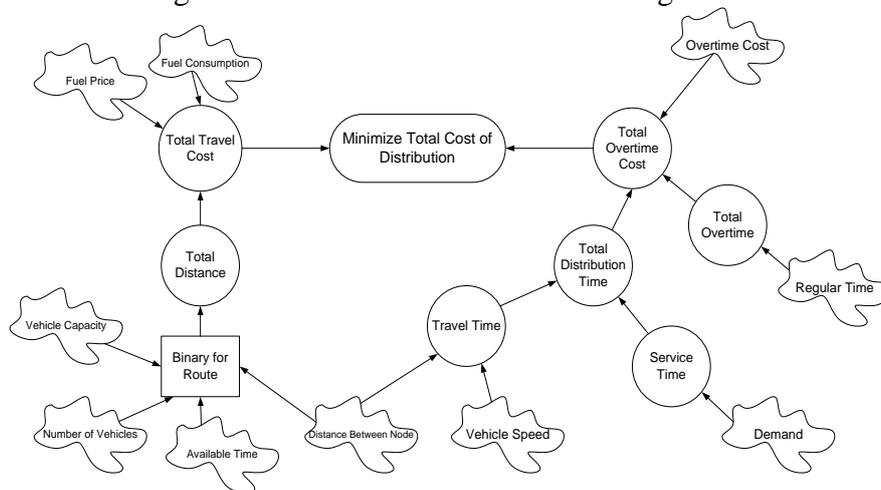


Fig. 1. Influence Diagram

Problem Formulation

This section will describe the notation begin with index, decision variables, parameters, variables, objective function, and constraints. Node 0 is denoted for central. Each route will start from central, visit customer node and return to central. There is an assumption for this model, if the demands in one period exceed vehicle capacity, then the excess will be transferred to the next distribution period.

Index:

- i = beginning position/node ; $i = \{0,1,\dots,N\}$
- j = destination position/node ; $j = \{0,1,\dots, N\}$
- k = vehicle ; $k = \{1,\dots,K\}$
- p = distribution period; $p = \{1,2,\dots, P\}$

Decision Variable:

$$X_{ijkp} = \begin{cases} 1, & \text{if vehicle } k \text{ is used from } \textit{node} \ i \text{ to } j \text{ in period } p \\ 0, & \text{if vehicle } k \text{ isn't used from } \textit{node} \ i \text{ to } j \text{ in period } p \end{cases}$$

Parameters:

- m = big integer number = 100000
- v = vehicle speed
- cb = fuel cost
- ck = fuel consumption of vehicle
- C_{fix} = overtime cost
- a_{kp} = customer's opening hour
- b_{kp} = customer's closing hour
- tw_{kp} = available working hours for vehicle k in period p

Variables:

- N = total location of central and customers
- K = total available vehicle
- P = distribution period
- C_{ijkp} = distribution cost from node i to node j by vehicle k in period p
- D_{ijp} = distance from node i to node j in period p
- M_{ip} = total demand at node i in period p
- T_{ijkp} = travel time from node i to node j for vehicle k in period p
- ST_{ip} = service time (unloading) at node i in period p
- Q_{kp} = capacity of vehicle k in period p
- AT_{ikp} = time when vehicle k arrived at node i in period p
- AT_{jkp} = time when vehicle k arrived at node j in period p
- SS_{ip} = time when node i begin to be served in period p
- J_{kp} = total service time vehicle k in period p
- $Y_{ikp} = \begin{cases} 1, & \text{if node } i \text{ to be served by vehicle } k \text{ in period } p \\ 0, & \text{if node } i \text{ not to be served by vehicle } k \text{ in period } p \end{cases}$

Objective Function:

Minimize $C_{ijkp} * X_{ijkp}$

Which:

$$C_{ijk} = \text{Travel Cost} + \text{Overtime Cost} = \left[\left(\sum_{i=0}^N \sum_{\substack{j=0 \\ j \neq i}}^N \sum_{k=1}^K \sum_{p=1}^P D_{ijp} * X_{ijkp} \right) * cb * ck \right] + \left[\left(\sum_{k=1}^K \sum_{p=1}^P J_{kp} - tw_{kp} \right) * C_{fix} \right] \tag{1}$$

Subject to:

The total number of products carried does not exceed the capacity of vehicle:

$$\sum_{i=1}^N M_{ip} * Y_{ikp} \leq Q_{kp} \quad \text{for } k = \{1,2\} ; p = \{1,2,3\}$$

The number of vehicles departs and returns to the central (node 0) does not exceed the number of vehicles available:

$$\sum_{k=1}^K Y_{ikp} \leq K \quad \text{for } i = 0 ; p = \{1,2,3\}$$

Each node (customer) is only visited by one vehicle:

$$\sum_{k=1}^K Y_{ikp} = 1 \quad \text{for } i = \{1,2,\dots,9\} ; p = \{1,2,3\}$$

The vehicle that enters the node must exit at the same node except the central:

$$\sum_{\substack{j=1 \\ j \neq i}}^N X_{ijkp} = Y_{ikp} \quad \text{for } i = \{0,1,\dots,9\} ; p = \{1,2,3\} ; k = \{1,2\}$$

$$\sum_{\substack{j=1 \\ j \neq i}}^N X_{jikp} = Y_{ikp} \quad \text{for } i = \{0,1,\dots,9\} ; p = \{1,2,3\} ; k = \{1,2\}$$

Travel time is distance divided by average speed:

$$T_{ijp} = \frac{d_{ijp}}{v}$$

Time window rules:

Beginning service time (SS_{ip}) is greater than customer's opening hours and smaller than customer's closing hours:

$$a_{kp} \geq SS_{ip} \quad \text{for } i = \{0,1,\dots,9\} ; p = \{1,2,3\}$$

$$SS_{ip} \leq b_{kp} \quad \text{for } i = \{0,1,\dots,9\} ; p = \{1,2,3\}$$

Vehicle arrival time at central (AT_{ikp}) is greater than customer's opening hour for all vehicles and periods:

$$AT_{ikp} \geq a_{kp} \quad \text{for } i = 0 ; k = \{1,2\} ; p = \{1,2,3\}$$

Beginning service time (SS_{ip}) is greater than vehicle arrival time at node i (AT_{ikp}) for all customers, vehicles, and periods:

$$SS_{ip} \geq AT_{ikp} \quad \text{for } i = \{1,2,\dots,9\} ; k = \{1,2\} ; p = \{1,2,3\}$$

Arrival time at node j (next node) is the sum of customer's opening hours (a_{kp}) with the travel time from central to node j (t_{ijkp}):

$$AT_{jkp} \geq a_{kp} + T_{ijkp} - M * (1 - X_{ijkp}) \quad \text{for } i = 1 ; j = \{1,2,\dots,9\} ; k = \{1,2\} ; p = \{1,2,3\}$$

Arrival time at node j (next node) is the sum of beginning service time at node i (SS_{ip}), service time (unloading) (ST_{ikp}), and travel time from node i (beginning node) to node j (t_{ijkp}):

$$at_{jkp} \geq SS_{ip} + st_{ip} + t_{ijkp} - M * (1 - X_{ijkp}) \quad \text{for } i = \{1,2,\dots,9\} ; j = \{0,1,\dots,9\} ; k = \{1,2\} ; p = \{1,2,3\} ; j \neq i$$

Numerical Example and Calculation

Numerical example in this study is a case at a washing jeans company. Currently, the company does not have a method for determining distribution routes and facing the difficulty in determining the

optimal distribution route. The driver have a full role in determining the route distribution based on experience. In determining the route, many aspects are considered such as the limited working hours and the varying number of customer’s demand making it more difficult to determine the route that give the minimum distance by meeting the vehicle capacity limits.

Actual route causes high cost and time needed to deliver products for customers. There is one central, nine location of customers, two vehicles, and three distribution periods with five regular working hours per day. Table 1 describe actual route with total travel time, total service time, total time needed, total overtime, total distance, and total distribution cost.

Table 1. Actual Route

Period	Vehicle	Route	Demand (unit)	Total Time (hours)	Overtime (hours)	Overtime Cost (IDR)	Total Distance (km)	Total Distance Cost (IDR)	Total Distribution Cost (IDR)
1	K1	0-9-8-7-4-6-0	1944	5.97	0.97	14600	37	27750	42350
	K2	0-5-3-2-1-0	1937	5.26	0.26	3925	32	24000	27925
2	K1	0-8-1-2-3-6-0	1737	5.57	0.57	8475	35.5	26625	35100
	K2	0-5-4-7-9-0	1718	4.71	0	0	26	19500	19500
3	K1	0-1-4-3-6-2-0	1788	5.18	0.18	2700	38	28500	31200
	K2	0-5-7-8-9-0	1519	5.03	0.03	475	33.3	24975	25450
Total				31.72	2.01	30175	201.8	151350	181525

In the same case, next calculation process try to use linear programming model that described before and helped by LINGO 12.0. Running time is about 387 hours and give a feasible solution. The result can be seen at Figure 2 and Table 2.

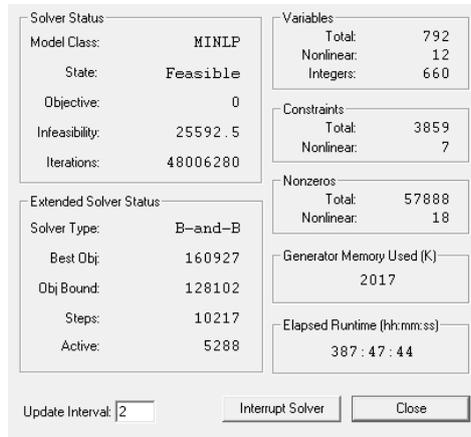


Fig. 2. LINGO Output

Table 2. Route by Linear Programming Model

Period	Vehicle	Route	Demand (unit)	Total Time (hours)	Overtime (hours)	Overtime Cost (IDR)	Total Distance (km)	Total Distance Cost (IDR)	Total Distribution Cost (IDR)
1	K1	0-3-7-1-9-0	1938	4.23	0	0	29.4	22050	22050
	K2	0-8-5-4-2-6-0	1943	4.08	0	0	34.3	25725	25725
2	K1	0-5-3-6-2-0	1472	6.25	1.25	18798	26.4	19800	38598
	K2	0-8-9-1-7-4-0	1983	5.37	0.37	5582	27	20250	25832
3	K1	0-5-3-6-2-4-0	1599	5.93	0.93	13998	26.4	19800	33798
	K2	0-8-9-7-1-0	1708	4.48	0	0	19.9	14925	14925
Total				30.34	2.55	38378	163.4	122550	160928

Table 3 describe about distribution cost comparison between actual route and route by Linear Programming (LP) model.

Table 3. Comparison

	Actual Route	Route by LP Model	Saving	
			Deviation	Percentage
Total Distance (km)	201.8	163.4	38.4	19.03%
Total Time Needed (hours)	31.72	30.34	1.38	4.35%
Total Overtime (hours)	2.01	2.56	-0.55	-27.18%
Total Travel Cost (IDR)	151350	122550	28800	19.03%
Total Overtime Cost (IDR)	30175	38378	-8203	-27.18%
Total Distribution Cost (IDR)	181525	160928	20597	11.35%

From the total distance, route by LP model can give saving about 38.4 km (19.03%) and total time needed about 1.38 hours (4.35%). From the overtime, actual route smaller than route by LP model because in actual route, total time needed for vehicle which not overtime near by regular working hours (five hours), but with LP model, total time needed is smaller than five hours. LP model give the best result because at the actual route, overtime often occur in every period but with LP model only in few period.

The negative deviation at total overtime (hours) occurred due to vehicles that do not work overtime, the total time needed is much smaller than working hours, which is 5 hours per day while the actual route, the total time needed for vehicles that do not work overtime is close to regular hours of 5 hours. Similarly with total overtime cost have a negative deviation as same as total overtime (hours).

Limitation which occur in calculation processing is running hours (387 hours) because of the complexity of the model. Based on the characteristic of LP model that increasing the size of the problem will increase the level of difficulty in computing and problem solving.

Conclusions

Determination distributing route could solve by linear programming model with branch and bound method. The model calculation could give minimum distance, time, and also minimum total distribution cost. Distribution cost in this model consists of travel cost and overtime cost.

The contribution given in this research is the development of Dell'Amico, et al (2007) model which describe about CPRVTW with some adjustments in index period and cost function such as overtime cost. The company which have characteristic such as demand, distribution route, and working hours per period could use this model in determining distribution route. In numerical example, based on calculation process, washing jeans company will get total distribution cost about IDR 160928 per week and actual route which give cost about IDR 181525 per week. The total saving cost about 20598 IDR per week or 11.35% per week.

Because of the limitation of this study about long running hours, the next further study could try metaheuristic method such as genetic algorithm, simulated annealing, ant colony, etc to find feasible solution.

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